

2.

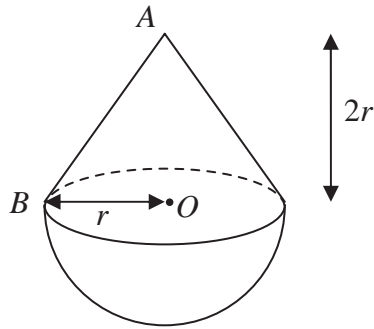


Figure 1

A toy is formed by joining a uniform solid hemisphere, of radius r and mass $4m$, to a uniform right circular solid cone of mass km . The cone has vertex A , base radius r and height $2r$. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the hemisphere is O and OB is a radius of its plane face as shown in Figure 1. The centre of mass of the toy is at O .

- (a) Find the value of k . **(4)**

A metal stud of mass λm is attached to the toy at A . The toy is now suspended by a light string attached to B and hangs freely at rest. The angle between OB and the vertical is 30° .

- (b) Find the value of λ . **(4)**



3.

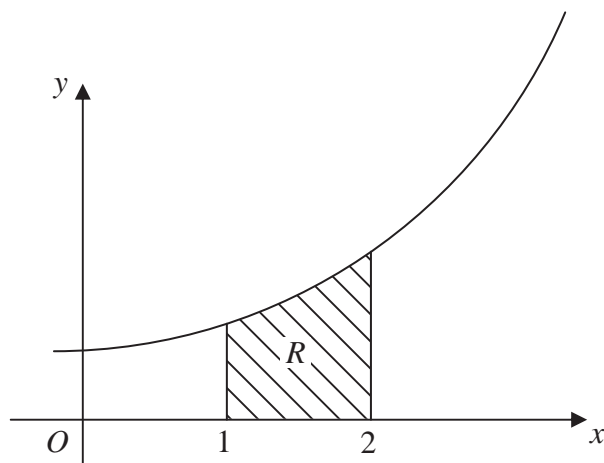


Figure 2

The region R is bounded by the curve with equation $y = e^x$, the line $x = 1$, the line $x = 2$ and the x -axis as shown in Figure 2. A uniform solid S is formed by rotating R through 2π about the x -axis.

(a) Show that the volume of S is $\frac{1}{2} \pi (e^4 - e^2)$. (4)

(b) Find, to 3 significant figures, the x -coordinate of the centre of mass of S . (6)



4. A particle P moves along the x -axis. At time t seconds its displacement, x metres, from the origin O is given by $x = 5 \sin \left(\frac{1}{3} \pi t \right)$.

(a) Prove that P is moving with simple harmonic motion. **(3)**

(b) Find the period and the amplitude of the motion. **(2)**

(c) Find the maximum speed of P . **(2)**

The points A and B on the positive x -axis are such that $OA = 2$ m and $OB = 3$ m.

(d) Find the time taken by P to travel directly from A to B . **(4)**



7.

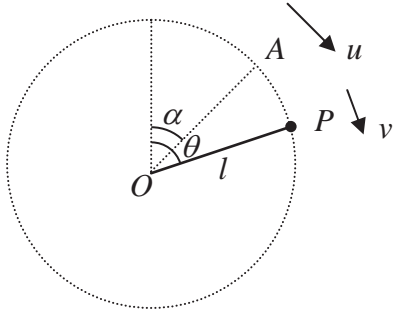


Figure 5

A particle *P* of mass *m* is attached to one end of a light rod of length *l*. The other end of the rod is attached to a fixed point *O*. The rod can turn freely in a vertical plane about *O*. The particle is projected with speed *u* from a point *A*, where *OA* makes an angle α with the upward vertical through *O* and $0 < \alpha < \frac{\pi}{2}$. When *OP* makes an angle θ with the upward vertical through *O* the speed of *P* is *v* as shown in Figure 5.

(a) Show that $v^2 = u^2 + 2gl(\cos \alpha - \cos \theta)$. (4)

It is given that $\cos \alpha = \frac{3}{5}$ and that *P* moves in a complete vertical circle.

(b) Show that $u > 2 \sqrt{\left(\frac{gl}{5}\right)}$. (4)

As the rod rotates the least tension in the rod is *T* and the greatest tension is *5T*.

(c) Show that $u^2 = \frac{33}{10} gl$. (9)



