

**STATISTICS (A) UNIT 2****TEST PAPER 5**

1. (a) Explain briefly why it is often useful to take a sample from a population. (2 marks)  
 (b) Suggest a suitable sampling frame for a local council to use to survey attitudes towards a proposed new shopping centre. (1 mark)
2. A certain type of lettuce seed has a 12% failure rate for germination. In a new sample of 25 genetically modified seeds, only 1 did not germinate.  
 Clearly stating your hypotheses, test, at the 1% significance level, whether the GM seeds are better. (6 marks)
3. A random variable  $X$  has a Poisson distribution with a mean,  $\lambda$ , which is assumed to equal 5.  
 (a) Find  $P(X = 0)$ . (1 mark)  
 (b) In 100 measurements, the value 0 occurs three times. Find the highest significance level at which you should reject the original hypothesis in favour of  $\lambda < 5$ . (8 marks)
4. The waiting time, in minutes, at a dentist is modelled by the continuous random variable  $T$  with probability density function
- $$f(t) = k(10 - t) \quad 0 \leq t \leq 10,$$
- $$f(t) = 0 \quad \text{otherwise.}$$
- (a) Sketch the graph of  $f(t)$  and find the value of  $k$ . (4 marks)  
 (b) Find the mean value of  $T$ . (4 marks)  
 (c) Find the 95th percentile of  $T$ . (3 marks)  
 (d) State whether you consider this function to be a sensible model for  $T$  and suggest how it could be modified to provide a better model. (2 marks)
5. A textbook contains, on average, 1.2 misprints per page. Assuming that the misprints are randomly distributed throughout the book,  
 (a) specify a suitable model for  $X$ , the random variable representing the number of misprints on a given page. (1 mark)  
 (b) Find the probability that a particular page has more than 2 misprints. (3 marks)  
 (c) Find the probability that Chapter 1, with 8 pages, has no misprints at all. (2 marks)  
 Chapter 2 is longer, at 20 pages.  
 (d) Use a suitable approximation to find the probability that Chapter 2 has less than ten misprints altogether. Explain what adjustment is necessary when making this approximation. (7 marks)

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6. On a production line, bags are filled with cement and weighed as they emerge. It is found that 20% of the bags are underweight. In a random sample consisting of  $n$  bags, the variance of the number of underweight bags is found to be 2.4.

- (a) Show that  $n = 15$ . **(2 marks)**
- (b) Use cumulative binomial probability tables to find the probability that, in a further random sample of 15 bags, the number that are underweight is
- (i) less than 3, **(3 marks)**
- (ii) at least 5. **(2 marks)**

Ten samples of 15 bags each are tested. Find the probability that

- (c) all these batches contain less than 5 underweight bags, **(3 marks)**
- (d) the fourth batch tested is the first to contain less than 5 underweight bags. **(3 marks)**
7. A continuous random variable  $X$  has a probability density function given by

$$f(x) = \frac{x^2}{312} \quad 4 \leq x \leq 10,$$

$$f(x) = 0 \quad \text{otherwise.}$$

- (a) Find  $E(X)$ . **(3 marks)**
- (b) Find the variance of  $X$ . **(4 marks)**
- (c) Find the cumulative distribution function  $F(x)$ , for all values of  $x$ . **(5 marks)**
- (d) Hence find the median value of  $X$ . **(3 marks)**
- (e) Write down the modal value of  $X$ . **(1 mark)**

It is sometimes suggested that, for most distributions,

$$2 \times (\text{median} - \text{mean}) \approx \text{mode} - \text{median}.$$

- (f) Show that this result is not satisfied in this case, and suggest a reason why. **(2 marks)**