

1. A particle  $P$  of mass  $m$  is above the surface of the Earth at distance  $x$  from the centre of the Earth. The Earth exerts a gravitational force on  $P$ . The magnitude of this force is inversely proportional to  $x^2$ .

At the surface of the Earth the acceleration due to gravity is  $g$ . The Earth is modelled as a sphere of radius  $R$ .

- (a) Prove that the magnitude of the gravitational force on  $P$  is  $\frac{mgR^2}{x^2}$ . (3)

A particle is fired vertically upwards from the surface of the Earth with initial speed  $3U$ . At a height  $R$  above the surface of the Earth the speed of the particle is  $U$ .

- (b) Find  $U$  in terms of  $g$  and  $R$ . (7)
- (Total 10 marks)**

2. At time  $t = 0$ , a particle  $P$  is at the origin  $O$  moving with speed  $2 \text{ m s}^{-1}$  along the  $x$ -axis in the positive  $x$ -direction. At time  $t$  seconds ( $t > 0$ ), the acceleration of  $P$  has magnitude  $\frac{3}{(t+1)^2} \text{ m s}^{-2}$  and is directed towards  $O$ .

- (a) Show that at time  $t$  seconds the velocity of  $P$  is  $\left(\frac{3}{t+1} - 1\right) \text{ m s}^{-1}$ . (5)

- (b) Find, to 3 significant figures, the distance of  $P$  from  $O$  when  $P$  is instantaneously at rest. (7)
- (Total 12 marks)**

3. A particle  $P$  of mass  $0.5$  kg is moving along the positive  $x$ -axis. At time  $t$  seconds,  $P$  is moving under the action of a single force of magnitude  $[4 + \cos(\pi t)]$  N, directed away from the origin. When  $t = 1$ , the particle  $P$  is moving away from the origin with speed  $6 \text{ m s}^{-1}$ .

Find the speed of  $P$  when  $t = 1.5$ , giving your answer to 3 significant figures.

(Total 7 marks)

4. A particle  $P$  of mass  $3$  kg is moving in a straight line. At time  $t$  seconds,  $0 \leq t \leq 4$ , the only force acting on  $P$  is a resistance to motion of magnitude  $\left(9 + \frac{15}{(t+1)^2}\right)$  N. At time  $t$  seconds the velocity of  $P$  is  $v \text{ m s}^{-1}$ . When  $t = 4$ ,  $v = 0$ .

Find the value of  $v$  when  $t = 0$ .

(Total 7 marks)

5. A particle  $P$  of mass  $0.1$  kg moves in a straight line on a smooth horizontal table. When  $P$  is a distance  $x$  metres from a fixed point  $O$  on the line, it experiences a force of magnitude  $\frac{16}{5x^2}$  N away from  $O$  in the direction  $OP$ . Initially  $P$  is at a point  $2$  m from  $O$  and is moving towards  $O$  with speed  $8 \text{ m s}^{-1}$ .

Find the distance of  $P$  from  $O$  when  $P$  first comes to rest.

(Total 8 marks)

6. An open container  $C$  is modelled as a thin uniform hollow cylinder of radius  $h$  and height  $h$  with a base but no lid. The centre of the base is  $O$ .

(a) Show that the distance of the centre of mass of  $C$  from  $O$  is  $\frac{1}{3}h$ .

(5)

The container is filled with uniform liquid. Given that the mass of the container is  $M$  and the mass of the liquid is  $M$ ,

- (b) find the distance of the centre of mass of the filled container from  $O$ .

(5)

(Total 10 marks)

7. A spacecraft  $S$  of mass  $m$  moves in a straight line towards the centre of the earth. The earth is modelled as a fixed sphere of radius  $R$ . When  $S$  is at a distance  $x$  from the centre of the earth, the force exerted by the earth on  $S$  is directed towards the centre of the earth and has magnitude  $\frac{k}{x^2}$ , where  $k$  is a constant.

- (a) Show that  $k = mgR^2$ .

