Stats 2 Hypothesis Testing Questions

6 In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9.

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving.

In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5.

(a) Investigate, at the 5% level of significance, the teachers' suspicion. *(6 marks)*

(b) Explain, in the context of this question, the meaning of a Type I error. *(2 marks)*

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8 Bottles of sherry nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of sherry in a bottle has changed.

In order to investigate this suspicion, a random sample of 12 bottles of sherry is taken and the volume of sherry in each bottle is measured.

The volumes, in millilitres, of sherry in these bottles are found to be

996	1006	1009	999	1007	1003
998	1010	997	996	1008	1007

Assuming that the volume of sherry in a bottle is normally distributed, investigate, at the 5% level of significance, whether the mean volume of sherry in a bottle differs from 1000 millilitres. *(10 marks)*

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6 The lifetime, X hours, of Everwhite camera batteries is normally distributed. The manufacturer claims that the mean lifetime of these batteries is 100 hours.

(a) The members of a photography club suspect that the batteries do not last as long as is claimed by the manufacturer. In order to investigate their suspicion, the members test a random sample of five of these batteries and find the lifetimes, in hours, to be as follows:

$$85 \qquad 92 \qquad 100 \qquad 95 \qquad 99$$

Test the members' suspicion at the 5% level of significance. *(9 marks)*

(b) The manufacturer, believing that the mean lifetime of these batteries has not changed from 100 hours, decides to determine the lifetime, x hours, of each of a random sample of 80 Everwhite camera batteries. The manufacturer obtains the following results, where $\bar{x}$ denotes the sample mean:

$$\sum x = 8080 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 6399$$

Test the manufacturer's belief at the 5% level of significance. *(8 marks)*

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3 The handicap committee of a golf club has indicated that the mean score achieved by the club's members in the past was 85.9.

A group of members believes that recent changes to the golf course have led to a change in the mean score achieved by the club's members and decides to investigate this belief.

A random sample of the scores, x, of 100 club members was taken and is summarised by

$$\sum x = 8350 \quad \text{and} \quad \sum (x - \bar{x})^2 = 15\,321$$

where $\bar{x}$ denotes the sample mean.

Test, at the 5% level of significance, the group's belief that the mean score of 85.9 has changed. *(8 marks)*

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5 Jasmine's French teacher states that a homework assignment should take, on average, 30 minutes to complete.

Jasmine believes that he is understating the mean time that the assignment takes to complete and so decides to investigate. She records the times, in minutes, that it takes for a random sample of 10 students to complete the French assignment, with the following results:

29	33	36	42	30	28	31	34	37	35

(a) Test, at the 1% level of significance, Jasmine's belief that her French teacher has understated the mean time that it should take to complete the homework assignment. *(7 marks)*

(b) State an assumption that you must make in order for the test used in part (a) to be valid. *(1 mark)*

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3 David is the professional coach at the golf club where Becki is a member. He claims that, after having a series of lessons with him, the mean number of putts that Becki takes per round of golf will reduce from her present mean of 36.

After having the series of lessons with David, Becki decides to investigate his claim.

She therefore records, for each of a random sample of 50 rounds of golf, the number of putts, x, that she takes to complete the round. Her results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 1730 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 784$$

Using a z-test and the 1% level of significance, investigate David's claim. *(8 marks)*

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8 A jam producer claims that the mean weight of jam in a jar is 230 grams.

(a) A random sample of 8 jars is selected and the weight of jam in each jar is determined. The results, in grams, are

220	228	232	219	221	223	230	229

Assuming that the weight of jam in a jar is normally distributed, test, at the 5% level of significance, the jam producer's claim. *(9 marks)*

(b) It is later discovered that the mean weight of jam in a jar is indeed 230 grams.

Indicate whether a Type I error, a Type II error or neither has occurred in carrying out the hypothesis test in part (a). Give a reason for your answer. *(2 marks)*

Stats 2 Hypothesis Testing Answers

6(a)	$H_0 : \mu = 65$ $H_1 : \mu < 65$	B1		1-tailed test
	$\bar{X} \sim N\left(65, \frac{81}{35}\right)$			
	$z_{crit} = -1.6449$	B1		
	$z = \dfrac{61.5 - 65}{9/\sqrt{35}} = -2.30$	M1A1		for σ^2/n used
	Reject H_0 at 5% level of significance	A1√		(on their z-values)
	Evidence to suggest students may be under-achieving	E1	6	
(b)	Reject H_0 when H_0 true $\Downarrow$ Conclude that students are under-achieving when in fact they are not	E1 E1	 2	
	Total		**8**	

8	$H_0 : \mu = 1000$ $H_1 : \mu \neq 1000$	B1		2-tailed test
	$\bar{x} = \dfrac{12036}{12} = 1003$	B1		
	$S = 5.444$	B1		$(S^2 = 29.6)$
	$v = 12 - 1 = 11$	B1		
	$t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}} = \dfrac{1003 - 1000}{5.444/\sqrt{12}} = 1.91$	M1 A1ft A1		
	$t_{crit} = \pm 2.201$	B1√		(on their v)
	Accept H_0	A1√		(on their t-values)
	Insufficient evidence to indicate a change in the mean content of sherry in a bottle	E1√	10	
	Total		**10**	

