Edexcel Maths S1

Topic Questions from Papers

Discrete Random Variables

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5. The random variable X has probability function

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, \\ k(x+1), & x = 4, 5, \end{cases}$$

where k is a constant.

(a) Find the value of k.

(2)

(b) Find the exact value of E(X).

(2)

(c) Show that, to 3 significant figures, Var(X) = 1.47.

(4)

(d) Find, to 1 decimal place, Var(4 – 3X).

(2)

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Question 5 continued

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4. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \qquad x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $E(X)$ and show that $Var(X) = 2$.

(3)

Find

(b) $E(3X - 2)$,

(2)

(c) $Var(4 - 3X)$.

(2)

N 2 2 3 3 7 A 0 1 2 2 0

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3. The random variable X has probability function

$$P(X = x) = \frac{(2x-1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X.

(3)

Find

(b) $P(2 < X \leqslant 5)$,

(2)

(c) the exact value of $E(X)$.

(2)

(d) Show that $Var(X) = 1.97$ to 3 significant figures.

(4)

(e) Find $Var(2 - 3X)$.

(2)

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Question 3 continued

(Total 13 marks)

Q3

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7. The random variable X has probability distribution

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

(a) Given that $E(X) = 4.5$, write down two equations involving p and q. **(3)**

Find

(b) the value of p and the value of q, **(3)**

(c) $P(4 < X \leqslant 7)$. **(2)**

Given that $E(X^2) = 27.4$, find

(d) $\text{Var}(X)$, **(2)**

(e) $E(19 - 4X)$, **(1)**

(f) $\text{Var}(19 - 4X)$. **(2)**

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Question 7 continued

Q7

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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3. The random variable X has probability distribution given in the table below.

x	-1	0	1	2	3
$P(X = x)$	p	q	0.2	0.15	0.15

Given that $E(X) = 0.55$, find

(a) the value of p and the value of q,
(5)

(b) $Var(X)$,
(4)

(c) $E(2X - 4)$.
(2)

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Question 3 continued

H 3 2 5 8 2 A 0 1 1 2 8

Turn over

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3. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
$P(X = x)$	0.4	0.3	0.2	0.1

(a) Find $E(X)$.

(2)

(b) Find $F(1.5)$.

(2)

(c) Show that $Var(X) = 1$

(4)

(d) Find $Var(5 - 3X)$.

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10. After 3 games he has a total of 6 points.
You may assume that games are independent.

(e) Find the probability that Rohit wins the prize.

(6)

N 3 2 6 8 0 A 0 1 0 2 4

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Question 3 continued

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6. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3 - x) & x = 0,1,2 \\ b & x = 3 \end{cases}$$

(a) Find $P(X = 2)$ and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that $E(X) = 1.6$

(b) Find the value of a and the value of b.

(5)

Find

(c) $P(0.5 < X < 3)$,

(2)

(d) $E(3X - 2)$.

(2)

(e) Show that the $\text{Var}(X) = 1.64$

(3)

(f) Calculate $\text{Var}(3X - 2)$.

(2)

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Question 6 continued

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5. The probability function of a discrete random variable X is given by

$$p(x) = kx^2 \qquad x = 1, 2, 3$$

where k is a positive constant.

(a) Show that $k = \frac{1}{14}$ **(2)**

Find

(b) $P(X \geqslant 2)$ **(2)**

(c) $E(X)$ **(2)**

(d) $Var(1-X)$ **(4)**

Leave blank

Question 5 continued

Q5

(Total 10 marks)

N 3 5 7 1 1 A 0 1 5 2 4

Turn over

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3. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a.
(2)

(b) Write down $E(X)$.
(1)

(c) Find $\text{Var}(X)$.
(3)

The random variable $Y = 6 - 2X$

(d) Find $\text{Var}(Y)$.
(2)

(e) Calculate $P(X \geqslant Y)$.
(3)

H 3 5 3 9 5 A 0 8 2 8

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Question 3 continued

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6. The discrete random variable X has the probability distribution

x	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$4k$

(a) Show that $k = 0.1$ **(1)**

Find

(b) $E(X)$ **(2)**

(c) $E(X^2)$ **(2)**

(d) $Var(2 - 5X)$ **(3)**

Two independent observations X_1 and X_2 are made of X.