(2)

Given that  $S$  starts from rest when its distance from the centre of the earth is  $2R$ , and that air resistance can be ignored,

- (b) find the speed of  $S$  as it crashes into the surface of the earth.

(7)

(Total 9 marks)

8. A particle  $P$  moves along the  $x$ -axis. At time  $t = 0$ ,  $P$  passes through the origin  $O$ , moving in the positive  $x$ -direction. At time  $t$  seconds, the velocity of  $P$  is  $v \text{ ms}^{-1}$  and  $OP = x$  metres. The acceleration of  $P$  is  $\frac{1}{12}(30 - x) \text{ m s}^{-2}$ , measured in the positive  $x$ -direction.

- (a) Give a reason why the maximum speed of  $P$  occurs when  $x = 30$ .

(1)

Given that the maximum speed of  $P$  is  $10 \text{ m s}^{-1}$ ,

- (b) find an expression for  $v^2$  in terms of  $x$ .

(5)

(Total 6 marks)

9. A particle moving in a straight line starts from rest at the point  $O$  at time  $t = 0$ . At time  $t$  seconds, the velocity  $v \text{ m s}^{-1}$  of the particle is given by

$$v = 3t(t - 4), \quad 0 \leq t \leq 5,$$

$$v = 75t^{-1}, \quad 5 \leq t \leq 10.$$

- (a) Sketch a velocity-time graph for the particle for  $0 \leq t \leq 10$ . (3)
- (b) Find the set of values of  $t$  for which the acceleration of the particle is positive. (2)
- (c) Show that the total distance travelled by the particle in the interval  $0 \leq t \leq 5$  is 39 m. (3)
- (d) Find, to 3 significant figures, the value of  $t$  at which the particle returns to  $O$ . (5)

(Total 13 marks)

10. A particle  $P$  moves along the  $x$ -axis. At time  $t$  seconds the velocity of  $P$  is  $v \text{ m s}^{-1}$  and its acceleration is  $2 \sin \frac{1}{2}t \text{ m s}^{-2}$ , both measured in the direction of  $Ox$ . Given that  $v = 4$  when  $t = 0$ ,

- (a) find  $v$  in terms of  $t$ , (4)
- (b) calculate the distance travelled by  $P$  between the times  $t = 0$  and  $t = \frac{\pi}{2}$ . (4)

(Total 8 marks)

11. At time  $t = 0$ , a particle  $P$  is at the origin  $O$ , moving with speed  $18 \text{ m s}^{-1}$  along the  $x$ -axis, in the positive  $x$ -direction. At time  $t$  seconds ( $t > 0$ ) the acceleration of  $P$  has magnitude

$$\frac{3}{\sqrt{t+4}} \text{ m s}^{-2} \text{ and is directed towards } O.$$

- (a) Show that, at time  $t$  seconds, the velocity of  $P$  is  $[30 - 6\sqrt{t+4}] \text{ m s}^{-1}$ . (5)

- (b) Find the distance of  $P$  from  $O$  when  $P$  comes to instantaneous rest.

(7)

(Total 12 marks)

12. A particle  $P$  moves along the  $x$ -axis. At time  $t$  seconds its acceleration is  $(-4e^{-2t}) \text{ m s}^{-2}$  in the direction of  $x$  increasing. When  $t = 0$ ,  $P$  is at the origin  $O$  and is moving with speed  $1 \text{ m s}^{-1}$  in the direction of  $x$  increasing.

- (a) Find an expression for the velocity of  $P$  at time  $t$ .

(3)

- (b) Find the distance of  $P$  from  $O$  when  $P$  comes to instantaneous rest.

(6)

(Total 9 marks)

13. A toy car of mass  $0.2 \text{ kg}$  is travelling in a straight line on a horizontal floor. The car is modelled as a particle. At time  $t = 0$  the car passes through a fixed point  $O$ . After  $t$  seconds the speed of the car is  $v \text{ m s}^{-1}$  and the car is at a point  $P$  with  $OP = x$  metres. The resultant force on the car is modelled as  $\frac{1}{10}x(4 - 3x) \text{ N}$  in the direction  $OP$ . The car comes to instantaneous rest when  $x = 6$ . Find

- (a) an expression for  $v^2$  in terms of  $x$ ,

(7)

- (b) the initial speed of the car.

