STATISTICS (C) UNIT 2 **TEST PAPER 8**

1. (i) Explain the difference between a discrete and a continuous variable. [2]

A random number generator on a calculator generates numbers, X, to 3 decimal places, in the range 0 to 1, e.g. 0·386. The variable X may be modelled by a continuous uniform distribution, having the probability density function $f(x)$, where

$$f(x) = 1 \qquad 0 < x < 1,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(ii) Explain why this model is not totally accurate. [1]

2. In an orchard, all the trees are either apple or pear trees. There are twice as many apple trees as pear trees.

Assuming that the types of tree are randomly distributed, use an appropriate normal distribution to find the probability that, in a random sample of 60 trees, there are equal numbers of apple and pear trees. [6]

3. A class of 100 children took an IQ test, and their scores can be summarised as

$$\sum x = 10\,560, \quad \sum x^2 = 1\,118\,320.$$

(i) Calculate unbiased estimates of the mean and standard deviation of the IQ of the population from which the class form a random sample. [4]

(ii) Using a 1% significance level, decide whether this result is evidence that the mean IQ of the population is greater than 104. [5]

4. A pharmaceutical company produces an ointment for earache that, in 80% of cases, relieves pain within 6 hours. A new drug is tried out on a sample of 60 people with earache, and 54 of them get better within 6 hours.

(i) Test, at the 5% significance level, the claim that the new treatment is better than the old one. State your hypotheses carefully. [7]

A rival company suggests that the sample does not give a conclusive result.

(ii) Might they be right, and how could a more conclusive statement be achieved? [2]

5. In a survey of 22 families, the number of children, X, in each family was given by the following table, where f denotes the frequency:

X	0	1	2	3	4	5
f	3	8	5	3	2	1

5. **continued ...**

(i) Calculate unbiased estimates of the mean and variance of X. [5]

(ii) Explain why these results suggest that X may follow a Poisson distribution. [1]

(iii) State another feature of the data that suggests a Poisson distribution. [1]

It is sometimes suggested that the number of children in a family follows a Poisson distribution with mean 2·4. Assuming that this is correct,

(iv) find the probability that a family has less than two children. [3]

6. A centre for receiving calls for the emergency services gets an average of 3·5 emergency calls every minute. Assuming that the number of calls per minute follows a Poisson distribution,

(i) find the probability that more than 6 calls arrive in any particular minute. [3]

Each operator takes a mean time of 2 minutes to deal with each call, and therefore seven operators are necessary to cope with the average demand.

(ii) Find how many operators are required for there to be a 99% probability that a call can be dealt with immediately. [3]

It is found from experience that a major disaster creates a surge of emergency calls. Taking the null hypothesis H_0 that there is no disaster,

(iii) find the number of calls that need to be received in one minute to disprove H_0 at the 0·1 % significance level. [4]

7. A light bulb manufacturer is investigating how long the bulbs survive.

(i) State the advantage of sampling in this case. [1]

Light bulbs produced in a certain factory have lifetimes, in hundreds of hours, whose distribution is modelled by the random variable X with probability density function

$$f(x) = \frac{2x(3-x)}{9}, \qquad 0 \le x \le 3;$$

$$f(x) = 0 \qquad \text{otherwise.}$$

(ii) Sketch f(x). [2]

(iii) Write down the mean lifetime of a bulb. [1]

(iv) Show that ten times as many bulbs fail before 200 hours as survive beyond 250 hours. [5]

(v) Given that a bulb lasts for 200 hours, find the probability that it will then last for at least another 50 hours. [2]

(vi) Give two reasons why the function f(x) is not a realistic model. [2]

STATISTICS 2 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	(i) A discrete variable can only have certain values (usually integers)	B1	
	A continuous variable can take any value (often in a certain range)	B1	
	(ii) X is continuous, but the calculator number is discrete, e.g. calculator cannot give $0{\cdot}385721\ldots$	B1	3
2.	No. of apples is $B(60, 2/3) \approx N(40, 13{\cdot}333)$, so $P(X = 30)$	B1 B1	
	$= P(29{\cdot}5 < X < 30{\cdot}5) = P(-10{\cdot}5/\sqrt{13{\cdot}333} < Z < -9{\cdot}5/\sqrt{13{\cdot}333})$	M1 A1	
	$= P(-2{\cdot}876 < Z < -2{\cdot}602) = 0{\cdot}9979 - 0{\cdot}9953 = 0{\cdot}0026$	M1 A1	6
3.	(i) Mean $= 10\,560 \div 100 = 105{\cdot}6$	B1	
	Variance $= 1\,118\,320/99 - (100/99) \times 105{\cdot}6^2 = 32{\cdot}16$	M1 A1	
	so standard deviation $= 5{\cdot}67$	A1	
	(ii) If pop. mean is 104, then sample means $\sim N(104, 32{\cdot}16/100)$	B1	
	so $P(\overline{X} < 105{\cdot}6) = P(Z > 1{\cdot}6/0{\cdot}567) = P(Z > 2{\cdot}822) = 0{\cdot}0024 < 1\%$	M1 A1 A1	
	Significant at 1% level : the population mean is greater than 104	A1	9
4.	(i) $H_0 : p = 0{\cdot}8 \quad H_1 : p > 0{\cdot}8$	B1 B1	
	Test statistic $z = (54/60 - 0{\cdot}8) / \sqrt{(0{\cdot}8 \times 0{\cdot}2/60)} = 1{\cdot}936$	M1 A1 A1	
	At 5% level, critical value is $1{\cdot}645$, so there is firm evidence that	M1	
	the new drug is better	A1	
	(ii) There might be a Type I error; more data would enable a more definite conclusion to be drawn	B2	9
5.	(i) Mean $= 40/22 = 1{\cdot}82$	M1 A1	
	Variance $= 112/21 - (22/21) \times 1{\cdot}818^2 = 1{\cdot}87$	M1 A1 A1	
	(ii) mean $\approx$ variance (iii) positive skew of data	B1 B1	
	(iv) $P(X < 2) = e^{-2{\cdot}4}(1 + 2{\cdot}4) = 0{\cdot}308$	M1 A1 A1	10
6.	(i) $X \sim Po(3{\cdot}5)$ $P(X > 6) = 1 - 0{\cdot}9347 = 0{\cdot}0653$	B1 M1 A1	
	(ii) $P(X \le 8) = 99{\cdot}01\%$, so the centre must be able to cope with	B1	
	8 calls, and therefore needs 16 operators	M1 A1	
	(iii) $P(X > 10) = 0{\cdot}1\%$, $P(X > 11) = 0{\cdot}03\%$, so need 11 calls	M1 M1 A1 A1	10
7.	(i) Otherwise would use all bulbs and none would be left to sell	B1	
	(ii) Graph drawn through (0, 0) and (3, 0) (iii) 150 hours	B2; B1	
	(iv) $P(X < 2) = \int_0^2 f(x)\,dx = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2 = \frac{2}{9}\left[6 - \frac{8}{3}\right] = 0{\cdot}741$	M1 A1	
	$P(X > 2{\cdot}5) = P(X < 0{\cdot}5) = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^{0.5} = \frac{2}{9}\left[\frac{3}{8} - \frac{1}{24}\right] = 0{\cdot}0741$	M1 A1 A1	
	(v) $0{\cdot}0741 \div (1 - 0{\cdot}741) = 0{\cdot}286$	M1 A1	
	(vi) Too rigid a cut-off; some bulbs might last longer than 300 hours; unlikely to be exactly symmetric about the mean	B2	13