STATISTICS (A) UNIT 2 **TEST PAPER 10**

1. Briefly explain what is meant by
   (a) a statistical model, **(2 marks)**
   (b) a sampling frame, **(1 mark)**
   (c) a sampling unit. **(1 mark)**

2. (a) Explain what is meant by the critical region of a statistical test. **(2 marks)**
   (b) Under a hypothesis H_0, an event A can happen with probability 4·2%. The event A does then happen. State, with justification, whether H_0 should be accepted or rejected at the 5% significance level. **(2 marks)**

3 (a) Briefly describe the main features of a binomial distribution. **(2 marks)**

I conduct an experiment by randomly selecting 10 cards, without replacement, from a normal pack of 52.

(b) Explain why the distribution of X, the number of hearts obtained, is not B(10, $\frac{1}{4}$). **(2 marks)**

After making the appropriate adjustment to the experiment, which should be stated, so that the distribution is B(10, $\frac{1}{4}$), find

(c) the probability of getting no hearts, **(3 marks)**

(d) the probability of getting 4 or more hearts. **(2 marks)**

(e) If the modified experiment is repeated 50 times, find the total number of hearts that you would you expect to have drawn. **(2 marks)**

4. A Geiger counter is observed in the presence of a radioactive source.
In 100 one-minute intervals, the number of counts recorded are as follows:

No of counts, X	0	1	2	3	4	5	6
Frequency	10	24	29	16	12	6	3

(a) Find the mean and variance of this data, and show that it supports the idea that the random variable X is following a Poisson distribution. **(5 marks)**

(b) Use a Poisson distribution with the mean found in part (a) to calculate, to 3 decimal places, the probability that more than 6 counts will be recorded in any particular minute. **(4 marks)**

(c) Find the number of one-minute intervals, in the sample of 100, in which more than 6 counts would be expected. **(2 marks)**

5. A continuous random variable X has the cumulative distribution function

$$\begin{aligned} F(x) &= 0 && x < 2, \\ F(x) &= k(x-a)^2 && 2 \le x \le 6, \\ F(x) &= 1 && x \ge 6. \end{aligned}$$

(a) Find the values of the constants a and k. **(4 marks)**

(b) Show that the median of X is $2(1 + \sqrt{2})$. **(4 marks)**

(c) Given that $X > 4$, find the probability that $X > 5$. **(6 marks)**

6. A small opinion poll shows that the Trendies have a 10% lead over the Oldies. The poll is based on a survey of 20 voters, in which the Trendies got 11 and the Oldies 9. The Oldies spokesman says that the result is consistent with a 10% lead for the Oldies, whilst the Trendies spokesperson says that this is impossible.

(a) At the 5% significance level, test which is right, stating your null hypothesis carefully. **(6 marks)**

(b) If it is indeed true that the Trendies are supported by 55% of the population, use a suitable approximation to find the probability that in a random sample of 200 voters they would obtain less than half of the votes. **(8 marks)**

7. A continuous random variable X has the probability density function

$$\begin{aligned} f(x) &= \frac{6x}{175} && 0 \le x < 5, \\ f(x) &= \frac{6x(10-x)}{875} && 5 \le x \le 10, \\ f(x) &= 0 && \text{otherwise.} \end{aligned}$$

(a) Verify that f is a probability density function. **(6 marks)**

(b) Write down the probability that $X < 1$. **(2 marks)**

(c) Find the cumulative distribution function of X, carefully showing how it changes for different domains. **(7 marks)**

(d) Find the probability that $2 < X < 7$. **(2 marks)**