(e) Show that $P(X_1 + X_2 = 4) = 0.1$ **(2)**

(f) Complete the probability distribution table for $X_1 + X_2$ **(2)**

y	2	3	4	5	6	7	8
$P(X_1 + X_2 = y)$	0.01	0.04	0.10		0.25	0.24	

(g) Find $P(1.5 < X_1 + X_2 \leqslant 3.5)$ **(2)**

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Question 6 continued

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2. The random variable $X \sim \mathrm{N}(\mu, 5^2)$ and $\mathrm{P}(X < 23) = 0.9192$

(a) Find the value of μ.

(4)

(b) Write down the value of $\mathrm{P}(\mu < X < 23)$.

(1)

Leave blank

3. The discrete random variable Y has probability distribution

y	1	2	3	4
$\mathrm{P}(Y = y)$	a	b	0.3	c

where a, b and c are constants.

The cumulative distribution function $\mathrm{F}(y)$ of Y is given in the following table

y	1	2	3	4
$\mathrm{F}(y)$	0.1	0.5	d	1.0

where d is a constant.

(a) Find the value of a, the value of b, the value of c and the value of d.

(5)

(b) Find $\mathrm{P}(3Y + 2 \geqslant 8)$.

(2)

Leave blank

3. The discrete random variable X can take only the values 2, 3, 4 or 6. For these values the probability distribution function is given by

x	2	3	4	6
$P(X = x)$	$\frac{5}{21}$	$\frac{2k}{21}$	$\frac{7}{21}$	$\frac{k}{21}$

where k is a positive integer.

(a) Show that $k = 3$

(2)

Find

(b) F(3)

(1)

(c) E(X)

(2)

(d) $E(X^2)$

(2)

(e) Var($7X - 5$)

(4)

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Question 3 continued

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1. A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{6}$ **(3)**

(b) Find $E(X)$ **(2)**

(c) Show that $E(X^2) = \frac{4}{3}$ **(2)**

(d) Find $\text{Var}(1-3X)$ **(3)**

Leave blank

2. The discrete random variable X can take only the values 1, 2 and 3. For these values the cumulative distribution function is defined by

$$F(x) = \frac{x^3 + k}{40} \qquad x = 1, 2, 3$$

(a) Show that $k = 13$

(2)

(b) Find the probability distribution of X.

(4)

Given that $\text{Var}(X) = \frac{259}{320}$

(c) find the exact value of $\text{Var}(4X - 5)$.

(2)

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6. A fair blue die has faces numbered 1, 1, 3, 3, 5 and 5. The random variable B represents the score when the blue die is rolled.

(a) Write down the probability distribution for B.
(2)

(b) State the name of this probability distribution.
(1)

(c) Write down the value of $E(B)$.
(1)

A second die is red and the random variable R represents the score when the red die is rolled.

The probability distribution of R is

r	2	4	6
$P(R = r)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(d) Find $E(R)$.
(2)

(e) Find $Var(R)$.
(3)

Tom invites Avisha to play a game with these dice.

Tom spins a fair coin with one side labelled 2 and the other side labelled 5. When Avisha sees the number showing on the coin she then chooses one of the dice and rolls it. If the number showing on the die is greater than the number showing on the coin, Avisha wins, otherwise Tom wins.

Avisha chooses the die which gives her the best chance of winning each time Tom spins the coin.

(f) Find the probability that Avisha wins the game, stating clearly which die she should use in each case.
(4)

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Question 6 continued

P 4 1 8 0 5 A 0 1 5 2 0

Turn over

Leave blank

2. The discrete random variable X takes the values 1, 2 and 3 and has cumulative distribution function $F(x)$ given by

x	1	2	3
$F(x)$	0.4	0.65	1

(a) Find the probability distribution of X.