(2)

(Total 9 marks)

14. A particle  $P$  moves on the positive  $x$ -axis. When the displacement of  $P$  from  $O$  is  $x$  metres, its acceleration is  $(6 - 4x) \text{ m s}^{-2}$ , measured in the direction of  $x$  increasing. Initially  $P$  is at  $O$  and the velocity of  $P$  is  $4 \text{ m s}^{-1}$  in the direction  $Ox$ .

Find the distance of  $P$  from  $O$  when  $P$  is instantaneously at rest.

(Total 6 marks)

1. (a)  $F = (-)\frac{k}{x^2}$  M1  
 $mg = (-)\frac{k}{R^2}$  M1  
 $F = \frac{mgR^2}{x^2}$  \* A1 3

(b)  $m\ddot{x} = -\frac{mgR^2}{x^2}$  M1  
 $v\frac{dv}{dx} = -\frac{gR^2}{x^2}$  M1  
 $\frac{1}{2}v^2 = \int\left(-\frac{gR^2}{x^2}\right)dx$  M1 dep on 1st M Mark  
 $\frac{1}{2}v^2 = \frac{gR^2}{x} (+c)$  A1  
 $x = R, v = 3U$   $\frac{9U^2}{2} = gR + c$  M1 dep on 3rd M Mark  
 $\frac{1}{2}v^2 = \frac{gR^2}{x} + \frac{9U^2}{2} - gR$   
 $x = 2R, v = U$   $\frac{1}{2}U^2 = \frac{gR^2}{2R} + \frac{9U^2}{2} - gR$  M1 dep on 3rd M Mark  
 $U^2 = \frac{gR}{8}$   
 $U = \sqrt{\frac{gR}{8}}$  A1 7

[10]

2. (a)  $\frac{d^2x}{dt^2} = -\frac{3}{(t+1)^2}$  M1  
 $\frac{dx}{dt} = \int -3(t+1)^{-2} dt$   
 $= 3(t+1)^{-1} (+c)$  M1 A1  
 $t = 0, v = 2$   $2 = 3 + c$   $c = -1$  M1  
 $\frac{dx}{dt} = \frac{3}{t+1} - 1$  \* A1 5

(b)

$$x = \int \left( \frac{3}{t+1} - 1 \right) dt \quad \text{M1}$$

$$= 3 \ln(t+1) - t \quad (+c') \quad \text{A1}$$

$t = 0, x = 0 \Rightarrow c' = 0$

$$x = 3 \ln(t+1) - t \quad \text{B1}$$

$$v = 0 \Rightarrow \frac{3}{t+1} = 1 \quad \text{M1}$$

$$t = 2 \quad \text{A1}$$

$$x = 3 \ln 3 - 2 \quad \text{M1}$$

$$= 1.295... \quad \text{A1}$$

$$= 1.30 \text{ m} \quad (\text{Allow } 1.3) \quad \text{A1}$$

7

**[12]**

3.  $0.5a = 4 + \cos(\pi t) \quad \text{B1}$

Integrating  $0.5v = 4t + \frac{\sin(\pi t)}{\pi} (+ C) \quad \text{M1 A1}$

Using boundary values

$$3 = 4 + C \Rightarrow C = -1 \quad \text{M1 A1}$$

When  $t = 1.5$

$$0.5v = 6 - \frac{1}{\pi} - 1 \quad \text{M1}$$

$$v \approx 9.36 \text{ (m s}^{-1}\text{)} \quad \text{cao} \quad \text{A1}$$

**[7]**

4. N2L  $3a = -\left(9 + \frac{15}{(t+1)^2}\right)$  B1

$3v = -9t + \frac{15}{t+1} (+A)$  M1 A1ft

$v = 0, t = 4 \Rightarrow 0 = -36 + 3 + A \Rightarrow A = 33$  M1 A1

$v = -3t + \frac{5}{t+1} + 11$

$t = 0 \Rightarrow v = 16$  M1 A1

[7]

