Paper Reference(s)

6679

Edexcel GCE

Mechanics M3
(New Syllabus)
Advanced/Advanced Subsidiary

Thursday 7 June 2001 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

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Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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N6993 83019/R6677 3/3/3/3/3/36, 100

1. A particle P moves along the x-axis in the positive direction. At time t seconds, the velocity of P is v m s^{-1} and its acceleration is $\frac{1}{2}e^{-\frac{1}{6}t}$ m s^{-2}. When t = 0 the speed of P is 10 m s^{-1}.

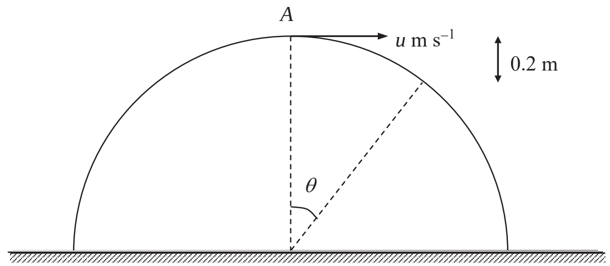
(*a*) Express v in terms of t. **(4)**

(*b*) Find, to 3 significant figures, the speed of P when t = 3. **(2)**

(*c*) Find the limiting value of v. **(1)**

2.

Figure 1



A smooth solid hemisphere, of radius 0.8 m and centre O, is fixed with its plane face on a horizontal table. A particle of mass 0.5 kg is projected horizontally with speed u m s^{-1} from the highest point A of the hemisphere. The particle leaves the hemisphere at the point B, which is a vertical distance of 0.2 m below the level of A. The speed of the particle at B is v m s^{-1} and the angle between OA and OB is θ, as shown in Fig. 1.

(*a*) Find the value of cos θ. **(1)**

(*b*) Show that v^2 = 5.88. **(3)**

(*c*) Find the value of u. **(3)**

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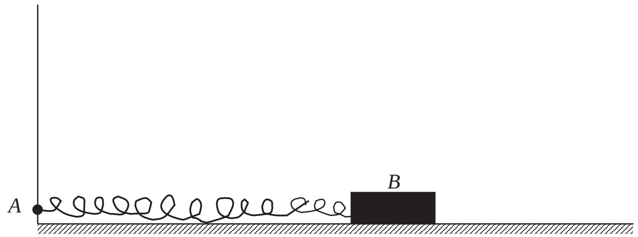
3. **Figure 2**



A light horizontal spring, of natural length 0.25 m and modulus of elasticity 52 N, is fastened at one end to a point *A*. The other end of the spring is fastened to a small wooden block *B* of mass 1.5 kg which is on a horizontal table, as shown in Fig. 2. The block is modelled as a particle.

The table is initially assumed to be smooth. The block is released from rest when it is a distance 0.3 m from *A*. By using the principle of the conservation of energy,

(*a*) find, to 3 significant figures, the speed of *B* when it is a distance 0.25 m from *A*.

(5)

It is now assumed that the table is rough and the coefficient of friction between *B* and the table is 0.6.

(*b*) Find, to 3 significant figures, the minimum distance from *A* at which *B* can rest in equilibrium.

(5)

4. A projectile *P* is fired vertically upwards from a point on the earth's surface. When *P* is at a distance x from the centre of the earth its speed is v. Its acceleration is directed towards the centre of the earth and has magnitude $\frac{k}{x^2}$, where k is a constant. The earth may be assumed to be a sphere of radius R.

(*a*) Show that the motion of *P* may be modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}.$$

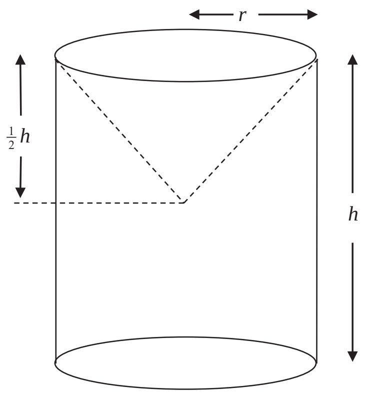
(4)

The initial speed of *P* is U, where $U^2 < 2gR$. The greatest distance of *P* from the centre of the earth is X.

(*b*) Find X in terms of U, R and g.

(6)

5. **Figure 3**



An ornament S is formed by removing a solid right circular cone, of radius r and height $\frac{1}{2}h$, from a solid uniform cylinder, of radius r and height h, as shown in Fig. 3.

(*a*) Show that the distance of the centre of mass S from its plane face is $\frac{17}{40}h$.

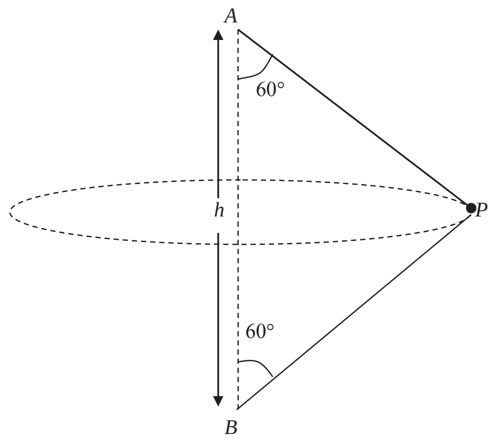
(7)

The ornament is suspended from a point on the circular rim of its open end. It hangs in equilibrium with its axis of symmetry inclined at an angle α to the horizontal. Given that $h = 4r$,

(*b*) find, in degrees to one decimal place, the value of α.

(4)

6. **Figure 4**



A particle P of mass m is attached to two light inextensible strings. The other ends of the string are attached to fixed points A and B. The point A is a distance h vertically above B. The system rotates about the line AB with constant angular speed ω. Both strings are taut and inclined at 60° to AB, as shown in Fig. 4. The particle moves in a circle of radius r.

(*a*) Show that $r = \frac{\sqrt{3}}{2}h$.

(2)

(*b*) Find, in terms of m, g, h and ω, the tension in AP and the tension in BP.

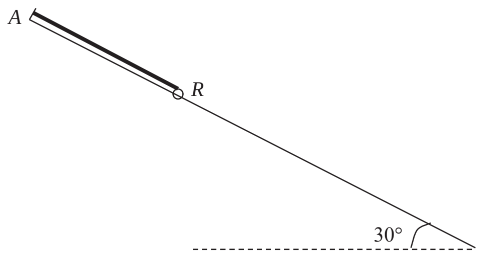
(8)

The time taken for P to complete one circle is T.

(*c*) Show that $T < \pi\sqrt{\left(\frac{2h}{g}\right)}$.

(4)

7. **Figure 5**



A small ring R of mass *in* is free to slide on a smooth straight wire which is fixed at an angle of 30° to the horizontal. The ring is attached to one end of a light elastic string of natural length a and modulus of elasticity λ. The other end of the string is attached to a fixed point A of the wire, as shown in Fig. 5. The ring rests in equilibrium at the point B, where $AB = \frac{9}{8}a$.

(*a*) Show that $\lambda = 4mg$.

(3)

The ring is pulled down to the point C, where $BC = \frac{1}{4}a$, and released from rest. At time t after R is released the extension of the string is $(\frac{1}{8}a + x)$.

(*b*) Obtain a differential equation for the motion of R while the string remains taut, and show that it represents simple harmonic motion with period $\pi\sqrt{\left(\frac{a}{g}\right)}$.

(6)

(*c*) Find, in terms of g, the greatest magnitude of the acceleration of R while the string remains taut.

(2)

(*d*) Find, in terms of a and g, the time taken for R to move from the point at which it first reaches maximum speed to the point where the string becomes slack for the first time.

(5)

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3
(New Syllabus)
Advanced/Advanced Subsidiary

Monday 14 January 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

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Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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N6993 83019/R6677 3/3/3/3/3/36, 100

1. A particle P of mass 0.2 kg moves away from the origin along the positive x-axis. It moves under the action of a force directed away from the origin O, of magnitude $\frac{5}{x+1}$ N, where $OP = x$ metres. Given that the speed of P is 5 m s^{-1} when $x = 0$, find the value of x, to 3 significant figures, when the speed of P is 15 m s^{-1}.

(8)

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2. One end of a light elastic string, of natural length 2 m and modulus of elasticity 19.6 N, is attached to a fixed point A. A small ball B of mass 0.5 kg is attached to the other end of the string. The ball is released from rest at A and first comes to instantaneous rest at the point C, vertically below A.

(*a*) Find the distance AC.

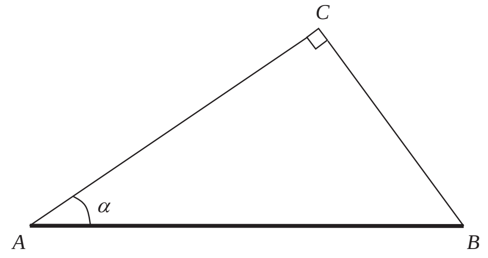
(6)

(*b*) Find the instantaneous acceleration of B at C.

(3)

3.

Figure 1



A rod AB, of mass $2m$ and length $2a$, is suspended from a fixed point C by two light strings AC and BC. The rod rests horizontally in equilibrium with AC making an angle α with the rod, where $\tan \alpha = \frac{3}{4}$, and with AC perpendicular to BC, as shown in Fig. 1.

(*a*) Give a reason why the rod cannot be uniform.

(1)

(*b*) Show that the tension in BC is $\frac{8}{5}mg$ and find the tension in AC.

(5)

The string BC is elastic, with natural length a and modulus of elasticity kmg, where k is constant.

(*c*) Find the value of k.

(4)

4.