6(a)	$\bar{x} = \dfrac{471}{5} = 94.2$	B1		
	$s = 6.058$	B1		Or $s^2 = 36.7$
	$\nu = 4$ 1-tailed test	B1		
	$t_{\text{crit}} = -2.132$	B1		Or on diagram
	$\text{H}_0 : \mu = 100$ $\text{H}_1 : \mu < 100$	B1		
	$t = \dfrac{94.2 - 100}{6.058/\sqrt{5}} = -2.14$	M1A1		$\dfrac{\text{their } \bar{x} - 100}{(\text{their } s)/\sqrt{5}}$
	Reject H_0 at 5% level of significance	A1√		On their t and critical value
	Evidence at the 5% level of significance to support the members' belief that the batteries last less than 100 hours.	E1√	9	
(b)	$\bar{x} = \dfrac{8080}{80} = 101$			
	$s^2 = \dfrac{6399}{79} = 81$ (or $\dfrac{6399}{80} = 79.9875$) $s = 9$ (or $s = 8.944$)	B1		For $s\left(\text{or } s^2\right)$ and $\bar{x}$
	$\text{H}_0 : \mu = 100$ $\text{H}_1 : \mu \neq 100$	B1		
	$\bar{X} \sim \text{N}\left(100, \dfrac{81 \text{ (or } 79.9875)}{80}\right)$ under H_0	B1		Or 100, $\dfrac{9}{\sqrt{80}}$ used
	$z = \dfrac{101 - 100}{9/\sqrt{80}} = 0.99$	M1 A1		Allow use of t method AWFW 0.99 to 1.00 (allow 1)
	2-tailed test $z_{\text{crit}} = \pm 1.96$	B1		Or z = 1.96
	Accept H_0 at 5% level of significance.	A1√		On their z and critical value Or t
	Sufficient evidence at the 5% level of significance to support the manufacturer's belief.	E1√	8	
	Total		**17**	

3	$\bar{x}=83.5$	B1		
	$s^2=\frac{1}{99}(15321)=154.76$			$(154<s^2\le 155)$
	$s=12.44$	B1		$(12.4\le s\le 12.45)$
	$H_0:\mu=85.9$ $H_1:\mu\ne 85.9$	B1		
	Under H_0, $\bar{X}\sim N\left(85.9,\frac{(12.44^2)}{100}\right)$			
	$z_{crit}=\pm 1.96$	B1		$z=1.96+2$ tail test used
	$z=\frac{83.5-85.9}{12.44/10}=-1.929$	M1		$\frac{(\text{their }\bar{x})-85.9}{(\text{their }s)/10}$
		A1		AWFW −1.94 to 1.92
	accept H_0, reject the claim	A1√		On their z
	Insufficient evidence to suggest that the mean has changed from 85.9 at the 5% level of significance.	E1√	8	
	Total		**8**	

5(a)	$H_0:\mu=30$ $H_1:\mu>30$	B1		
	$\bar{x}=33.5$ and $s=4.25$ $(s^2=18.06)$	B1B1		σ=4.03 $(\sigma^2=16.25)$
	Under H_0 $\bar{X}\sim N\left(30,\frac{(4.25^2)}{10}\right)$			↓
	$t=\frac{33.5-30}{4.25/\sqrt{10}}=2.60$	M1A1		$\frac{33.5-30}{4.03/\sqrt{9}}$ $(2.6-2.61)$
	$t_{crit}=2.821$	B1		
	do not reject H_0 Insufficient evidence at the 1% level of significance that Jasmine's teacher is underestimating the time that it takes to complete the homework assignments.	E1√	7	
(b)	Times are Normally distributed	B1	1	
	Total		**8**	

3	$H_0 : \mu = 36$ $H_1 : \mu < 36$	B1		
	$\bar{x} = \frac{1730}{50} = 34.6$	B1		
	$s^2 = \frac{784}{49} = 16$	B1		
	Test statistic: $z = \frac{34.6 - 36}{4/\sqrt{50}} = -2.47$	M1 A1		(–2.48 to –2.47)
	$z_{\text{crit}} = -2.3263$	B1		
	Reject H_0	A1√		
	Sufficient evidence at the 1% level of significance to support David's claim	E1√	8	
	Total		**8**	

8(a)	$\bar{x} = 225.25$	B1		
	$s = 5.06 \quad (s^2 = 25.6)$	B1		$(\sigma = 4.74)$, $(\sigma^2 = 22.4)$
	$H_0 : \mu = 230$ $H_1 : \mu \neq 230$	B1		both
	$\nu = 8 - 1 = 7$	B1		
	$t_{\text{crit}} = \pm 2.365$	B1		accept $t_{\text{crit}} = -2.365$
	Test statistic: $t = \frac{225.25 - 230}{5.064/\sqrt{8}} = -2.65$	M1		$\frac{225.25 - 230}{4.74/\sqrt{7}} = -2.65$
		A1		(–2.66 to –2.65)
	Reject H_0 at 5% level	A1√		
	No evidence to support the producer's claim	E1√	9	
(b)	We have rejected H_0 when in fact H_0 may be true. This indicates that a Type I error may have been made.	B2	2	
	Total		**11**	