5.  $m 'a' = \pm \frac{16}{5x^2}$ , with acceleration in any form (e.g.  $\frac{d^2x}{dt^2}, v \frac{dv}{dx}, \frac{dv}{dt}$ ) B1

Uses  $a = v \frac{dv}{dx}$  to obtain  $k v \frac{dv}{dx} = \pm k' \frac{32}{x^2}$  M1

Separates variables,  $k \int v dv = k' \int \frac{32}{x^2} dx$  dM1

Obtains  $\frac{1}{2} v^2 = \mp \frac{32}{x} (+C)$  or equivalent e.g.  $\frac{0.1}{2} v^2 = -\frac{16}{5x} (+C)$  A1

Substituting  $x = 2$  if + used earlier or  $-2$  if  $-$  used in d.e. M1 A1

$x = 2, v = \pm 8 \Rightarrow 32 = -16 + C \Rightarrow C = 48$   
(or value appropriate to their correct equation)

$v = 0 \Rightarrow \frac{32}{x} = 48 \Rightarrow x = \frac{2}{3} \text{ m}$

(N.B.  $-\frac{2}{3}$  is not acceptable for final answer) M1 A1 cao 8

N.B.  $\frac{d}{dx} \left( \frac{1}{2} m v^2 \right) = \frac{16}{5x^2}$ , is also a valid approach.

Last two method marks are independent of earlier marks and of each other

[8]

6.	(a)		Base	Cylinder	Container			
		Mass ratios	$\pi h^2$	$2\pi h^2$	$3\pi h^2$	Ratio of 1 : 2 : 3	B1	
		$\bar{y}$	0	$\frac{h}{2}$	$\bar{y}$		B1	
		$3\pi h^2 \times \bar{y} = 2\pi h^2 \times \frac{h}{2}$		M1A1				
		Leading to $\bar{y} = \frac{1}{3}h^*$			cso	A1	5	

(b)		Liquid	Container	Total			
	Mass ratios	$M$	$M$	$2M$	Ratio of 1 : 1 : 2	B1	
	$\bar{y}$	$\frac{h}{2}$	$\frac{h}{3}$	$\bar{y}$		B1	
	$2M \times \bar{y} = M \times \frac{h}{2} + M \times \frac{h}{3}$					M1A1	
		$\bar{y} = \frac{5}{12}h$				A1	5

[10]

7.	(a)	At surface				
		$\frac{k}{R^2} = mg \Rightarrow k = mgR^2 *$		cso	M1A1	2

(b)	N2L	$m\ddot{x} = -\frac{mgR^2}{x^2}$			
	$v \frac{dv}{dx} = -\frac{gR^2}{x^2}$ or $\frac{d}{dx} \left( \frac{1}{2} v^2 \right) = -\frac{gR^2}{x^2}$			M1	
	$\int v dv = -gR^2 \int \frac{1}{x^2} dx$ or $\frac{1}{2} v^2 = -gR^2 \int \frac{1}{x^2} dx$			M1	
	$\frac{1}{2} v^2 = \frac{gR^2}{x} (+C)$			A1	
	$x = 2R, v = 0 \Rightarrow C = -\frac{gR}{2}$			M1A1	
	$v^2 = \frac{2gR^2}{x} - gR$				
	At $x = R, v^2 = \frac{2gR^2}{R} - gR$			M1	
	$v = \sqrt{(gR)}$			A1	7

[9]

8. (a) Maximum speed when accel. = 0 (o.e.) B1 1

Allow “acceln > 0 for  $x < 30$ , acceln < 0 for  $x > 30$ ”  
 Also “accelerating for  $x < 30$ , decelerating for  $x > 30$ ”  
 But “acceln < 0 for  $x > 30$ ” only is B0