Figure 2

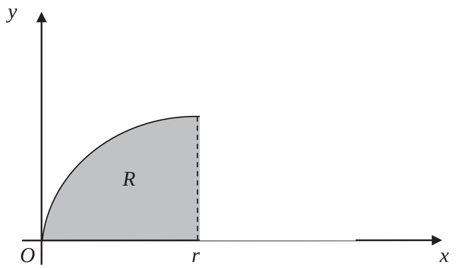


Figure 2 shows the region R bounded by the curve with equation $y^2 = rx$, where r is a positive constant, the x-axis and the line $x = r$. A uniform solid of revolution S is formed by rotating R through one complete revolution about the x-axis.

(a) Show that the distance of the centre of mass of S from O is $\frac{2}{3}r$.

(6)

The solid is placed with its plane face on a plane which is inclined at an angle α to the horizontal. The plane is sufficiently rough to prevent S from sliding. Given that S does not topple,

(b) find, to the nearest degree, the maximum value of α.

(4)

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5. A cyclist is travelling around a circular track which is banked at 25° to the horizontal. The coefficient of friction between the cycle's tyres and the track is 0.6. The cyclist moves with constant speed in a horizontal circle of radius 40 m, without the tyres slipping.

Find the maximum speed of the cyclist.

(10)

---

6. The points O, A, B and C lie in a straight line, in that order, where OA = 0.6 m, OB = 0.8 m and OC = 1.2 m. A particle P, moving along this straight line, has a speed of $\left(\frac{3}{10}\sqrt{3}\right)$ m s^{-1} at A, $\left(\frac{1}{5}\sqrt{5}\right)$ m s^{-1} at B and is instantaneously at rest at C.

(a) Show that this information is consistent with P performing simple harmonic motion with centre O.

(5)

Given that P is performing simple harmonic motion with centre O,

(b) show that the speed of P at O is 0.6 m s^{-1},

(2)

(c) find the magnitude of the acceleration of P as it passes A,

(2)

(d) find, to 3 significant figures, the time taken for P to move directly from A to B.

(4)

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7. **Figure 3**

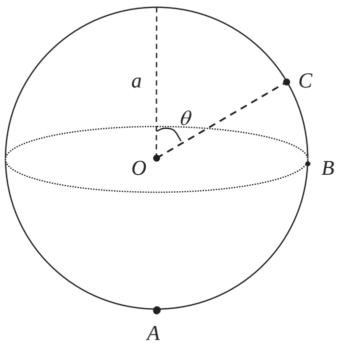

Figure 3 shows a fixed hollow sphere of internal radius a and centre *O*. A particle *P* of mass *m* is projected horizontally from the lowest point *A* of a sphere with speed $\sqrt{\left(\frac{7}{2}ag\right)}$. It moves in a vertical circle, centre *O*, on the smooth inner surface of the sphere. The particle passes through the point *B*, which is in the same horizontal plane as *O*. It leaves the surface of the sphere at the point *C*, where *OC* makes an angle θ with the upward vertical.

(*a*) Find, in terms of *m* and *g*, the normal reaction between *P* and the surface of the sphere at *B*.
(4)

(*b*) Show that $\theta = 60°$.
(7)

After leaving the surface of the sphere, *P* meets it again at the point *A*.

(*c*) Find, in terms of *a* and *g*, the time *P* takes to travel from *C* to *A*.
(4)

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Thursday 30 May 2002 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

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Advice to Candidates

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N10810

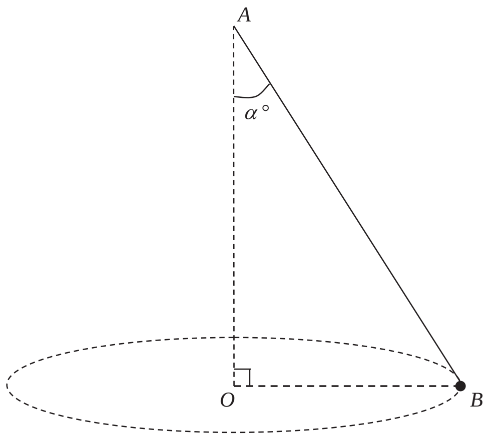
1. A particle P moves in a straight line with simple harmonic motion about a fixed centre O with period 2 s. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, $v = 0$ and P is at a point A where $OA = 0.25$ m.

Find the smallest positive value of t for which $AP = 0.375$ m.

(6)

2.

Figure 1



A metal ball B of mass m is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point A. The ball B moves in a horizontal circle with centre O vertically below A, as shown in Fig. 1. The string makes a constant angle $\alpha°$ with the downward vertical and B moves with constant angular speed $\sqrt{(2gk)}$, where k is a constant. The tension in the string is $3mg$. By modelling B as a particle. find

(*a*) the value of α,

(4)

(*b*) the length of the string.

(5)

---

3. A particle P of mass 2.5 kg moves along the positive x-axis. It moves away from a fixed origin O, under the action of a force directed away from O. When $OP = x$ metres the magnitude of the force is $2e^{-0.1x}$ newtons and the speed of P is v m s^{-1}. When $x = 0$, $v = 2$. Find

(*a*) v^2 in terms of x,

(6)

(*b*) the value of x when $v = 4$.

(3)

(*c*) Give a reason why the speed of P does not exceed $\sqrt{20}$ m s^{-1}.

(1)

---

4. A light elastic string AB of natural length 1.5 m has modulus of elasticity 20 N. The end A is fixed to a point on a smooth horizontal table. A small ball S of mass 0.2 kg is attached to the end B. Initially S is at rest on the table with $AB = 1.5$ m. The ball S is then projected horizontally directly away from A with a speed of 5 m s^{-1}. By modelling S as a particle,

(*a*) find the speed of S when $AS = 2$ m.

(5)

When the speed of S is 1.5 m s^{-1}, the string breaks.

(*b*) Find the tension in the string immediately before the string breaks.

(5)

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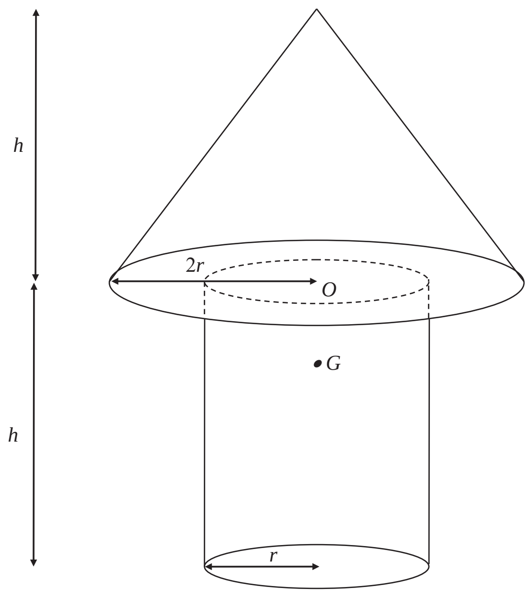
5. **Figure 2**



A model tree is made by joining a uniform solid cylinder to a uniform solid cone made of the same material. The centre O of the base of the cone is also the centre of one end of the cylinder, as shown in Fig. 2. The radius of the cylinder is r and the radius of the base of the cone is $2r$. The height of the cone and the height of the cylinder are each h. The centre of mass of the model is at the point G.

(*a*) Show that $OG = \frac{1}{14}h$.

(8)

The model stands on a desk top with its plane face in contact with the desk top. The desk top is tilted until it makes an angle α with the horizontal, where $\tan \alpha = \frac{7}{26}$. The desk top is rough enough to prevent slipping and the model is about to topple.

(*b*) Find r in terms of h.

(4)

6. A light elastic string, of natural length $4a$ and modulus of elasticity $8mg$, has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the string and hangs in equilibrium at the point O.

(*a*) Find the distance AO.

(2)

The particle is now pulled down to a point C vertically below O, where $OC = d$. It is released from rest. In the subsequent motion the string does not become slack.

(*b*) Show that P moves with simple harmonic motion of period $\pi\sqrt{\left(\frac{2a}{g}\right)}$.

(7)

The greatest speed of P during this motion is $\frac{1}{2}\sqrt{(ga)}$.

(*c*) Find d in terms of a.

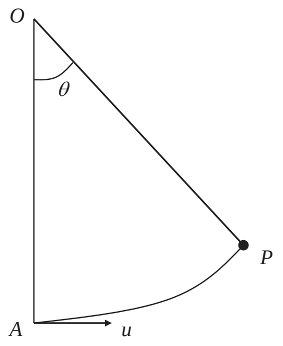
(3)

Instead of being pulled down a distance d, the particle is pulled down a distance a. Without further calculation,

(*d*) describe briefly the subsequent motion of P.

(2)

7. **Figure 3**



A particle of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging at the point A, which is vertically below O. It is projected horizontally with speed u. When the particle is at the point P, $\angle AOP = \theta$, as shown in Fig. 3. The string oscillates through an angle α on either side of OA where $\cos \alpha = \frac{2}{3}$.

(*a*) Find u in terms of g and l.

(4)

When $\angle AOP = \theta$, the tension in the string is T.

(*b*) Show that $T = \frac{mg}{3}(9 \cos \theta - 4)$.

(6)

(*c*) Find the range of values of T.

