STATISTICS 2 (A) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	(a) If every rope were tested to breaking point, none would be left	B2	
	(b) e.g. a production list of all the ropes manufactured	B1	3
2.	$X \sim \text{Po}(\lambda)$ Under H_0, $P(X \leq 2) > 1\%$, $P(X \leq 1) < 1\%$	M1 A1 A1	
	$X = 0$ or $X = 1$ will lead to rejection of H_0 at 1% level	M1 A1	5
3.	(a) Continuous uniform U[15, 30] Graph drawn	B2 B2	
	(b) $P(X > 20) = \frac{10}{15} = \frac{2}{3}$	M1 A1 A1	7
4.	(a) $X \sim B(50, p)$ $H_0 : p = 0{\cdot}1$, $H_1 : p > 0{\cdot}1$	B1 B1	
	Under H_0, $P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0{\cdot}9421 = 0{\cdot}0579$	M1 A1 A1	
	$> 5\%$, so do not reject H_0	A1	
	(b) Need $P(X \geq n) < 0{\cdot}01$, so $n = 11$ Need 11 faulty	M1 M1 A1	9
5.	(a) Mean $= 40/22 = 1{\cdot}82$ Variance $= 112/22 - 1{\cdot}818^2 = 1{\cdot}79$	M1 A1 M1 A1	
	(b) mean $\approx$ variance	B1	
	(c) positive skewness	B1	
	(d) $P(X < 2) = e^{-2{\cdot}4}(1 + 2{\cdot}4) = 0{\cdot}308$	M1 A1 A1	
	(e) $^{22}C_{11}\,(0{\cdot}308)^{11}(0{\cdot}692)^{11} = 0{\cdot}0293$	M1 A1 A1	12
6.	(a) No. disapproving $= X \sim B(10, 0{\cdot}3)$ $P(X \leq 4) = 0{\cdot}850$	B1 M1 A1	
	(b) $P(X \leq 3) - P(X \leq 2) = 0{\cdot}6496 - 0{\cdot}3828 = 0{\cdot}267$	M1 A1 A1	
	(c) No. approving is $X \sim B(20, p)$ $H_0 : p = 0{\cdot}7$, $H_1 : p < 0{\cdot}7$	B1 B1	
	Under H_0, $P(X \leq 9) = 0{\cdot}0171 < 5\%$ so reject H_0, i.e. conclude	M1 A1	
	that less than 70% actually do approve	A1	
	(d) No. of approvals is $B(500, 0{\cdot}45) \approx N(225, 123{\cdot}75)$, so	M1 A1	
	$P(X < 250) = P(X < 249{\cdot}5) = P(Z < 24{\cdot}5/11{\cdot}12)$	M1 A1 A1	
	$= P(Z < 2{\cdot}20) = 0{\cdot}986$	M1 A1	18
7.	(a) Graph sketched : straight lines joining (0, 0), $(1, \frac{2}{3})$ and (3, 0)	B3	
	(b) $E(X) = \int_0^1 \frac{2}{3}x^2\,dx + \int_1^3 x - \frac{1}{3}x^2\,dx = \left[\frac{2x^3}{9}\right]_0^1 + \left[\frac{x^2}{2} - \frac{x^3}{9}\right]_1^3$	M1 A1 M1 A1	
	$= \frac{2}{9} + \frac{9}{2} - 3 - \frac{1}{2} + \frac{1}{9} = 1\frac{1}{3}$	A1	
	(c) $E(X^2) = \int_0^1 \frac{2}{3}x^3\,dx + \int_1^3 x^2 - \frac{1}{3}x^3\,dx = \left[\frac{x^4}{6}\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^4}{12}\right]_1^3$	M1 A1 M1 A1	
	$= \frac{1}{6} + 9 - \frac{81}{12} - \frac{1}{3} + \frac{1}{12} = 2\frac{1}{6}$ s.d. $= \sqrt{0{\cdot}389} = 0{\cdot}624$	A1 M1 A1	
	(d) $F(x) = \int_0^x \frac{2}{3}u\,du = \frac{x^2}{3}$ $(0 \leq x < 1)$	M1 A1	
	$F(x) = \frac{1}{3} + \int_1^x 1 - \frac{1}{3}u\,du = [u - \frac{1}{6}u^2]_1^x + \frac{1}{3} = x - \frac{1}{6}x^2 - \frac{1}{2}$	M1 A1 M1 A1	
	$(1 \leq x \leq 3)$		21