

Edexcel Maths S1

Topic Questions from Papers

Mathematical Models

6. (a) Give two reasons to justify the use of statistical models.

(2)

It has been suggested that there are 7 stages involved in creating a statistical model. They are summarised below, with stages 3, 4 and 7 missing.

Stage 1. The recognition of a real-world problem.

Stage 2. A statistical model is devised.

Stage 3.

Stage 4.

Stage 5. Comparisons are made against the devised model.

Stage 6. Statistical concepts are used to test how well the model describes the real-world problem.

Stage 7.

(b) Write down the missing stages.

(3)



Statistics S1

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $P(X = x_i)$

$$\text{Expectation (mean): } E(X) = \mu = \sum x_i P(X = x_i)$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 P(X = x_i) = \sum x_i^2 P(X = x_i) - \mu^2$$

$$\text{For a function } g(X): E(g(X)) = \sum g(x_i) P(X = x_i)$$

Continuous distributions

Standard continuous distribution:

| Distribution of X | P.D.F. | Mean | Variance |
|---------------------------|---|-------|------------|
| Normal $N(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | μ | σ^2 |

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\} \{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right) \left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

THE NORMAL DISTRIBUTION FUNCTION

The function tabulated below is $\Phi(z)$, defined as $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}t^2} dt$.