(4)

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 13 January 2003 − Afternoon

Time: 1 hour 30 minutes

Materials required for examination
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Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

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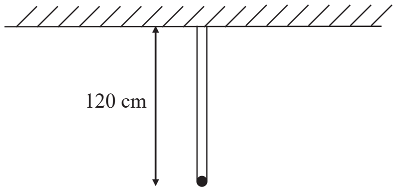
N10814A

1. **Figure 1**



A particle of mass 5 kg is attached to one end of two light elastic strings. The other ends of the strings are attached to a hook on a beam. The particle hangs in equilibrium at a distance 120 cm below the hook with both strings vertical, as shown in Fig. 1. One string has natural length 100 cm and modulus of elasticity 175 N. The other string has natural length 90 cm and modulus of elasticity λ newtons.

Find the value of λ.

(5)

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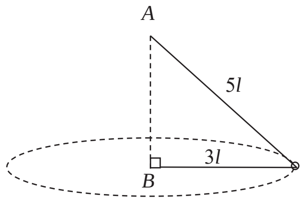
2. **Figure 2**



A light inextensible string of length $8l$ has its ends fixed to two points A and B, where A is vertically above B. A small smooth ring of mass m is threaded on the string. The ring is moving with constant speed in a horizontal circle with centre B and radius $3l$, as shown in Fig. 2. Find

(*a*) the tension in the string,

(3)

(*b*) the speed of the ring.

(5)

(*c*) State briefly in what way your solution might no longer be valid if the ring were firmly attached to the string.

(1)

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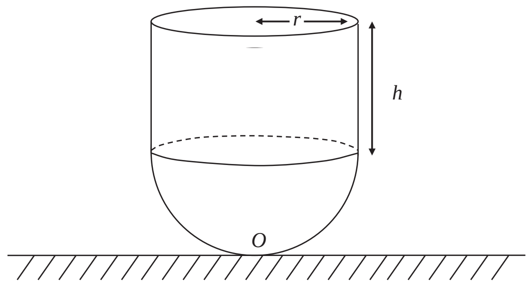
3. **Figure 3**



A child's toy consists of a uniform solid hemisphere attached to a uniform solid cylinder. The plane face of the hemisphere coincides with the plane face of the cylinder, as shown in Fig. 3. The cylinder and the hemisphere each have radius r, and the height of the cylinder is h. The material of the hemisphere is 6 times as dense as the material of the cylinder. The toy rests in equilibrium on a horizontal plane with the cylinder above the hemisphere and the axis of the cylinder vertical.

(*a*) Show that the distance d of the centre of mass of the toy from its lowest point O is given by

$$d = \frac{h^2 + 2hr + 5r^2}{2(h + 4r)}.$$

(7)

When the toy is placed with any point of the curved surface of the hemisphere resting on the plane it will remain in equilibrium.

(*b*) Find h in terms of r.

(3)

---

4. A piston P in a machine moves in a straight line with simple harmonic motion about a point O, which is the centre of the oscillations. The period of the oscillations is π s. When P is 0.5 m from O, its speed is 2.4 m s^{-1}. Find

(*a*) the amplitude of the motion,

(4)

(*b*) the maximum speed of P during the motion,

(1)

(*c*) the maximum magnitude of the acceleration of P during the motion,

(1)

(*d*) the total time, in s to 2 decimal places, in each complete oscillation for which the speed of P is greater than 2.4 m s^{-1}.

(5)

---

5. A car of mass 800 kg moves along a horizontal straight road. At time t seconds, the resultant force acting on the car has magnitude $\frac{48\,000}{(t+2)^2}$ newtons, acting in the direction of the motion of the car. When $t = 0$, the car is at rest.

(*a*) Show that the speed of the car approaches a limiting value as t increases and find this value.

(6)

(*b*) Find the distance moved by the car in the first 6 seconds of its motion.

(6)

---

6. A light elastic string has natural length 4 m and modulus of elasticity 58.8 N. A particle P of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a vertical point A. The particle is released from rest at A and falls vertically.

(*a*) Find the distance travelled by P before it immediately comes to instantaneous rest for the first time.

(7)

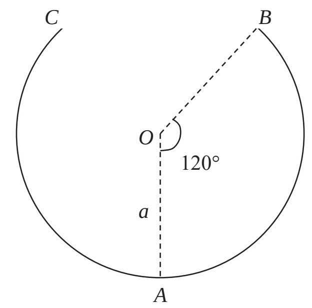
The particle is now held at a point 7 m vertically below A and released from rest.

(*b*) Find the speed of the particle when the string first becomes slack.

(5)

7.

Figure 4



Part of a hollow spherical shell, centre O and radius a, is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point A is the lowest point of the bowl. The point B is on the rim of the bowl and $\angle AOB = 120°$, as shown in Fig. 4. A smooth small marble of mass m is placed inside the bowl at A and given an initial horizontal speed u. The direction of motion of the marble lies in the vertical plane AOB. The marble stays in contact with the bowl until it reaches B. When the marble reaches B, its speed is v.

(*a*) Find an expression for v^2.

(3)

(*b*) For the case when $u^2 = 6ga$, find the normal reaction of the bowl on the marble as the marble reaches B.

(3)

(*c*) Find the least possible value of u for the marble to reach B.

(3)

The point C is the other point on the rim of the bowl lying in the vertical plane OAB.

(*d*) Find the value of u which will enable the marble to leave the bowl at B and meet it again at the point C.

(7)

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 19 May 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle P of mass m is held at a point A on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{3}$. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $4mg$. The other end of the string is attached to a fixed point O on the plane, where $OA = \frac{3}{2}a$. The particle P is released from rest and comes to rest at a point B, where $OB < a$.

Using the work-energy principle, or otherwise, calculate the distance AB.

(6)

---

2. A car moves round a bend which is banked at a constant angle of 10° to the horizontal. When the car is travelling at a constant speed of 18 m s^{-1}, there is no sideways frictional force on the car. The car is modelled as a particle moving in a horizontal circle of radius r metres.

Calculate the value of r.

(6)

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3. A toy car of mass 0.2 kg is travelling in a straight line on a horizontal floor. The car is modelled as a particle. At time $t = 0$ the car passes through a fixed point O. After t seconds the speed of the car is v m s^{-1} and the car is at a point P with $OP = x$ metres. The resultant force on the car is modelled as $\frac{1}{10}x(4 - 3x)$ N in the direction OP. The car comes to instantaneous rest when $x = 6$. Find

(*a*) an expression for v^2 in terms of x,

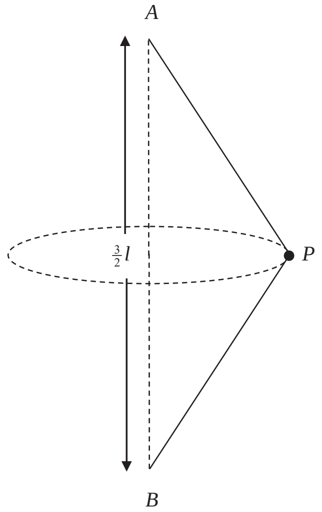
(7)

(*b*) the initial speed of the car.

(2)

4.

Figure 1



A particle P of mass m is attached to the ends of two light inextensible strings AP and BP each of length l. The ends A and B are attached to fixed points, with A vertically above B and $AB = \frac{3}{2}l$, as shown in Fig. 1. The particle P moves in a horizontal circle with constant angular speed ω. The centre of the circle is the mid-point of AB and both strings remain taut.

(*a*) Show that the tension AP is $\frac{1}{6}m(3l\omega^2 + 4g)$.

(7)

(*b*) Find, in terms of m, l, ω and g, an expression for the tension in BP.

(2)

(*c*) Deduce that $\omega^2 \geq \dfrac{4g}{3l}$.

(2)

5. A particle P of mass 0.8 kg is attached to one end A of a light elastic spring OA, of natural length 60 cm and modulus of elasticity 12 N. The spring is placed on a smooth horizontal table and the end O is fixed. The particle P is pulled away from O to a point B, where OB = 85 cm, and is released from rest.

(*a*) Prove that the motion of P is simple harmonic with period $\dfrac{2\pi}{5}$ s.

(5)

(*b*) Find the greatest magnitude of the acceleration of P during the motion.

(2)

Two seconds after being released from rest, P passes through the point C.

(*c*) Find, to 2 significant figures, the speed of P as it passes through C.

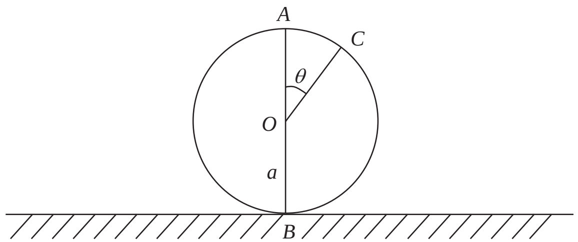
(5)

(d) State the direction in which P is moving 2 s after being released.

(1)

6.

Figure 2



A particle is at the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed u, where $u < \sqrt{(ag)}$. The particle leaves the sphere at the point C, where OC makes an angle θ with the upward vertical, as shown in Fig. 2.

(*a*) Find an expression for $\cos\theta$ in terms of u, g and a.

(7)

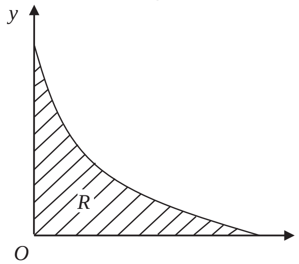
The particle strikes the plane with speed $\sqrt{\left(\dfrac{9ag}{2}\right)}$.

(*b*) Find, to the nearest degree, the value of θ.

(7)

7.

