

STATISTICS 2 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. (a) A mathematical description which allocates a probability p to each value of a variable X reflecting the relative likelihood of that value occurring B2
 (b) A list of all the members of a population B1
 (c) The individual members of population, available for sampling B1 4
2. (a) The values of X which will cause the null hypothesis to be rejected B2
 (b) If H_0 is correct, then $P(A) = 4.2\%$. This is less than 5% , so if it does happen, H_0 should be rejected B1 4
3. (a) n repeated experiments, each with the same two possible outcomes with constant probabilities of success or fail B2
 (b) If cards not replaced, probabilities change after each draw B2
 (c) Replace card each time, to get $B(10, 0.25)$ $P(X = 0) = 0.0563$ B1 M1 A1
 (d) $P(X \geq 4) = 1 - 0.7759 = 0.224$ M1 A1
 (e) $500 \times 0.25 = 125$ M1 A1 11
4. (a) Mean = $226/100 = 2.26$ Variance = $734/100 - 2.26^2 = 2.23$ M1 A1 M1 A1
 Since mean \approx variance, distribution could be Poisson B1
 (b) In $Po(2.26)$, $P(X > 6) = 0.00623 + 0.00176 + 0.00044... = 0.009$ M1 M1 A1 A1
 (c) In a sample of 100, expected no. = $100 \times 0.009 = 0.9$ intervals M1 A1 11
5. (a) $k(2 - a)^2 = 0$, so $a = 2$ $k(6 - a)^2 = 1$, so $k = \frac{1}{16}$ M1 A1 M1 A1
 (b) Need $\frac{1}{16}(x - 2)^2 = \frac{1}{2}$ $x - 2 = \sqrt{8} = 2\sqrt{2}$ $x = 2 + 2\sqrt{2}$ M1 M1 A1 A1
 (c) $P(X > 4) = 1 - F(4) = 1 - \frac{1}{4} = \frac{3}{4}$ M1 A1
 $P(X > 5) = 1 - F(5) = 1 - \frac{9}{16} = \frac{7}{16}$ $P(X > 5 | X > 4) = \frac{7}{12}$ M1 A1 M1 A1 14
6. (a) If Oldies correct, take $H_0 : P(\text{Trendie}) = 0.45$. Assuming this, distribution of Trendies is $X \sim (20, 0.45)$; then $P(X \geq 11) = 1 - 0.7507 = 0.249 > 5\%$, so do not reject H_0 B1
 M1 A1
 M1 A1 A1
 (b) No. of Trendies is $B(200, 0.55) \approx N(110, 49.5)$ B1 M1 A1
 so $P(X < 100) = P(X < 99.5) = P(Z < -10.5/\sqrt{49.5})$ M1 A1 M1 A1
 $= P(Z < -1.49) = 0.0681$ A1 14
7. (a) $\int_0^{10} f(x) dx = \frac{3}{7} + \frac{6}{875} \int_5^{10} 10x - x^2 dx = \frac{3}{7} + \frac{6}{875} \left[5x^2 - \frac{1}{3}x^3 \right]_5^{10}$ M1 A1 A1
 $= \frac{3}{7} + \frac{6}{875} \left[500 - \frac{1000}{3} - 125 + \frac{125}{3} \right] = \frac{3}{7} + \frac{4}{7} = 1$ M1 A1
 Also $f(x) \geq 0$ for all x , so f is a p.d.f. A1
- (b) $P(X < 1) = \frac{1}{2} \times 1 \times \frac{6}{175} = \frac{3}{175}$ M1 A1
- (c) $F(x) = 0$ ($x < 0$) $F(x) = \frac{3}{175}x^2$ ($0 \leq x \leq 5$) B1 M1 A1
 $F(x) = \frac{3}{7} + \int_5^x \frac{6}{875} (10u - u^2) du = \frac{30x^2 - 2x^3 - 125}{875}$ ($5 < x \leq 10$) M1 A1 A1
 $F(x) = 1$ ($x > 10$) B1
- (d) $F(7) - F(2) = \frac{659}{875} - \frac{12}{175} = \frac{599}{875}$ M1 A1 17