(b)  $\frac{1}{12}(30-x) = v \frac{dv}{dx}$  (acceln = ... + attempt to integrate) M1

Use of  $v \frac{dv}{dx} : \frac{v^2}{2} = \frac{1}{12} \left( 30x - \frac{x^2}{2} \right) (+c)$  M1A1

Substituting  $x = 30$ ,  $v = 10$  and finding  $c (= 12.5)$ , or limits M1

$v^2 = 25 + 5x - \frac{1}{12}x^2$  (o.e.) A1 5

1<sup>st</sup> M1 will be generous for wrong form of acceln (e.g.  $dv/dx$ )!

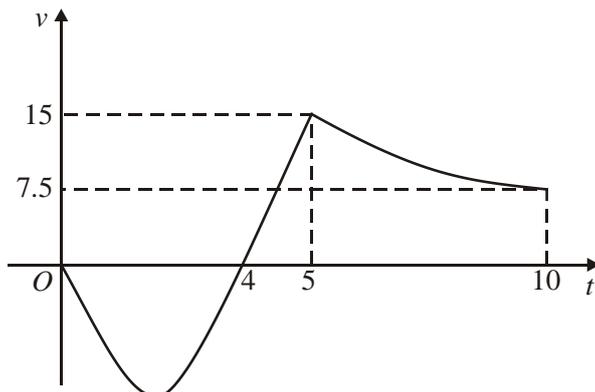
3<sup>rd</sup> M1 If use limits, they must use them in correct way with correct values

Final A1. Have to accept any expression, but it must be for  $v^2$  explicitly (not  $1/2v^2$ ), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above,

or e.g.  $\frac{1}{12}(300 + 60x - x^2)$ ; also

$100 - \frac{1}{12}(30-x)^2$

9. (a)



- Parabola B1  
 Hyperbola B1  
 Points B1 3

(b) Identifying the minimum point of the parabola and 5 as the end points. M1  
 $2 < t < 5$  A1 2

(c) Splitting the integral into two part, with limits 0 and 4, and 4 and 5, and evaluating both integrals M1

$$\int_0^4 3t(t-4) dt = [t^3 - 6t^2]_0^4 = -32 \quad \int_4^5 3t(t-4) dt = [t^3 - 6t^2]_4^5 = 7 \quad \text{Both A1}$$

Total distance = 39 (m) cs0 A1 3

(d)  $\int_5^{t_1} \frac{75}{t} dt = 32 - 7$  M1 A1

$$75[\ln t]_5^{t_1} = 25 \quad \text{A1}$$

$$\ln \frac{t_1}{5} = \frac{1}{3} \Rightarrow t_1 = 5e^{\frac{1}{3}} \quad \text{M1}$$

$\approx 6.98$  cao A1 5

[13]

10. (a)  $\frac{dv}{dt} = 2\sin \frac{1}{2}t \Rightarrow v = A - 4 \cos \frac{1}{2}t$  M1 A1

$$v = 4, t = 0 \Rightarrow 4 = A - 4 \Rightarrow A = 8 \quad \text{M1}$$

$$v = 8 - 4 \cos \frac{1}{2}t \quad \text{A1 4}$$

(b)  $\int \left(8 - 4 \cos \frac{1}{2}t\right) dt = 8t - 8 \sin \frac{1}{2}t$  ft constants M1 A1ft

$$[\dots]_0^{\pi/2} = 4(\pi - \sqrt{2}) \quad \text{awrt 6.9 M1 A1 4}$$

[8]

11. (a)  $\frac{dv}{dt} = -\frac{3}{\sqrt{t+4}}$  M1  
 $v = -3 \int (t+4)^{-\frac{1}{2}} dt$   
 $v = -6(t+4)^{\frac{1}{2}} + c$  M1A1  
 $t = 0, v = 18: 18 = -6 \times 2 + c \Rightarrow c = 30$  M1  
 $v = 30 - 6\sqrt{t+4}$  (\*) A1 c.s.o. 5