Figure 3



The shaded region R is bounded by part of the curve with equation $y = \frac{1}{2}(x - 2)^2$, the x-axis and the y-axis, as shown in Fig. 3. The unit of length on both axes is 1 cm. A uniform solid S is made by rotating R through 360° about the x-axis. Using integration,

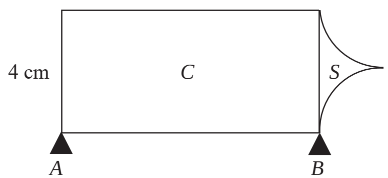
(*a*) calculate the volume of the solid S, leaving your answer in terms of π,

(4)

(*b*) show that the centre of mass of S is $\frac{1}{3}$ cm from its plane face.

(7)

Figure 4



A tool is modelled as having two components, a solid uniform cylinder C and the solid S. The diameter of C is 4 cm and the length of C is 8 cm. One end of C coincides with the plane face of S. The components are made of different materials. The weight of C is $10W$ newtons and the weight of S is $2W$ newtons. The tool lies in equilibrium with its axis of symmetry horizontal on two smooth supports A and B, which are at the ends of the cylinder, as shown in Fig. 4.

(*c*) Find the magnitude of the force of the support A on the tool.

(5)

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 12 January 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

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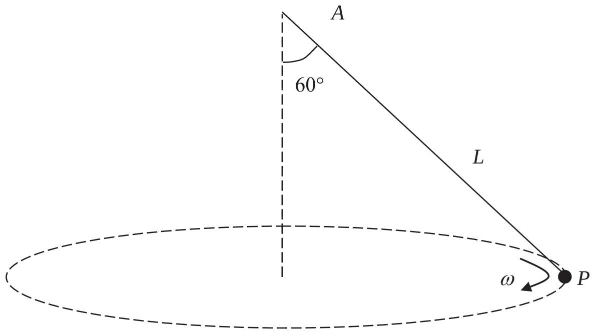
N13892A

Figure 1



A particle P of mass m is attached to one end of a light string. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of 60° to the vertical, as shown in Fig. 1. The length of the string is L.

(*a*) Show that the tension in the string is $2mg$.

(1)

(*b*) Find ω in terms of g and L.

(4)

The string is elastic and has natural length $\frac{3}{5}L$.

(*c*) Find the modulus of elasticity of the string.

(2)

---

2. A particle P moves along the x-axis. At time t seconds its acceleration is $(-4e^{-2t})$ m s^{-2} in the direction of x increasing. When $t = 0$, P is at the origin O and is moving with speed 1 m s^{-1} in the direction of x increasing.

(*a*) Find an expression for the velocity of P at time t.

(3)

(*b*) Find the distance of P from O when P comes to instantaneous rest.

(6)

---

3. Above the earth's surface, the magnitude of the force on a particle due to the earth's gravity is inversely proportional to the square of the distance of the particle from the centre of the earth. Assuming that the earth is a sphere of radius R, and taking g as the acceleration due to gravity at the surface of the earth,

(*a*) prove that the magnitude of the gravitational force on a particle of mass m when it is a distance x $(x \geq R)$ from the centre of the earth is $\dfrac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the earth with initial speed u, where $u^2 = \frac{3}{2}gR$. Ignoring air resistance,

(*b*) find, in terms of g and R, the speed of the particle when it is at a height $2R$ above the surface of the earth.

(7)

---

4. A particle P of mass m is attached to one end of a light elastic string of length a and modulus of elasticity $\frac{1}{2}mg$. The other end of the string is fixed at the point A which is at a height $2a$ above a smooth horizontal table. The particle is held on the table with the string making an angle β with the horizontal, where $\tan \beta = \frac{3}{4}$.

(*a*) Find the elastic energy stored in the string in this position.

(3)

The particle is now released. Assuming that P remains on the table,

(*b*) find the speed of P when the string is vertical.

(3)

By finding the vertical component of the tension in the string when P is on the table and AP makes an angle θ with the horizontal,

(*c*) show that the assumption that P remains in contact with the table is justified.

(5)

---

5. A piston in a machine is modelled as a particle of mass 0.2 kg attached to one end A of a light elastic spring, of natural length 0.6 m and modulus of elasticity 48 N. The other end B of the spring is fixed and the piston is free to move in a horizontal tube which is assumed to be smooth. The piston is released from rest when AB = 0.9 m.

(*a*) Prove that the motion of the piston is simple harmonic with period $\frac{\pi}{10}$ s.

(5)

(*b*) Find the maximum speed of the piston.

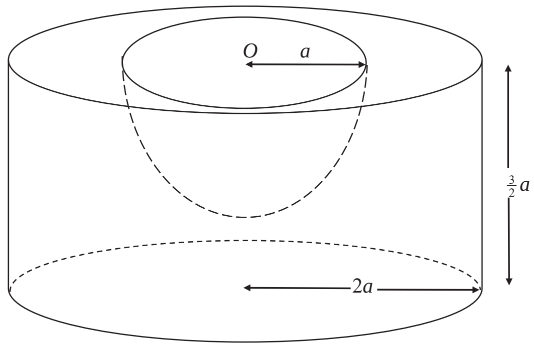
(2)

(*c*) Find, in terms of π, the length of time during each oscillation for which the length of the spring is less than 0.75 m.

(5)

6.

Figure 2



A uniform solid cylinder has radius $2a$ and height $\frac{3}{2}a$. A hemisphere of radius a is removed from the cylinder. The plane face of the hemisphere coincides with the upper plane face of the cylinder, and the centre O of the hemisphere is also the centre of this plane face, as shown in Fig. 2. The remaining solid is S.

(*a*) Find the distance of the centre of mass of S from O.

(6)

The lower plane face of S rests in equilibrium on a desk lid which is inclined at an angle θ to the horizontal. Assuming that the lid is sufficiently rough to prevent S from slipping, and that S is on the point of toppling when $\theta = \alpha$,

(*b*) find the value of α.

(3)

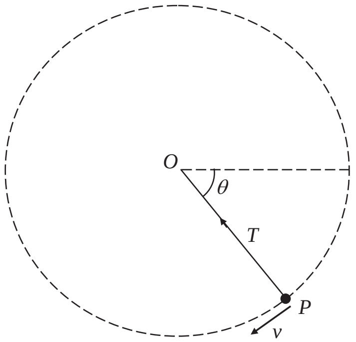
Given instead that the coefficient of friction between S and the lid is 0.8, and that S is on the point of sliding down the lid when $\theta = \beta$,

(*c*) find the value of β.

(3)

7.

Figure 3



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at a point O. The particle is held with the string taut and OP horizontal. It is then projected vertically downwards with speed u, where $u^2 = \frac{3}{2}ga$. When OP has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Fig. 3.

(*a*) Find an expression for v^2 in terms of a, g and θ.
(2)

(*b*) Find an expression for T in terms of m, g and θ.
(3)

(*c*) Prove that the string becomes slack when $\theta = 210°$.
(2)

(*d*) State, with a reason, whether P would complete a vertical circle if the string were replaced by a light rod.
(2)

After the string becomes slack, P moves freely under gravity and is at the same level as O when it is at the point A.

(*e*) Explain briefly why the speed of P at A is $\sqrt{\left(\frac{3}{2}ga\right)}$.
(1)

The direction of motion of P at A makes an angle φ with the horizontal.

(*f*) Find φ.
(4)

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 24 May 2004 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

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Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

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N17132A

1. A circular flywheel of diameter 7 cm is rotating about the axis through its centre and perpendicular to its plane with constant angular speed 1000 revolutions per minute.

Find, in m s^{-1} to 3 significant figures, the speed of a point on the rim of the flywheel.

(3)

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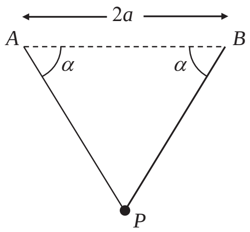
2. **Figure 1**



Two light elastic strings each have natural length a and modulus of elasticity λ. A particle P of mass m is attached to one end of each string. The other ends of the strings are attached to points A and B, where AB is horizontal and $AB = 2a$. The particle is held at the mid-point of AB and released from rest. It comes to rest for the first time in the subsequent motion when PA and PB make angles α with AB, where $\tan \alpha = \frac{4}{3}$, as shown in Fig. 1.

Find λ in terms of m and g.

(7)

---

3. A particle P of mass m kg slides from rest down a smooth plane inclined at 30° to the horizontal. When P has moved a distance x metres down the plane, the resistance to the motion of P from non-gravitational forces has magnitude mx^2 newtons.

Find

(*a*) the speed of P when $x = 2$,

(7)

(*b*) the distance P has moved when it comes to rest for the first time.

(3)

---

4. A rough disc rotates in a horizontal plane with constant angular velocity ω about a fixed vertical axis. A particle P of mass m lies on the disc at a distance $\frac{4}{3}a$ from the axis. The coefficient of friction between P and the disc is $\frac{3}{5}$. Given that P remains at rest relative to the disc,

(*a*) prove that $\omega^2 \leq \dfrac{9g}{20a}$.

(4)

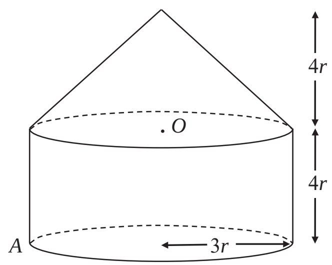
The particle is now connected to the axis by a horizontal light elastic string of natural length a and modulus of elasticity $2mg$. The disc again rotates with constant angular velocity ω about the axis and P remains at rest relative to the disc at a distance $\frac{4}{3}a$ from the axis.

(*b*) Find the greatest and least possible values of ω^2.

(7)

5.

Figure 2



A toy is formed by joining a uniform solid right circular cone, of base radius $3r$ and height $4r$, to a uniform solid cylinder, also of radius $3r$ and height $4r$. The cone and the cylinder are made from the same material, and the plane face of the cone coincides with a plane face of the cylinder, as shown in Fig. 2. The centre of this plane face is O.

