

STATISTICS 2 (A) TEST PAPER 3 : ANSWERS AND MARK SCHEME

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|---|----------------------------------|----|
| 1. (a) The set of all items considered | B1 | |
| (b) The individual elements of the population | B1 | |
| (c) e.g. A list of customers from a holiday company | B1 | |
| (d) e.g. A list from the local social services office. | B1 | 4 |
| 2. (a) $k = \frac{1}{10}$, so $F(x) = 0$ ($x < 5$), $F(x) = \frac{x-5}{10}$ ($5 \leq x \leq 15$)
$F(x) = 1$ ($x > 15$). | B1 B1 M1 A1
B1 | |
| (b) $P(X < 8) = F(8) = \frac{3}{10}$ | B1 | 6 |
| 3. (a) $X \sim B(20, p)$ $H_0 : p = \frac{1}{2}, H_1 : p \neq \frac{1}{2}$
Assuming H_0 , $P(X \geq 16 \text{ or } X \leq 4) = 0.0059 \times 2 = 0.0118$
$> 1\%$, so do not reject H_0 at 1% level. | B1 B1
M1 M1 A1
A1 | |
| (b) For significance at 0.1% level, would need $X \leq 2$ or $X \geq 18$ | B1 B1 | 8 |
| 4. (a) $X \sim B(5, p)$
$P(X \geq 3) = \binom{5}{3}p^3(1-p)^2 + \binom{5}{4}p^4(1-p) + \binom{5}{5}p^5$
$= p^3 + 5p^4 - 5p^5 + 10p^3(1-2p+p^2)$
$= 6p^3 - 15p^4 + 10p^5 = p^3(6p^2 - 15p + 10)$ | B1
M1 A1 A1
A1
M1 A1 | |
| (b) Put $p = 0.6$ to get $P(X \geq 3) = 0.683$ | M1 A1 | 9 |
| 5. (a) No. of left-handed is $B(10, 0.32)$ $P(X = 0) = 0.68^{10} = 0.0211$ | M1 M1 A1 | |
| (b) $P(X = 1) = 10 \times 0.68^9 \times 0.32 = 0.0995$
$P(X \geq 2) = 1 - 0.0211 - 0.0995 = 0.879$ | M1 A1
M1 A1 | |
| (c) Now no. of left-handed is $B(35, 0.32) \approx N(11.2, 7.616)$
$P(5 < X < 15) = P(5.5 < X < 14.5) = P(-2.07 < Z < 1.20)$
$= 0.8849 - 0.0193 = 0.866$
Continuity correction, going from discrete to continuous variable | M1 A1
M1 A1 A1
M1 A1
B1 | 15 |
| 6. (a) Poisson : $Po(1.5)$ | B1 B1 | |
| (b) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9344 = 0.0656$ | M1 A1 A1 | |
| (c) Counts in 5 minutes are $Po(7.5)$, so $P(X \leq 3) = 0.0591$ | B1 M1 A1 | |
| (d) Mean = $84/50 = 1.68$ Var. = $226/50 - 1.68^2 = 1.698$
Mean \approx variance; this supports Poisson model, but mean > 1.5 | B1 M1 A1
B1 | |
| (e) With mean = 1.68, $P(X = 2) = e^{-1.68}(1.68^2/2!) = 0.263$ | M1 M1 A1 | 15 |
| 7. (a) When $F(t) = 0.5$, $t^2(3t^2 - 16t + 24) = 8$
When $t = 0.77$, L.H.S. = $7.98 \approx 8$ | M1
M1 A1 | |
| (b) $F(1.2) = 0.8208$ $F(1.5) = 0.9492$
$P(\text{Wait } 1.5 \mid \text{wait } 1.2) = (1 - 0.9492) \div (1 - 0.8208) = 0.283$ | B1 B1
M1 A1 A1 | |
| (c) $f(t) = \frac{1}{16}(12t^3 - 48t^2 + 48t)$ ($0 \leq t \leq 2$), $f(t) = 0$ otherwise
Graph sketched | M1 A1 A1
B1 | |
| (d) Mode is at max. point, where $f'(t) = 0$ $36t^2 - 96t + 48 = 0$
$12(3t - 2)(t - 2) = 0$ Mode is $t = \frac{2}{3}$ ($6\frac{2}{3}$ minutes) | M1 A1
M1 A1 A1 | |
| (e) Unlikely to be no delays above 20 minutes | B1 | 18 |