(b)  $x = \int 30 - 6(t+4)^{\frac{1}{2}} dt$  M1  
 $= 30t - 4(t+4)^{\frac{3}{2}} + D$  A1  
 $t = 0, x = 0: 0 = 0 - 4 \times 8 + D \Rightarrow D = 32$  M1  
 $v = 0 \Rightarrow 30 - 6\sqrt{t+4} = 0 \Rightarrow t = 21$  M1 A1  
 $t = 21, x = 30 \times 21 - 4 \times 53 + 32$  M1  
 $= 162$  (a) A1 7

[12]

12. (a) Integration of  $-4e^{-2t}$  w.r.t.  $t$  to give  $v = 2e^{-2t} (+c)$  B1  
 Using initial conditions to find  $c$  (-1) or  $v - 1 = [f(t)]_0^t$  M1  
 $v = 2e^{-2t} - 1 \text{ ms}^{-1}$  A1 3

(b) Finding  $t$  when  $v = 0$ ;  $[T = \frac{1}{2} \ln 2, 0.347]$  M1  
**Integrating**  $v$  w.r.t  $t$ ;  $x = -e^{-2t} - t (+c)$  M1; A1ft  
 Using  $t = 0, x = 0$  and finding value for  $c$  ( $c = 1$ ) M1  
 Substituting  $T$  in equation for  $x$  and finding value for  $x$  M1  
 [Def. integral:  $x = [-e^{-2t} - t]_0^T$  M1; correct use of limits M1]  
 $x = \frac{1}{2}(1 - \ln 2) \text{ m}$  (or equiv. **two terms**) or **0.15** or **0.153** m(awrt) A1 6

[9]

13. (a)  $\frac{1}{10}x(4 - 3x) = 0.2a$  M1 A1
- $\frac{1}{10}x(4 - 3x) = 0.2v \frac{dv}{dx}$  or  $\frac{1}{10}x(4 - 3x) = 0.2 \frac{d(\frac{1}{2}v^2)}{dx}$  M1
- Integrating :  $v^2 = 2x^2 - x^3 + C$  or equivalent M1 A1
- Substituting  $x = 6, v = 0$  to find candidate's "C" M1
- $v^2 = 2x^2 - x^3 + 144$  A1 7
- (b) Substituting  $x = 0$  and finding  $v; v = 12 \text{ (m s}^{-1}\text{)}$  M1; A1 ft 2

[9]

14.  $v \frac{dv}{dx} = 6 - 4x$  M1
- $\int v dv = \int 6 - 4x \Rightarrow \frac{1}{2}v^2 = 6x - 2x^2 + c$  M1 A1
- $x = 0, v = 4 \Rightarrow c = 8$  A1
- $v = 0 \Rightarrow 8 + 6x - 2x^2 = 0$  M1
- $4 + 3x - x^2 = 0$
- $(4 - x)(1 + x) = 0$
- $(x > 0 \Rightarrow) x = 4$  A1

[6]

1. Part (a) should have posed no problems for the majority of candidates as it was a piece of standard bookwork. However, many candidates had little or no idea of how to proceed, but still managed to arrive at the printed result.

In part (b) many forgot the initial minus sign and ended up with a negative  $u^2$ . They did not seem to realise that the most likely error is a missing minus and instead, obtained a real value for the square root of their negative answer. Several tried to use energy with PE as  $mgh$  even though the result from part (a) should have told them that the force was variable and so this approach was invalid. There were a few “suvat” equations which again were invalid and a few successful energy attempts using integration. The majority of successful candidates adopted an indefinite integral approach; a few using definite integrals got the limits the wrong way round.