(*a*) Find the distance of the centre of mass of the toy from O.

(5)

The point A lies on the edge of the plane face of the cylinder which forms the base of the toy. The toy is suspended from A and hangs in equilibrium.

(*b*) Find, in degrees to one decimal place, the angle between the axis of symmetry of the toy and the vertical.

(4)

The toy is placed with the curved surface of the cone on horizontal ground.

(*c*) Determine whether the toy will topple.

(4)

6.

Figure 3

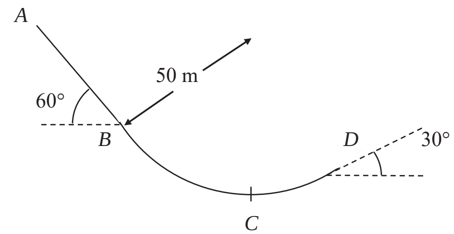


Figure 3 represents the path of a skier of mass 70 kg moving on a ski-slope *ABCD*. The path lies in a vertical plane. From A to B, the path is modelled as a straight line inclined at 60° to the horizontal. From B to D, the path is modelled as an arc of a vertical circle of radius 50 m. The lowest point of the arc *BD* is C.

At B, the skier is moving downwards with speed 20 m s^{-1}. At D, the path is inclined at 30° to the horizontal and the skier is moving upwards. By modelling the slope as smooth and the skier as a particle, find

(*a*) the speed of the skier at C,

(3)

(*b*) the normal reaction of the slope on the skier at C,

(3)

(*c*) the speed of the skier at D,

(3)

(*d*) the change in the normal reaction of the slope on the skier as she passes B.

(4)

The model is refined to allow for the influence of friction on the motion of the skier.

(*e*) State briefly, with a reason, how the answer to part (*b*) would be affected by using such a model. (No further calculations are expected.)

(2)

---

7. A particle P of mass 0.3 kg is attached to one end of a light elastic spring. The other end of the spring is attached to a fixed point O on a smooth horizontal table. The spring has natural length 2 m and modulus of elasticity 21.6 N. The particle P is placed on the table at the point A, where OA = 2 m. The particle P is now pulled away from O to the point B, where OAB is a straight line with OB = 3.5 m. It is then released from rest.

(*a*) Prove that P moves with simple harmonic motion of period $\frac{\pi}{3}$ s.
(4)

(*b*) Find the speed of P when it reaches A.
(2)

The point C is the mid-point of AB.

(*c*) Find, in terms of π, the time taken for P to reach C for the first time.
(3)

Later in the motion, P collides with a particle Q of mass 0.2 kg which is at rest at A.

After the impact, P and Q coalesce to form a single particle R.

(*d*) Show that R also moves with simple harmonic motion and find the amplitude of this motion.
(7)

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced Level

Wednesday 12 January 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

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When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
The total mark for this paper is 75.

Advice to Candidates

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1. A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is attached to a fixed point A. The particle is moving, with the string taut, in a horizontal circle with centre O vertically below A. The particle is moving with constant angular speed 2.7 rad s^{-1}. Find

(*a*) the tension in the string,

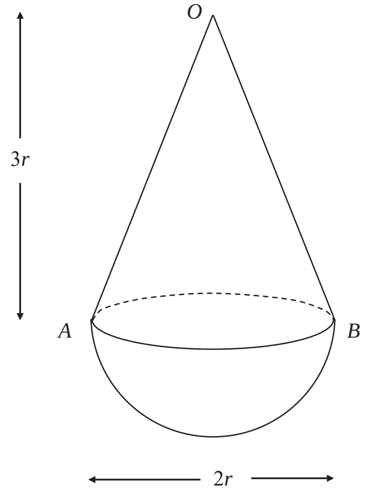
(4)

(*b*) the angle, to the nearest degree, that AP makes with the downward vertical.

(3)

---

2. **Figure 1**



A child's toy consists of a uniform solid hemisphere, of mass M and base radius r, joined to a uniform solid right circular cone of mass m, where $2m < M$. The cone has vertex O, base radius r and height $3r$. Its plane face, with diameter AB, coincides with the plane face of the hemisphere, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of the toy from AB is

$$\frac{3(M-2m)}{8(M+m)}r.$$

(5)

The toy is placed with OA on a horizontal surface. The toy is released from rest and does not remain in equilibrium.

(*b*) Show that $M > 26m$.

(4)

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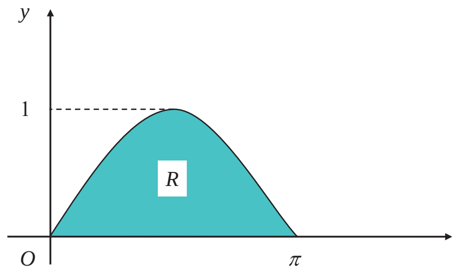
3. **Figure 2**



A uniform lamina occupies the region R bounded by the x-axis and the curve

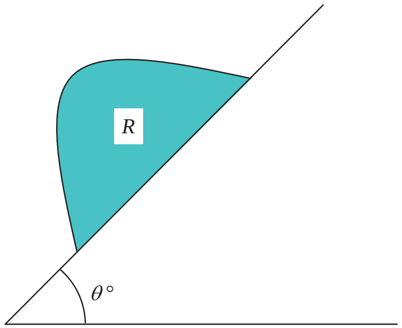
$$y = \sin x, \qquad 0 \le x \le \pi,$$

as shown in Figure 2.

(*a*) Show, by integration, that the y-coordinate of the centre of mass of the lamina is $\frac{\pi}{8}$.

(6)

Figure 3



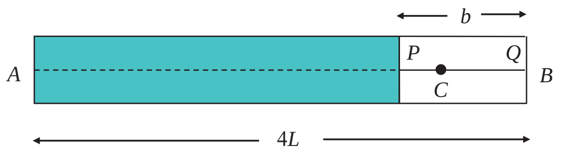
A uniform prism S has cross-section R. The prism is placed with its rectangular face on a table which is inclined at an angle $\theta°$ to the horizontal. The cross-section R lies in a vertical plane as shown in Figure 3. The table is sufficiently rough to prevent S sliding. Given that S does not topple,

(*b*) find the largest possible value of θ.

(3)

4.

Figure 4



In a game at a fair, a small target C moves horizontally with simple harmonic motion between the points A and B, where $AB = 4L$. The target moves inside a box and takes 3 s to travel from A to B. A player has to shoot at C, but C is only visible to the player when it passes a window PQ, where $PQ = b$. The window is initially placed with Q at the point as shown in Figure 4. The target C takes 0.75 s to pass from Q to P.

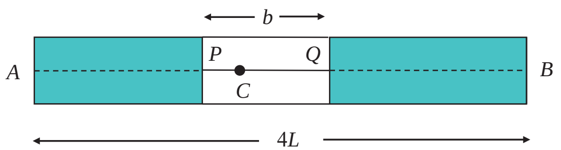
(*a*) Show that $b = (2 - \sqrt{2})L$.

(5)

(*b*) Find the speed of C as it passes P.

(2)

Figure 5



For advanced players, the window PQ is moved to the centre of AB so that $AP = QB$, as shown in Figure 5.

(*c*) Find the time, in seconds to 2 decimal places, taken for C to pass from Q to P in this new position.

(3)

---

5. At time $t = 0$, a particle P is at the origin O, moving with speed 18 m s^{-1} along the x-axis, in the positive x-direction. At time t seconds $(t > 0)$ the acceleration of P has magnitude $\frac{3}{\sqrt{(t+4)}}$ m s^{-2} and is directed towards O.

(*a*) Show that, at time t seconds, the velocity of P is $[30 - 6\sqrt{(t+4)}]$ m s^{-1}.

(5)

(*b*) Find the distance of P from O when P comes to instantaneous rest.

(7)

---

6. A light spring of natural length L has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the spring. The particle is moving vertically. As it passes through the point B below A, where $AB = L$, its speed is $\sqrt{(2gL)}$. The particle comes to instantaneous rest at a point C, $4L$ below A.

(*a*) Show that the modulus of elasticity of the spring is $\frac{8mg}{9}$.

(4)

At the point D the tension in the spring is mg.

(*b*) Show that P performs simple harmonic motion with centre D.

(5)

(*c*) Find, in terms of L and g,

(i) the period of the simple harmonic motion,

(ii) the maximum speed of P.

(5)

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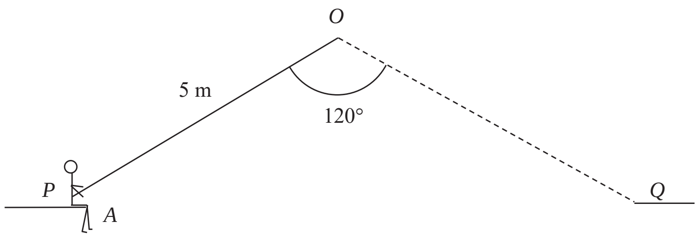
7. **Figure 6**



A trapeze artiste of mass 60 kg is attached to the end A of a light inextensible rope OA of length 5 m. The artiste must swing in an arc of a vertical circle, centre O, from a platform P to another platform Q, where PQ is horizontal. The other end of the rope is attached to the fixed point O which lies in the vertical plane containing PQ, with $\angle POQ = 120°$ and $OP = OQ = 5$ m, as shown in Figure 6.