| z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ |
|------|-----------|------|-----------|------|-----------|------|-----------|------|-----------|
| 0.00 | 0.5000 | 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 |
| 0.01 | 0.5040 | 0.51 | 0.6950 | 1.01 | 0.8438 | 1.51 | 0.9345 | 2.02 | 0.9783 |
| 0.02 | 0.5080 | 0.52 | 0.6985 | 1.02 | 0.8461 | 1.52 | 0.9357 | 2.04 | 0.9793 |
| 0.03 | 0.5120 | 0.53 | 0.7019 | 1.03 | 0.8485 | 1.53 | 0.9370 | 2.06 | 0.9803 |
| 0.04 | 0.5160 | 0.54 | 0.7054 | 1.04 | 0.8508 | 1.54 | 0.9382 | 2.08 | 0.9812 |
| 0.05 | 0.5199 | 0.55 | 0.7088 | 1.05 | 0.8531 | 1.55 | 0.9394 | 2.10 | 0.9821 |
| 0.06 | 0.5239 | 0.56 | 0.7123 | 1.06 | 0.8554 | 1.56 | 0.9406 | 2.12 | 0.9830 |
| 0.07 | 0.5279 | 0.57 | 0.7157 | 1.07 | 0.8577 | 1.57 | 0.9418 | 2.14 | 0.9838 |
| 0.08 | 0.5319 | 0.58 | 0.7190 | 1.08 | 0.8599 | 1.58 | 0.9429 | 2.16 | 0.9846 |
| 0.09 | 0.5359 | 0.59 | 0.7224 | 1.09 | 0.8621 | 1.59 | 0.9441 | 2.18 | 0.9854 |
| 0.10 | 0.5398 | 0.60 | 0.7257 | 1.10 | 0.8643 | 1.60 | 0.9452 | 2.20 | 0.9861 |
| 0.11 | 0.5438 | 0.61 | 0.7291 | 1.11 | 0.8665 | 1.61 | 0.9463 | 2.22 | 0.9868 |
| 0.12 | 0.5478 | 0.62 | 0.7324 | 1.12 | 0.8686 | 1.62 | 0.9474 | 2.24 | 0.9875 |
| 0.13 | 0.5517 | 0.63 | 0.7357 | 1.13 | 0.8708 | 1.63 | 0.9484 | 2.26 | 0.9881 |
| 0.14 | 0.5557 | 0.64 | 0.7389 | 1.14 | 0.8729 | 1.64 | 0.9495 | 2.28 | 0.9887 |
| 0.15 | 0.5596 | 0.65 | 0.7422 | 1.15 | 0.8749 | 1.65 | 0.9505 | 2.30 | 0.9893 |
| 0.16 | 0.5636 | 0.66 | 0.7454 | 1.16 | 0.8770 | 1.66 | 0.9515 | 2.32 | 0.9898 |
| 0.17 | 0.5675 | 0.67 | 0.7486 | 1.17 | 0.8790 | 1.67 | 0.9525 | 2.34 | 0.9904 |
| 0.18 | 0.5714 | 0.68 | 0.7517 | 1.18 | 0.8810 | 1.68 | 0.9535 | 2.36 | 0.9909 |
| 0.19 | 0.5753 | 0.69 | 0.7549 | 1.19 | 0.8830 | 1.69 | 0.9545 | 2.38 | 0.9913 |
| 0.20 | 0.5793 | 0.70 | 0.7580 | 1.20 | 0.8849 | 1.70 | 0.9554 | 2.40 | 0.9918 |
| 0.21 | 0.5832 | 0.71 | 0.7611 | 1.21 | 0.8869 | 1.71 | 0.9564 | 2.42 | 0.9922 |
| 0.22 | 0.5871 | 0.72 | 0.7642 | 1.22 | 0.8888 | 1.72 | 0.9573 | 2.44 | 0.9927 |
| 0.23 | 0.5910 | 0.73 | 0.7673 | 1.23 | 0.8907 | 1.73 | 0.9582 | 2.46 | 0.9931 |
| 0.24 | 0.5948 | 0.74 | 0.7704 | 1.24 | 0.8925 | 1.74 | 0.9591 | 2.48 | 0.9934 |
| 0.25 | 0.5987 | 0.75 | 0.7734 | 1.25 | 0.8944 | 1.75 | 0.9599 | 2.50 | 0.9938 |
| 0.26 | 0.6026 | 0.76 | 0.7764 | 1.26 | 0.8962 | 1.76 | 0.9608 | 2.55 | 0.9946 |
| 0.27 | 0.6064 | 0.77 | 0.7794 | 1.27 | 0.8980 | 1.77 | 0.9616 | 2.60 | 0.9953 |
| 0.28 | 0.6103 | 0.78 | 0.7823 | 1.28 | 0.8997 | 1.78 | 0.9625 | 2.65 | 0.9960 |
| 0.29 | 0.6141 | 0.79 | 0.7852 | 1.29 | 0.9015 | 1.79 | 0.9633 | 2.70 | 0.9965 |
| 0.30 | 0.6179 | 0.80 | 0.7881 | 1.30 | 0.9032 | 1.80 | 0.9641 | 2.75 | 0.9970 |
| 0.31 | 0.6217 | 0.81 | 0.7910 | 1.31 | 0.9049 | 1.81 | 0.9649 | 2.80 | 0.9974 |
| 0.32 | 0.6255 | 0.82 | 0.7939 | 1.32 | 0.9066 | 1.82 | 0.9656 | 2.85 | 0.9978 |
| 0.33 | 0.6293 | 0.83 | 0.7967 | 1.33 | 0.9082 | 1.83 | 0.9664 | 2.90 | 0.9981 |
| 0.34 | 0.6331 | 0.84 | 0.7995 | 1.34 | 0.9099 | 1.84 | 0.9671 | 2.95 | 0.9984 |
| 0.35 | 0.6368 | 0.85 | 0.8023 | 1.35 | 0.9115 | 1.85 | 0.9678 | 3.00 | 0.9987 |
| 0.36 | 0.6406 | 0.86 | 0.8051 | 1.36 | 0.9131 | 1.86 | 0.9686 | 3.05 | 0.9989 |
| 0.37 | 0.6443 | 0.87 | 0.8078 | 1.37 | 0.9147 | 1.87 | 0.9693 | 3.10 | 0.9990 |
| 0.38 | 0.6480 | 0.88 | 0.8106 | 1.38 | 0.9162 | 1.88 | 0.9699 | 3.15 | 0.9992 |
| 0.39 | 0.6517 | 0.89 | 0.8133 | 1.39 | 0.9177 | 1.89 | 0.9706 | 3.20 | 0.9993 |
| 0.40 | 0.6554 | 0.90 | 0.8159 | 1.40 | 0.9192 | 1.90 | 0.9713 | 3.25 | 0.9994 |
| 0.41 | 0.6591 | 0.91 | 0.8186 | 1.41 | 0.9207 | 1.91 | 0.9719 | 3.30 | 0.9995 |
| 0.42 | 0.6628 | 0.92 | 0.8212 | 1.42 | 0.9222 | 1.92 | 0.9726 | 3.35 | 0.9996 |
| 0.43 | 0.6664 | 0.93 | 0.8238 | 1.43 | 0.9236 | 1.93 | 0.9732 | 3.40 | 0.9997 |
| 0.44 | 0.6700 | 0.94 | 0.8264 | 1.44 | 0.9251 | 1.94 | 0.9738 | 3.50 | 0.9998 |
| 0.45 | 0.6736 | 0.95 | 0.8289 | 1.45 | 0.9265 | 1.95 | 0.9744 | 3.60 | 0.9998 |
| 0.46 | 0.6772 | 0.96 | 0.8315 | 1.46 | 0.9279 | 1.96 | 0.9750 | 3.70 | 0.9999 |
| 0.47 | 0.6808 | 0.97 | 0.8340 | 1.47 | 0.9292 | 1.97 | 0.9756 | 3.80 | 0.9999 |
| 0.48 | 0.6844 | 0.98 | 0.8365 | 1.48 | 0.9306 | 1.98 | 0.9761 | 3.90 | 1.0000 |
| 0.49 | 0.6879 | 0.99 | 0.8389 | 1.49 | 0.9319 | 1.99 | 0.9767 | 4.00 | 1.0000 |
| 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 | | |

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p ; that is, $P(Z > z) = 1 - \Phi(z) = p$.

| p | z | p | z |
|--------|--------|--------|--------|
| 0.5000 | 0.0000 | 0.0500 | 1.6449 |
| 0.4000 | 0.2533 | 0.0250 | 1.9600 |
| 0.3000 | 0.5244 | 0.0100 | 2.3263 |
| 0.2000 | 0.8416 | 0.0050 | 2.5758 |
| 0.1500 | 1.0364 | 0.0010 | 3.0902 |
| 0.1000 | 1.2816 | 0.0005 | 3.2905 |