2. For the majority of candidates, this was a very straightforward and quick question. Unusually, almost all candidates remembered the  $+c$  in both parts and found it correctly. The fact that the answer to (a) was given allowed many candidates who would otherwise have missed the required negative in the acceleration to correct their work. However, a substantial minority chose instead to “correct” their work with a further, highly visible deliberate mistake. It is not sensible to choose to do this when they clearly know that what they are writing is wrong. Sign errors are so common in Mechanics that they should have had plenty of experience in finding their source; throwing good marks after bad is never wise. The most common “corrections”

were  $\int \frac{3}{(t+1)^2} dt = \frac{3}{t+1}$  which was nearly always accompanied by a crossed out minus sign,

showing that the correct integration was known but ignored and using  $v = -2$  when  $t = 0$  to get  $v = -\frac{3}{t+1} + 1$  followed by a deliberately fudged rearrangement or a vague comment that “ $v$  is in the other direction” to effect the necessary sign change.

One surprising aspect of both parts was that a significant minority needed to use a substitution to integrate, letting  $u = t + 1$  to deal with both  $\int \frac{3}{(t+1)^2} dt$  and  $\int \frac{3}{(t+1)} dt$ . Fortunately there was little or no reference to “suvat” equations from candidates in this question. Some did ignore the instruction to give the final answer to 3 significant figures.

3. This was a straightforward non-uniform motion problem and so made a good opening question for the paper. Nearly all candidates set up a correct differential equation and attempted the required integration. Inevitably some integrated their acceleration as  $\frac{1}{2}v^2$ . Most who made this fundamental error had not previously written the acceleration in its differential form; had they done so they would probably not have chosen  $v \frac{dv}{dx}$  and so not made the mistake. Overall, the most common error arose in the integration of  $\cos(\pi t)$ . Many thought this integrated to  $\pi \sin(\pi t)$  and there were the inevitable sign errors as well. A few candidates did not make their method clear, by writing  $F = 4 + \cos(\pi t)$  without preceding it with  $F = ma$ . If their subsequent mental arithmetic led them to  $a = 8 + 2\cos(\pi t)$  this was not a problem, but a slip which then produced  $a = 2 + \frac{1}{2}\cos(\pi t)$  or any other incorrect expression caused a serious loss of marks. Fortunately only a few attempts at this question involved the use of the constant acceleration equations.

4. This was a straightforward opening question. As the positive direction was not defined by the question, a final answer of +16 or -16 was acceptable, provided the working shown supported the answer given. There were a few cases of  $v = \frac{dv}{dx}$  or even  $v = \frac{dv}{dt}$  being used for the acceleration; this rarely prevented candidates from integrating the other side of their equation with respect to  $t$ . A few candidates omitted the mass from their equation and there was a scattering of errors in integration and algebra when obtaining the constant.
5. The most common solution involved using “ $F = ma$ ” and replacing  $a$  by  $v \frac{dv}{dx}$ . Use of  $\frac{dv}{dx}$  instead of  $v \frac{dv}{dx}$  for acceleration was far less common than often in the past, and errors in the integration were rare. Those who used a minus sign in their initial equation rarely managed consistency throughout the question. (They should have used  $x = -2$  with  $v = +/- 8$  as their boundary condition) A small minority did not realise the need for calculus and tried to use constant acceleration formulae. Another group of candidates used the “work done approach” but a significant proportion of these candidates were not successful with this approach. It was surprising that so many candidates reaching the final line of the solution, could not solve the equation  $\frac{32}{x} = 48$  correctly, obtaining 1.5 instead of  $\frac{2}{3}$  as their answer for  $x$ .
6. This question was, on the whole, well done with a great many candidates gaining full marks. Part (a) caused more problems than part (b) but many candidates who had difficulty with (a) went on to complete (b) successfully. There were a significant number of very weak attempts at (a) which tried to involve integration, often apparently trying to prove the result for the centre of mass of a cone. Among successful solutions, the most popular method was to treat the container as a cylinder without ends and combine this with one circular end. However the alternative involving the removal of a lid from a container closed at both ends was also quite common and was usually successful. Problems with (b) were rare and tended to arise only where candidates attempted to consider the curved surface, base and liquid as three separate items, ignoring the given  $M$  and often taking the masses as  $2\pi h^2$ ,  $\pi h^2$  and  $\pi h^3$ . Some, however, completed this method successfully by using the mass of the liquid as  $3\pi h^2$ . Geometrical solutions to either part of the question were uncommon.
7. There were some unnecessarily complicated solutions to part (a), often involving the general inverse square law  $F = \frac{Gmm'}{r^2}$ . Candidates did not always make it clear that they were using the fact that the force on the rocket was  $mg$  at the surface of the earth, i.e. when  $x = R$ . It was not unusual to see this part of the question omitted. In part (b), the sign in Newton’s Second Law was more often incorrect than correct. Provided an appropriate form was used for the acceleration, candidates could usually integrate correctly. Those who then used a constant of integration almost invariably used the boundary condition,  $x = 2R$  when  $v = 0$ , correctly and, if they had the initial sign correct, gained full marks. Those who used definite integration often had trouble getting their limits the right way round. Although the use of definite integration is obviously a completely valid method, the evidence of the responses seen does suggest that those using this method are less successful in producing correct solutions. It was disappointing that