As part of her act, the artiste projects herself from P with speed $\sqrt{15}$ m s^{-1} in a direction perpendicular to the rope OA and in the plane POQ. She moves in a circular arc towards Q. At the lowest point of her path she catches a ball of mass m kg which is travelling towards her with speed 3 m s^{-1} and parallel to QP. After catching the ball, she comes to rest at the point Q.

By modelling the artiste and the ball as particles and ignoring her air resistance, find

(*a*) the speed of the artiste immediately before she catches the ball,
(4)

(*b*) the value of m,
(7)

(*c*) the tension in the rope immediately after she catches the ball.
(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Tuesday 7 June 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

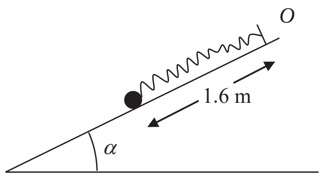
Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. **Figure 1**



A particle of mass 0.8 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point O on a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The particle is held on the plane at a point which is 1.6 m down a line of greatest slope of the plane from O, as shown in Figure 1. The particle is then released from rest.

Find the initial acceleration of the particle.

(Total 6 marks)

---

2. A closed container C consists of a thin uniform hollow hemispherical bowl of radius a, together with a lid. The lid is a thin uniform circular disc, also of radius a. The centre O of the disc coincides with the centre of the hemispherical bowl. The bowl and its lid are made of the same material.

(a) Show that the centre of mass of C is at a distance $\frac{1}{3}a$ from O.

(4)

The container C has mass M. A particle of mass $\frac{1}{2}M$ is attached to the container at a point P on the circumference of the lid. The container is then placed with a point of its curved surface in contact with a horizontal plane. The container rests in equilibrium with P, O and the point of contact in the same vertical plane.

(b) Find, to the nearest degree, the angle made by the line PO with the horizontal.

(5)

(Total 9 marks)

---

3. A light elastic string has natural length $2l$ and modulus of elasticity $4mg$. One end of the string is attached to a fixed point A and the other end to a fixed point B, where A and B lie on a smooth horizontal table, with $AB = 4l$. A particle P of mass m is attached to the mid-point of the string.

The particle is released from rest at the point of the line AB which is $\frac{5l}{3}$ from B. The speed of P at the mid-point of AB is V.

(a) Find V in terms of g and L.

(7)

(b) Explain why V is the maximum speed of P.

(2)

(Total 9 marks)

---

4. A particle P of mass m moves on the smooth inner surface of a spherical bowl of internal radius r. The particle moves with constant angular speed in a horizontal circle, which is at a depth $\frac{1}{2}r$ below the centre of the bowl.

(a) Find the normal reaction of the bowl on P.

(4)

(b) Find the time for P to complete one revolution of its circular path.

(6)

(Total 10 marks)

---

5. A smooth solid sphere, with centre O and radius a, is fixed to the upper surface of a horizontal table. A particle P is placed on the surface of the sphere at a point A, where OA makes an angle α with the upward vertical, and $0 < \alpha < \frac{\pi}{2}$. The particle is released from rest. When OP makes an angle θ with the upward vertical, and P is still on the surface of the sphere, the speed of P is v.

(a) Show that $v^2 = 2ga(\cos \alpha - \cos \theta)$.

(4)

Given that $\cos \alpha = \frac{3}{4}$, find

(b) the value of θ when P loses contact with the sphere,

(5)

(c) the speed of P as it hits the table.

(4)

(Total 13 marks)

---

6. The rise and fall of the water level in a harbour is modelled as simple harmonic motion. On a particular day the maximum and minimum depths of water in the harbour are 10 m and 4 m and these occur at 1100 hours and 1700 hours respectively.

(*a*) Find the speed, in m h^{-1}, at which the water level in the harbour is falling at 1600 hours on this particular day.

(9)

(*b*) Find the total time, between 1100 hours and 2300 hours on this particular day, for which the depth of water in the harbour is less than 5.5 m.

(5)

(Total 14 marks)

---

7. A particle *P* of mass $\frac{1}{3}$ kg moves along the positive *x*-axis under the action of a single force. The force is directed towards the origin *O* and has magnitude $\frac{k}{(x+1)^2}$ N, where $OP = x$ metres and *k* is a constant. Initially *P* is moving away from *O*. At $x = 1$ the speed of *P* is 4 m s^{-1}, and at $x = 8$ the speed of *P* is $\sqrt{2}$ m s^{-1}.

(*a*) Find the value of *k*.

(10)

(*b*) Find the distance of *P* from *O* when *P* first comes to instantaneous rest.

(4)

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Wednesday 25 January 2006 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

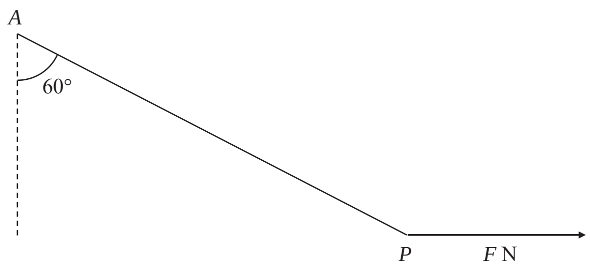
A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N21136A

1. **Figure 1**



A particle P of mass 0.8 kg is attached to one end of a light inelastic string, of natural length 1.2 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point A. A horizontal force of magnitude F newtons is applied to P. The particle P in in equilibrium with the string making an angle 60° with the downward vertical, as shown in Figure 1.

Calculate

(*a*) the value of F,

(3)

(*b*) the extension of the string,

(3)

(*c*) the elasticity stored in the string.

(2)

---

2. A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} and its acceleration is $2 \sin \frac{1}{2} t$ m s^{-2}, both measured in the direction of Ox. Given that $v = 4$ when $t = 0$,

(*a*) find v in terms of t,

(4)

(*b*) calculate the distance travelled by P between the times $t = 0$ and $t = \frac{\pi}{2}$.

(4)

---

3. A rocket is fired vertically upwards with speed U from a point on the Earth's surface. The rocket is modelled as a particle P of constant mass m, and the Earth as a fixed sphere of radius R. At a distance x from the centre of the Earth, the speed of P is v. The only force acting on P is directed towards the centre of the Earth and has magnitude $\frac{cm}{x^2}$, where c is a constant.

(*a*) Show that $v^2 = U^2 + 2c\left(\frac{1}{x} - \frac{1}{R}\right)$.

(5)

The kinetic energy of P at $x = 2R$ is half of its kinetic energy at $x = R$.

(*b*) Find c in terms of U and R.

(3)

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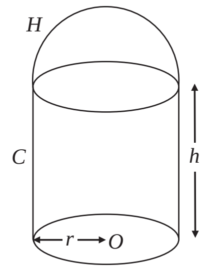
4. **Figure 2**



A body consists of a uniform solid circular cylinder C, together with a uniform solid hemisphere H which is attached to C. The plane face of H coincides with the upper plane face of C, as shown in Figure 2. The cylinder C has base radius r, height h and mass $3M$. The mass of H is $2M$. The point O is the centre of the base of C.

(*a*) Show that the distance of the centre of mass of the body from O is

$$\frac{14h + 3r}{20}.$$

(5)

The body is placed with its plane face on a rough plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{4}{3}$. The plane is sufficiently rough to prevent slipping. Given that the body is on the point of toppling,

(*b*) find h in terms of r.

(4)

---

5. A light elastic string of natural length l has one end attached to a fixed point A. A particle P of mass m is attached tot he other end of the string and hangs in equilibrium at the point O, where $AO = \frac{5}{4}l$.

(*a*) Find the modulus of the elasticity of the string.
(2)

The particle P is then pulled down and released from rest. At time t the length of the string is $\frac{5l}{4} + x$.

(*b*) Prove that, while the string is taut,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{4gx}{l}.$$
(5)

When P is released, $AP = \frac{7}{4}l$. The point B is a distance l vertically below A.

(*c*) Find the speed of P at B.
(4)

(*d*) Describe briefly the motion of P after it has passed through B for the first time until it next passes through O.
(2)

---

6. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m which is hanging freely at rest at point B. The particle P is projected horizontally from B with speed $\sqrt{(3gl)}$. When AP makes an angle θ with the downward vertical and the string remains taut, the tension in the string is T.

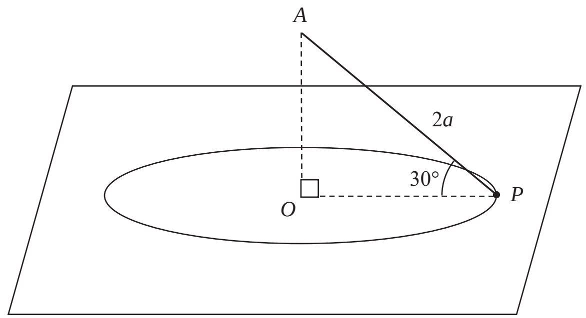
(*a*) Show that $T = mg(1 + 3\cos\theta)$.
(6)

(*b*) Find the speed of P at the instant when the string becomes slack.
(3)

(*c*) Find the maximum height above the level of B reached by P.
(5)

7.

Figure 3



A particle P of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is fixed to a point A which is vertically above the point O on a smooth horizontal table. The particle P remains in contact with the surface of the table and moves in a circle with centre O and with angular speed $\sqrt{\left(\frac{kg}{3a}\right)}$, where k is a constant. Throughout the motion the string remains taut and $\angle APO = 30°$, as shown in Figure 3.

(*a*) Show that the tension in the string is $\frac{2kmg}{3}$.
(3)

(*b*) Find, in terms of m, g and k, the normal reaction between P and the table.
(3)

(*c*) Deduce the range of possible values of k.
(2)

The angular speed of P is changed to $\sqrt{\left(\frac{2g}{a}\right)}$. The particle P now moves in a horizontal circle above the table. The centre of this circle is X.