(3)

(b) Write down the value of F(1.8).

(1)

Q2

(Total 4 marks)

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7. The score S when a spinner is spun has the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	0.2	0.2	0.1	0.3	0.2

(a) Find $E(S)$.

(2)

(b) Show that $E(S^2) = 10.4$

(2)

(c) Hence find $Var(S)$.

(2)

(d) Find

(i) $E(5S - 3)$,

(ii) $Var(5S - 3)$.

(4)

(e) Find $P(5S - 3 > S + 3)$

(3)

The spinner is spun twice.

The score from the first spin is S_1 and the score from the second spin is S_2

The random variables S_1 and S_2 are independent and the random variable $X = S_1 \times S_2$

(f) Show that $P(\{S_1 = 1\} \cap X < 5) = 0.16$

(2)

(g) Find $P(X < 5)$.

(3)

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Question 7 continued

Q7

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

END

P 4 3 9 5 6 A 0 2 4 2 4

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5. A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$P(X = x)$	a	a	a	b	b	0.3

(a) Given that $E(X) = 4.2$ find the value of a and the value of b.

(5)

(b) Show that $E(X^2) = 20.4$

(1)

(c) Find $Var(5 - 3X)$

(3)

A biased die with five faces is rolled. The discrete random variable Y represents the score which is uppermost. The cumulative distribution function of Y is shown in the table below.

y	1	2	3	4	5
$F(y)$	$\frac{1}{10}$	$\frac{2}{10}$	$3k$	$4k$	$5k$

(d) Find the value of k.

(1)

(e) Find the probability distribution of Y.

(3)

Each die is rolled once. The scores on the two dice are independent.

(f) Find the probability that the sum of the two scores equals 2

(2)

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Question 5 continued

Statistics S1

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A)$$

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\,\mathrm{P}(A)}{\mathrm{P}(B \mid A)\,\mathrm{P}(A) + \mathrm{P}(B \mid A')\,\mathrm{P}(A')}$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $\mathrm{P}(X = x_i)$

Expectation (mean): $\mathrm{E}(X) = \mu = \Sigma x_i\,\mathrm{P}(X = x_i)$

Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma(x_i - \mu)^2\,\mathrm{P}(X = x_i) = \Sigma\,x_i^2\,\mathrm{P}(X = x_i) - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \Sigma \mathrm{g}(x_i)\,\mathrm{P}(X = x_i)$

Continuous distributions

Standard continuous distribution:

Distribution of X	P.D.F.	Mean	Variance
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

THE NORMAL DISTRIBUTION FUNCTION

The function tabulated below is $\Phi(z)$, defined as $\Phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{z} e^{-\frac{1}{2}t^2}\, dt$.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.02	0.9783
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.04	0.9793
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.06	0.9803
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.08	0.9812
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.10	0.9821
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.12	0.9830
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.14	0.9838
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.16	0.9846
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.18	0.9854
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.20	0.9861
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.22	0.9868
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.24	0.9875
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.26	0.9881
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.28	0.9887
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.30	0.9893
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.32	0.9898
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.34	0.9904
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.36	0.9909
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.38	0.9913
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.40	0.9918
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.42	0.9922
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.44	0.9927
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.46	0.9931
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.48	0.9934
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.50	0.9938
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.55	0.9946
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.60	0.9953
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.65	0.9960
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.70	0.9965
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.75	0.9970
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.80	0.9974
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.85	0.9978
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.90	0.9981
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.95	0.9984
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	3.00	0.9987
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686	3.05	0.9989
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693	3.10	0.9990
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699	3.15	0.9992
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706	3.20	0.9993
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	3.25	0.9994
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719	3.30	0.9995
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726	3.35	0.9996
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732	3.40	0.9997
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738	3.50	0.9998
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	3.60	0.9998
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750	3.70	0.9999
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756	3.80	0.9999
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761	3.90	1.0000
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767	4.00	1.0000
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772		

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p; that is, $P(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905