OCR Maths S1

Topic Questions from Papers

Discrete Random Variables
1 The table below shows the probability distribution of the random variable $X$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{5}$</td>
<td>$k$</td>
<td>$\frac{2}{5}$</td>
<td>$\frac{1}{10}$</td>
</tr>
</tbody>
</table>

(i) Find the value of the constant $k$. [2]

(ii) Calculate the values of $E(X)$ and $\text{Var}(X)$. [5]

(Q4, Jan 2005)

2 The probability distribution of a discrete random variable, $X$, is given in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{4}{7}$</td>
<td>$p$</td>
<td>$q$</td>
</tr>
</tbody>
</table>

It is given that the expectation, $E(X)$, is $1\frac{1}{4}$.

(i) Calculate the values of $p$ and $q$. [5]

(ii) Calculate the standard deviation of $X$. [4]

(Q5, June 2006)

3 Part of the probability distribution of a variable, $X$, is given in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{3}{10}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{2}{5}$</td>
<td></td>
</tr>
</tbody>
</table>

(i) Find $P(X = 0)$. [2]

(ii) Find $E(X)$. [2]

(Q1, Jan 2007)
The table shows the probability distribution for a random variable $X$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Calculate $E(X)$ and $\text{Var}(X)$. \[ \text{(Q1, June 2007)} \]

Each time a certain triangular spinner is spun, it lands on one of the numbers 0, 1 and 2 with probabilities as shown in the table.

<table>
<thead>
<tr>
<th>Number</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The spinner is spun twice. The total of the two numbers on which it lands is denoted by $X$.

(i) Show that $P(X = 2) = 0.18$. \[ \text{[3]} \]

The probability distribution of $X$ is given in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.49</td>
<td>0.28</td>
<td>0.18</td>
<td>0.04</td>
<td>0.01</td>
</tr>
</tbody>
</table>

(ii) Calculate $E(X)$ and $\text{Var}(X)$. \[ \text{[5]} \] \[ \text{(Q1, Jan 2009)} \]
Last year Eleanor played 11 rounds of golf. Her scores were as follows:

79, 71, 80, 67, 67, 74, 66, 65, 71, 66, 64.

(i) Calculate the mean of these scores and show that the standard deviation is 5.31, correct to 3 significant figures. [4]

(ii) Find the median and interquartile range of the scores. [4]

This year, Eleanor also played 11 rounds of golf. The standard deviation of her scores was 4.23, correct to 3 significant figures, and the interquartile range was the same as last year.

(iii) Give a possible reason why the standard deviation of her scores was lower than last year although her interquartile range was unchanged. [1]

In golf, smaller scores mean a better standard of play than larger scores. Ken suggests that since the standard deviation was smaller this year, Eleanor’s overall standard has improved.

(iv) Explain why Ken is wrong. [1]

(v) State what the smaller standard deviation does show about Eleanor’s play. [1]

(Q6, June 2009)

A certain four-sided die is biased. The score, $X$, on each throw is a random variable with probability distribution as shown in the table. Throws of the die are independent.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P($X = x$)</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

(i) Calculate $E(X)$ and $\text{Var}(X)$. [5]

The die is thrown 10 times.

(ii) Find the probability that there are not more than 4 throws on which the score is 1. [2]

(iii) Find the probability that there are exactly 4 throws on which the score is 2. [3]

(Q4, Jan 2010)
Each of four cards has a number printed on it as shown.

\[
\begin{array}{cccc}
1 & 2 & 3 & 3 \\
\end{array}
\]

Two of the cards are chosen at random, without replacement. The random variable \(X\) denotes the sum of the numbers on these two cards.

(i) Show that \(P(X = 6) = \frac{1}{6}\) and \(P(X = 4) = \frac{1}{3}\). \[3\]

(ii) Write down all the possible values of \(X\) and find the probability distribution of \(X\). \[4\]

(iii) Find \(E(X)\) and \(\text{Var}(X)\). \[5\]

(Q5, June 2010)

The probability distribution of a discrete random variable, \(X\), is shown below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X = x))</td>
<td>(a)</td>
<td>(1 - a)</td>
</tr>
</tbody>
</table>

(i) Find \(E(X)\) in terms of \(a\). \[2\]

(ii) Show that \(\text{Var}(X) = 4a(1 - a)\). \[3\]

(Q7, Jan 2011)

The probability distribution of a random variable \(X\) is shown in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X = x))</td>
<td>0.1</td>
<td>0.3</td>
<td>(2p)</td>
<td>(p)</td>
</tr>
</tbody>
</table>

(i) Find \(p\). \[2\]

(ii) Find \(E(X)\). \[2\]

(Q1, Jan 2012)
A bag contains 4 blue discs and 6 red discs. Chloe takes a disc from the bag. If this disc is red, she takes 2 more discs. If not, she takes 1 more disc. Each disc is taken at random and no discs are replaced.

(i) Complete the probability tree diagram in your Answer Book, showing all the probabilities. [2]

(ii) Show that \( P(X = 1) = \frac{3}{5} \). [2]

(iii) Calculate \( E(X) \) and \( \text{Var}(X) \). [5]

(Q5, June 2011)

The masses, \( x \text{ kg} \), of 50 bags of flour were measured and the results were summarised as follows.

\[
\begin{align*}
  n &= 50 \\
  \Sigma(x - 1.5) &= 1.4 \\
  \Sigma(x - 1.5)^2 &= 0.05
\end{align*}
\]

Calculate the mean and standard deviation of the masses of these bags of flour. [6]

(Q2, June 2012)
When a four-sided spinner is spun, the number on which it lands is denoted by $X$, where $X$ is a random variable taking values 2, 4, 6 and 8. The spinner is biased so that $P(X = x) = kx$, where $k$ is a constant.

(i) Show that $P(X = 6) = \frac{3}{10}$. [2]

(ii) Find $E(X)$ and $\text{Var}(X)$. [5]

(Q1, Jan 2013)