(*d*) Show that X is the mid-point of OA.
(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Monday 12 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N24324A

1. A uniform solid is formed by rotating the region enclosed between the curve with equation $y = \sqrt{x}$, the x-axis and the line $x = 4$, through one complete revolution about the x-axis. Find the distance of the centre of mass of the solid from the origin O.

(5)

2. A bowl consists of a uniform solid metal hemisphere, of radius a and centre O, from which is removed the solid hemisphere of radius $\frac{1}{2}a$ with the same centre O.

(*a*) Show that the distance of the centre of mass of the bowl from O is $\dfrac{45}{112}a$.

(5)

The bowl is fixed with its plane face uppermost and horizontal. It is now filled with liquid. The mass of the bowl is M and the mass of the liquid is kM, where k is a constant. Given that the distance of the centre of mass of the bowl and liquid together from O is $\dfrac{17}{48}a$,

(*b*) find the value of k.

(5)

3. A particle P of mass 0.2 kg oscillates with simple harmonic motion between the points A and B, coming to rest at both points. The distance AB is 0.2 m, and P completes 5 oscillations every second.

(*a*) Find, to 3 significant figures, the maximum resultant force exerted on P.

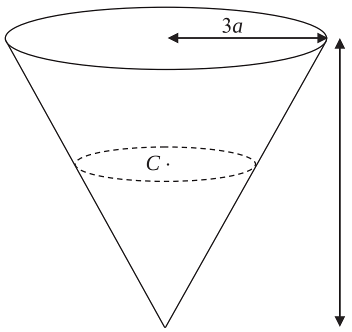
(6)

When the particle is at A, it is struck a blow in the direction BA. The particle now oscillates with simple harmonic motion with the same frequency as previously but twice the amplitude.

(*b*) Find, to 3 significant figures, the speed of the particle immediately after it has been struck.

(5)

4. **Figure 1**



A hollow cone, of base radius $3a$ and height $4a$, is fixed with its axis vertical and vertex V downwards, as shown in Figure 1. A particle moves in a horizontal circle with centre C, on the smooth inner surface of the cone with constant angular speed $\sqrt{\frac{8g}{9a}}$.

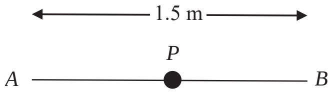
Find the height of C above V.

(11)

---

5. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N. A particle P of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B, where AB is horizontal and $AB = 1.5$ m.

Figure 2



The particle is held at the mid-point of AB. The particle is released from rest, as shown in Figure 2.

(*a*) Find the speed of P when it has fallen a distance of 1 m.

(6)

Given instead that P hangs in equilibrium vertically below the mid-point of AB, with $\angle APB = 2\alpha$,

(*b*) show that $\tan\alpha + 5\sin\alpha = 5$.

(6)

---

6. A particle moving in a straight line starts from rest at the point O at time $t = 0$. At time t seconds, the velocity v m s^{-1} of the particle is given by

$$v = 3t(t-4), \quad 0 \le t \le 5,$$
$$v = 75t^{-1}, \quad 5 \le t \le 10.$$

(*a*) Sketch a velocity-time graph for the particle for $0 \le t \le 10$.

(3)

(*b*) Find the set of values of t for which the acceleration of the particle is positive.

(2)

(*c*) Show that the total distance travelled by the particle in the interval $0 \le t \le 5$ is 39 m.

(3)

(*d*) Find, to 3 significant figures, the value of t at which the particle returns to O.

(5)

---

7. One end of a light inextensible string of length l is attached to a particle P of mass m. The other end is attached to a fixed point A. The particle is hanging freely at rest with the string vertical when it is projected horizontally with speed $\sqrt{\frac{5gl}{2}}$.

(*a*) Find the speed of P when the string is horizontal.

(4)

When the string is horizontal it comes into contact with a small smooth fixed peg which is at the point B, where AB is horizontal, and $AB < l$. Given that the particle then describes a complete semicircle with centre B,

(*b*) find the least possible value of the length AB.

(9)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Thursday 25 January 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

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N23822A

1. A particle P moves along the x-axis. At time $t = 0$, P passes through the origin O, moving in the positive x-direction. At time t seconds, the velocity of P is $v \text{ m s}^{-1}$ and $OP = x$ metres. The acceleration of P is $\frac{1}{12}(30 - x) \text{ m s}^{-2}$, measured in the positive x-direction.

(*a*) Give a reason why the maximum speed of P occurs when $x = 30$.

(1)

Given that the maximum speed of P is 10 m s^{-1},

(*b*) find an expression for v^2 in terms of x.

(5)

2.

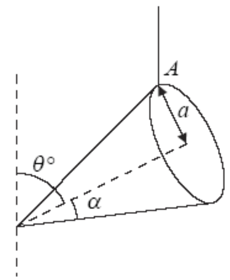


Figure 1

A uniform solid right circular cone has base radius a and semi-vertical angle α, where $\tan \alpha = \frac{1}{3}$. The cone is freely suspended by a string attached at a point A on the rim of its base, and hangs in equilibrium with its axis of symmetry making an angle of $\theta°$ with the upward vertical, as shown in Figure 1.

Find, to one decimal place, the value of θ.

(5)

3. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point O on a rough horizontal table. The particle is projected along the surface of the table from O with speed $\sqrt{(2ag)}$. At its furthest point from O, the particle is at the point A, where $OA = \frac{4}{3}a$.

(*a*) Find, in terms of m, g and a, the elastic energy stored in the string when P is at A.

(3)

(*b*) Using the work-energy principle, or otherwise, find the coefficient of friction between P and the table.

(6)

4.

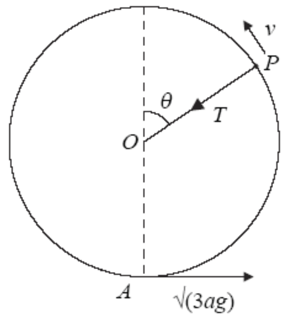


Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a point O. The point A is vertically below O, and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{(3ag)}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v, as shown in Figure 2.

(*a*) Find, in terms of a, g and θ, an expression for v^2.

(3)

(*b*) Show that $T = (1 - 3\cos\theta)mg$.

(3)

The string becomes slack when P is at the point B.

(*c*) Find, in terms of a, the vertical height of B above A.

(2)

After the string becomes slack, the highest point reached by P is C.

(*d*) Find, in terms of a, the vertical height of C above B.

(5)

5.

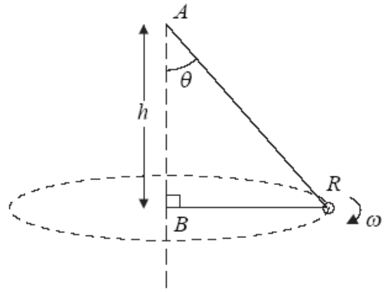


Figure 3

One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where $AB = h$. A small smooth ring R of mass m is threaded on the string. The ring R moves in a horizontal circle with centre B, as shown in Figure 3. The upper section of the string makes a constant angle θ with the downward vertical and R moves with constant angular speed ω. The ring is modelled as a particle.

(*a*) Show that $\omega^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$.

(7)

(*b*) Deduce that $\omega > \sqrt{\frac{2g}{h}}$.

(2)

Given that $\omega = \sqrt{\frac{3g}{h}}$,

(*c*) find, in terms of m and g, the tension in the string.

(4)

6.

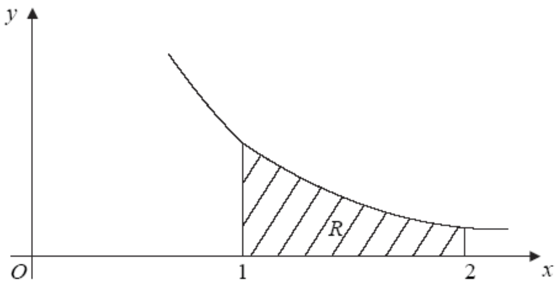


Figure 4

The shaded region R is bounded by the curve with equation $y = \dfrac{1}{2x^2}$, the x-axis and the lines $x = 1$ and $x = 2$, as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid S has the shape made by rotating R through 360° about the x-axis.

(*a*) Show that the centre of mass of S is $\frac{2}{7}$ m from its larger plane face.

(6)

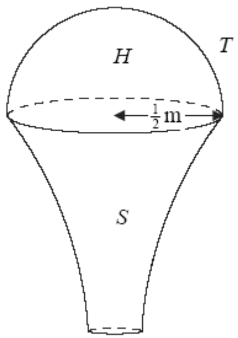


Figure 5

A sporting trophy T is a uniform solid hemisphere H joined to the solid S. The hemisphere has radius $\frac{1}{2}$ m and its plane face coincides with the larger plane face of S, as shown in Figure 5. Both H and S are made of the same material.

(*b*) Find the distance of the centre of mass of T from its plane face.

(7)

7. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A. In its equilibrium position, P is 0.85 m vertically below A.

(*a*) Show that $\lambda = 39.2$.

(2)

The particle is now displaced to a point B, 0.95 m vertically below A, and released from rest.

(*b*) Prove that, while the string remains stretched, P moves with simple harmonic motion of period $\dfrac{\pi}{7}$ s.

(6)

(*c*) Calculate the speed of P at the instant when the string first becomes slack.

(3)

The particle first comes to instantaneous rest at the point C.

(*d*) Find, to 3 significant figures, the time taken for P to move from B to C.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Thursday 14 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26116A

1. The rudder on a ship is modelled as a uniform plane lamina having the same shape as the region R which is enclosed between the curve with equation $y = 2x - x^2$ and the x-axis.

