STATISTICS 2 (A) TEST PAPER 5 : ANSWERS AND MARK SCHEME

1.	(a) Quicker and cheaper than a census	B2	
	(b) Use the electoral roll	B1	3
2.	$X \sim B(25, p)$ $\quad H_0: p = 0{\cdot}12 \quad H_1: p < 0{\cdot}12$	B1 B1	
	Under H_0, $P(X = 0 \text{ or } 1) = 0{\cdot}88^{25} + 25 \times 0{\cdot}88^{24} \times 0{\cdot}12 = 0{\cdot}181 > 1\%$	M1 M1 A1	
	Therefore, no significant evidence in favour of H_1 at the 1% level	A1	6
3.	(a) If mean = 5, $X \sim Po(5)$ $\quad P(X = 0) = 0{\cdot}0067$	B1	
	(b) $X \sim Po(\lambda)$ $\quad H_0: \lambda = 5 \quad H_1: \lambda < 5$	B1 B1	
	Under H_0, no. of '0's in 100 measurements $\sim Po(0{\cdot}67)$	M1 A1	
	$P(X \geq 3) = 1 - e^{-0{\cdot}67}(1 + 0{\cdot}67 + 0.67^2/2!) = 0{\cdot}031 = 3{\cdot}1\%$	M1 A1 A1	
	Reject H_0 at the 5% significance level, but not at the 1% level.	A1	9
4.	(a) Graph : straight line from (0, 10k) to (10, 0); on x-axis otherwise	B2	
	$\frac{1}{2} \times 10 \times 10k = 1 \qquad k = \frac{1}{50}$	M1 A1	
	(b) $E(T) = \int_0^{10} t\, f(t)\, dt = \frac{1}{50}\int_0^{10} 10t - t^2\, dt = \frac{1}{50}\left[5t^2 - \frac{t^3}{3}\right]_0^{10} = 3\frac{1}{3}$	M1 A1 M1 A1	
	(c) From graph, $\frac{1}{2}(10 - p)\left(\frac{10-p}{50}\right) = \frac{5}{100} \qquad (10 - p)^2 = 5 \qquad p = 7{\cdot}76$	M1 A1 A1	
	(d) Some wait more than 10 mins; more gradual slope needed	B1 B1	13
5.	(a) Poisson : $X \sim Po(1{\cdot}2)$	B1	
	(b) $P(X > 2) = 1 - e^{-1{\cdot}2} - 1{\cdot}2e^{-1{\cdot}2} - 1{\cdot}2^2\, e^{-1{\cdot}2}/2! = 0{\cdot}121$	M1 A1 A1	
	(c) $P(X = 0) = e^{-1{\cdot}2} = 0{\cdot}301 \qquad P(0 \text{ in Ch. } 1) = 0{\cdot}301^8 = 0{\cdot}0000677$	M1 A1	
	(d) Total for Ch. 2 $\sim Po(24) \approx N(24, 24)$ $\qquad$ Then $P(X < 10)$	M1 A1	
	$= P(X < 9{\cdot}5) = P(Z < -14{\cdot}5/4{\cdot}90) = P(Z < -2{\cdot}96) = 0{\cdot}0015$	M1 A1 M1 A1	
	Continuity correction needed, from discrete to continuous	B1	13
6.	(a) $B(n, 0{\cdot}2)$: $\text{Var} = n(0{\cdot}2)(0{\cdot}8) = 2{\cdot}4 \qquad n = 15$	M1 A1	
	(b) $X \sim B(15, 0{\cdot}2)$ $\qquad$ (i) $P(X < 3) = P(X \leq 2) = 0{\cdot}398$	B1 M1 A1	
	(ii) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0{\cdot}8358 = 0{\cdot}164$	M1 A1	
	(c) $X \sim B(10, 0{\cdot}8358)$: $P(X = 10) = 0{\cdot}8358^{10} = 0{\cdot}166$	B1 M1 A1	
	(d) $0{\cdot}1642^3 \times 0{\cdot}8358 = 0{\cdot}00370$	M1 A1 A1	13
7.	(a) $E(X) = \int_4^{10} \frac{x^3}{312}\, dx = \left[\frac{x^4}{1248}\right]_4^{10} = 7{\cdot}81$	M1 A1 A1	
	(b) $\text{Var}(X) = \int_4^{10} \frac{x^4}{312}\, dx - 7{\cdot}81^2 = \left[\frac{x^5}{1560}\right]_4^{10} - 7{\cdot}808^2 = 2{\cdot}49$	M1 A1 M1 A1	
	(c) $F(x) = 0 \ (x < 4)$, $\quad F(x) = \int_4^x \frac{u^2}{312}\, du = \frac{x^3 - 64}{936} \ (4 \leq x \leq 10)$,	B1 M1 A1 A1	
	$F(x) = 1 \ (x > 10)$	B1	
	(d) For median m, $\frac{m^3 - 64}{936} = \frac{1}{2} \qquad m^3 = 532 \qquad m = 8{\cdot}10$	M1 A1 A1	
	(e) By inspection, mode = 10	B1	
	(f) $2(8{\cdot}1 - 7{\cdot}81) = 0{\cdot}58 \qquad 10 - 8{\cdot}1 = 1{\cdot}9 \qquad$ Not very	B1	
	similar; mode is an extreme point and is not centrally located	B1	18