STATISTICS 2 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	(a) One-tailed : is a parameter greater (or less) than a given value?	B1	
	Two-tailed : is a parameter different from a given value?	B1	
	(b) One-tailed, as testing for 'warmer' rather than 'different'	B1 B1	4
2.	(a) (i) Sample is quick, does not use all population, but may be unreliable	B2	
	(ii) Census is accurate, but may be slow and very expensive	B2	
	(b) Sample : e.g. lifetime of light bulbs	B1	
	Census : e.g. government statistics	B1	6
3.	(a) $X \sim B(25, p)$ $\quad H_0 : p = 0{\cdot}8 \quad H_1 : p > 0{\cdot}8$	B1 B1	
	Assuming H_0, P(24 or more people recovering within 6 hours) =		
	$P(X \le 1)$ in $B(25, 0{\cdot}2) = 0{\cdot}0274 < 5\%$ so reject H_0 at 5% level	M1 M1 A1 A1	
	(b) Yes : at 1% level, do not reject H_0, i.e. new drug is no better	M1 A1	
	Do more tests, to get more conclusive answer	B1	9
4.	(a) $X \sim Po(3.5)$ $\quad P(X > 6) = 1 - 0{\cdot}9347 = 0{\cdot}0653$	B1 M1 A1	
	(b) $P(X \le 8) = 99{\cdot}01\%$, so the centre must be able to cope with	B1	
	8 calls, and therefore needs 16 operators	M1 A1	
	(c) $P(X > 10) = 0{\cdot}1\%$, $P(X > 11) = 0{\cdot}03\%$, so need 11 calls	M1 A1 A1	9
5.	(a) $f(x) = \frac{1}{2a}$, $a < x \le 3a$. $\quad E(X) = \int_a^{3a} \frac{x}{2a}\,dx = \left[\frac{x^2}{4a}\right]_a^{3a} = \frac{8a^2}{4a} = 2a$	B1 M1 A1 A1	
	$E(X^2) = \int_a^{3a} \frac{x^2}{2a}\,dx = \left[\frac{x^3}{6a}\right]_a^{3a} = \frac{13a^2}{3}$ $\quad Var(X) = \frac{a^2}{3}$	M1 A1 A1	
	(b) $P(\mid X - \mu \mid < \sigma) = P(\mid X - 2a \mid < \frac{a}{\sqrt{3}}) = \frac{1}{2a} \times 2\frac{a}{\sqrt{3}} = 0{\cdot}577$	M1 A1 A1	
	Normal : $P(\mid X - \mu \mid < \sigma) = P(\mid Z \mid < 1) = 2(0{\cdot}3413) = 0{\cdot}683$	M1 A1 A1	13
6.	(a) Must land on board, so $F(r) = 0\ (r < 0)$, $F(r) = 1\ (r > a)$	B1	
	By definition, $F(r) = P(X < r) = \frac{\pi r^2}{\pi a^2} = \frac{r^2}{a^2}$ $\quad (0 \le r \le a)$	M1 A1 A1	
	(b) $f(r) = F'(r) = \frac{2r}{a^2}$ $(0 \le r \le a)$; $f(r) = 0$ otherwise	M1 A1 M1 B1	
	$E(R) = \int_0^a \frac{2r^2}{a^2}\,dr = \frac{2}{a^2}\left[\frac{r^3}{3}\right]_0^a = \frac{2a}{3}$	M1 A1 A1	
	(c) $f(x) = F'(x) = \frac{2}{a} - \frac{2x}{a^2}$ $(0 \le x \le a)$; $f(x) = 0$ otherwise	M1 A1 B1	
	$f(x)$ decreases from $x = 0$ to $x = a$, so more likely to land near O	M1 A1	16
7.	(a) No. of pears is $B(10, 0{\cdot}2)$	B1	
	$P(X = 5) = 0{\cdot}9936 - 0{\cdot}9672 = 0{\cdot}0264$	M1 A1	
	(b) $P(X < 3) = P(X \le 2) = 0{\cdot}678$	M1 A1 A1	
	(c) $E(X) = 60 \times 0{\cdot}2 = 12$	B1	
	(d) $\sqrt{(12 \times 0{\cdot}8)} = \sqrt{9{\cdot}6} = 3{\cdot}10$	B1	
	Same answer for s.d. of apples (just interchange 0·2 and 0·8)	B1	
	(e) In $B(60, 0{\cdot}2)$, $P(X = 35) = {}^{60}C_{35}(0{\cdot}2)^{35}(0{\cdot}8)^{25} =$ [redacted] 6.7×10^{-11}	B1 M1 A1 A1	
	(f) $B(60, 0{\cdot}2) \approx N(12, 9{\cdot}6)$ $\quad P(X > 15) = P(X > 15{\cdot}5)$	B1 M1 A1	
	$= P(Z > 3{\cdot}5/\sqrt{9{\cdot}6}) = P(Z > 1{\cdot}13) = 0{\cdot}129$	M1 A1	18

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