candidates who obtained an answer  $v = \sqrt{-gR}$  often attempted to explain this in terms of a loss of energy instead of realising that this was an obviously incorrect answer and using this as a cue to check their work.

8. This proved to be a friendly starter question for almost all candidates and virtually all used the correct form of the expression for the acceleration in part (b) and integrated successfully. The initial conditions were also generally accurately applied, with only a few attempting to use a possible value for  $v$  when  $x = 0$ . In part (a), some explanations were not clear, and the most common cause for the loss of the mark was when candidates simply stated what would be happening after  $x = 30$ , and said nothing about the situation when  $x < 30$ .
9. This question was predominately a pure mathematics problem involving curve sketching and integration. Too many candidates saw it as just that and did not interpret their sketch in terms of the mechanics of the situation by identifying gradients with accelerations and areas with displacements, and not distances. Sketches in part (a) were often poor, with the join between the curves often incorrectly smoothed. This is, perhaps, understandable but the number of candidates who were unable to sketch correctly a factorised quadratic and a straight forward hyperbola, both GCSE topics, was disappointing. Surprisingly few used their graph to identify the time interval over which the acceleration was positive. However they were usually able to establish a method using differentiation even if there were errors of detail. For example, the acceleration at  $t = 2$  is 0, which is not positive. In part (c), nearly all candidates knew that integration was involved and the majority realised that they had to integrate separately from  $t = 0$  to  $t = 4$  and from  $t = 4$  to  $t = 5$ . However there was much adjustment of the signs to obtain the printed answer without any justification being given. In part (d), the use of indefinite integration was commoner than the use of definite integration but the latter was generally more successful. Those using indefinite integration often had difficulty with the boundary conditions. The use of  $s = 25$ , instead of  $s = -25$ , or  $s = 0$  at  $t = 0$ , was common.
10. This too was a straightforward question and full marks were common. Most errors arose from difficulties with the constants during integration, principally wrong signs in integrating sin and cos and multiplications instead of divisions by  $\frac{1}{2}$ . Unfortunately, candidates who made mistakes in the first integration invariably repeated them in the second, ending up with only half marks. A small but significant minority integrated  $dv / dt$  as  $v^2$  in (a) and were then faced with an integral they couldn't do in (b). A very few of the most able candidates were aware that they were calculating a displacement which might not equal the distance and investigated whether the particle stopped during the given time period. This showed insight but earned no extra credit.
11. This was easily the highest scoring question. There were often sign errors in part (a), where some candidates opted to use a substitution but the second part was usually completely correct.
12. The vast majority of candidates knew how to tackle this question and it proved to be a good source of marks. Even where there were errors in integration candidates often still managed to score six marks.

- 13.** This proved a very good source of marks for the majority of candidates. The most common error was omission of the mass, but the mark scheme allowed 7 out of the 9 marks in this case.

Most candidates did use  $v \frac{dv}{dx}$  for acceleration, and apart from some errors in integration loss of marks was more usually from slips in manipulating the equations.

- 14.** No Report available for this question.