(*a*) Show that the area of R is $\frac{4}{3}$.

(4)

(*b*) Find the coordinates of the centre of mass of the lamina.

(5)

---

2. An open container C is modelled as a thin uniform hollow cylinder of radius h and height h with a base but no lid. The centre of the base is O.

(*a*) Show that the distance of the centre of mass of C from O is $\frac{1}{3}h$.

(5)

The container is filled with uniform liquid. Given that the mass of the container is M and the mass of the liquid is M,

(*b*) find the distance of the centre of mass of the filled container from O.

(5)

---

3. A spacecraft S of mass m moves in a straight line towards the centre of the earth. The earth is modelled as a fixed sphere of radius R. When S is at a distance x from the centre of the earth, the force exerted by the earth on S is directed towards the centre of the earth and has magnitude $\dfrac{k}{x^2}$, where k is a constant.

(*a*) Show that $k = mgR^2$.

(2)

Given that S starts from rest when its distance from the centre of the earth is $2R$, and that air resistance can be ignored,

(*b*) find the speed of S as it crashes into the surface of the earth.

(7)

---

4. A light inextensible string of length l has one end attached to a fixed point A. The other end is attached to a particle P of mass m. The particle moves with constant speed v in a horizontal circle with the string taut. The centre of the circle is vertically below A and the radius of the circle is r.

Show that

$$gr^2 = v^2\sqrt{(l^2 - r^2)}.$$

(9)

---

5. A particle P moves on the x-axis with simple harmonic motion about the origin O as centre. When P is a distance 0.04 m from O, its speed is 0.2 m s^{-1} and the magnitude of its acceleration is 1 m s^{-2}.

(*a*) Find the period of the motion.

(3)

The amplitude of the motion is a metres.

Find

(*b*) the value of a,

(3)

(*c*) the total time, within one complete oscillation, for which the distance OP is greater than $\frac{1}{2}a$ metres.

(5)

---

6. A particle P is free to move on the smooth inner surface of a fixed thin hollow sphere of internal radius a and centre O. The particle passes through the lowest point of the spherical surface with speed U. The particle loses contact with the surface when OP is inclined at an angle α to the upward vertical.

(*a*) Show that $U^2 = ag(2 + 3\cos\alpha)$.

(7)

The particle has speed W as it passes through the level of O. Given that $\cos\alpha = \frac{1}{\sqrt{3}}$,

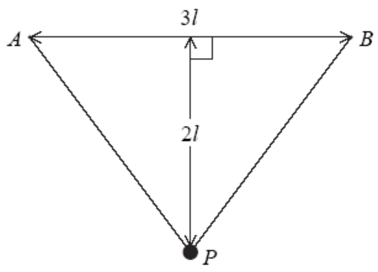
(*b*) show that

$$W^2 = ag\sqrt{3}.$$

(5)

---

7. **Figure 1**



A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in Figure 1,

(*a*) show that $\lambda = \frac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(*b*) Show that P comes to instantaneous rest on the line AB.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced

Thursday 24 January 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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Advice to Candidates

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N26363A

1. A light elastic string of natural length 0.4 m has one end *A* attached to a fixed point. The other end of the string is attached to a particle *P* of mass 2 kg. When *P* hangs in equilibrium vertically below *A*, the length of the string is 0.56 m.

(*a*) Find the modulus of elasticity of the string.
(3)

A horizontal force is applied to *P* so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(*b*) Find the angle θ.
(3)

---

2. A particle *P* of mass 0.1 kg moves in a straight line on a smooth horizontal table. When *P* is a distance *x* metres from a fixed point *O* on the line, it experiences a force of magnitude $\frac{16}{5x^2}$ N away from *O* in the direction *OP*. Initially *P* is at a point 2m from *O* and is moving towards *O* with speed 8 m s^{-1}.

Find the distance of *P* from *O* when *P* first comes to rest.
(8)

3.

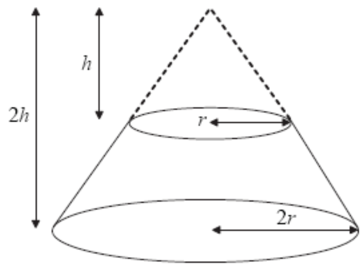

Figure 1

A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h, which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$.

(5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle $\theta°$ to the horizontal. The table is sufficiently rough to prevent S from slipping.

Given that $h = 2r$,

(b) find the greatest value of θ for which S does not topple.

(3)

---

4. A particle P of mass m lies on a smooth plane inclined at an angle 30° to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(a) find V in terms of a and g,

(6)

(b) find, in terms of a and g, the speed of P when the string first becomes slack.

(4)

---

5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1}.

Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of mg, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

6.

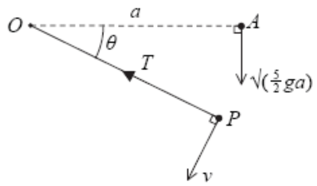

Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time $t = 0$, P is projected vertically downwards with speed $\sqrt{(\frac{5}{2}ga)}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Figure 2.

(*a*) Show that $v^2 = \frac{ga}{2}(5 + 4\sin\theta)$.

(3)

(*b*) Find T in terms of m, g and θ.

(3)

The string becomes slack when $\theta = \alpha$.

(*c*) Find the value of α.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(*d*) find the tension in the string when P is at the point C.

(6)

7. A particle P of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point A. When P hangs freely below A in equilibrium, P is at the point E, 1.2 m below A. The particle is now pulled down to a point B which is 0.4 m vertically below E and released from rest.

(*a*) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(*b*) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(*c*) Find the speed of P when the string first becomes slack.

(3)

(*d*) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Friday 6 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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Answers without working may not gain full credit.

N29501A

1.

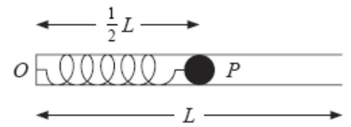

Figure 1

A light elastic spring, of natural length L and modulus of elasticity λ, has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L.

The tube is placed horizontally and P is held inside the tube with $OP = \frac{1}{2}L$, as shown in Figure 1. The particle P is released and passes through the open end of the tube with speed $\sqrt{(2gL)}$.

(*a*) Show that $\lambda = 8mg$.
(4)

The tube is now fixed vertically and P is held inside the tube with $OP = \frac{1}{2}L$ and P above O. The particle P is released and passes through the open top of the tube with speed u.

(*b*) Find u.
(5)

---

2. A particle P moves with simple harmonic motion and comes to rest at two points A and B which are 0.24 m apart on a horizontal line. The time for P to travel from A to B is 1.5 s. The midpoint of AB is O. At time $t = 0$, P is moving through O, towards A, with speed u m s^{-1}.

(*a*) Find the value of u.
(4)

(*b*) Find the distance of P from B when $t = 2$ s.
(5)

(*c*) Find the speed of P when $t = 2$ s.
(2)

3.

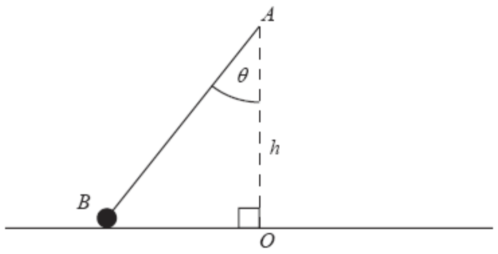

Figure 2

Figure 2 shows a particle B, of mass m, attached to one end of a light elastic string. The other end of the string is attached to a fixed point A, at a distance h vertically above a smooth horizontal table. The particle moves on the table in a horizontal circle with centre O, where O is vertically below A. The string makes a constant angle θ with the downward vertical and B moves with constant angular speed ω about OA.

(*a*) Show that $\omega^2 \leq \dfrac{g}{h}$.

(8)

The elastic string has natural length h and modulus of elasticity 2 mg.

Given that $\tan \theta = \dfrac{3}{4}$,

(*b*) find ω in terms of g and h.

(5)

4.

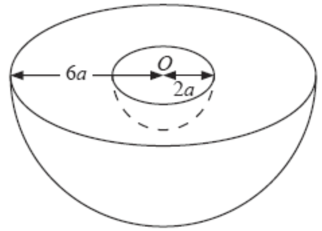

Figure 3

A uniform solid hemisphere, of radius $6a$ and centre O, has a solid hemisphere of radius $2a$, and centre O, removed to form a bowl B as shown in Figure 3.

(*a*) Show that the centre of mass of B is $\dfrac{30}{13}a$ from O.

(5)

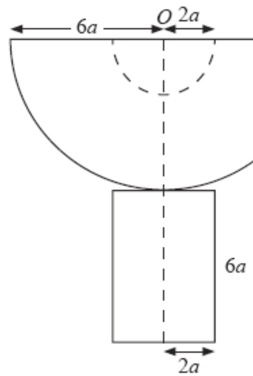

Figure 4

The bowl B is fixed to a plane face of a uniform solid cylinder made from the same material as B. The cylinder has radius $2a$ and height $6a$ and the combined solid S has an axis of symmetry which passes through O, as shown in Figure 4.

(*b*) Show that the centre of mass of S is $\dfrac{201}{61}a$ from O.

(4)

The plane surface of the cylindrical base of S is placed on a rough plane inclined at 12° to the horizontal. The plane is sufficiently rough to prevent slipping.

(*c*) Determine whether or not S will topple.

(4)

5. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is released from rest with the string taut and OP horizontal.

(*a*) Find the tension in the string when OP makes an angle of 60° with the downward vertical.

(6)

A particle Q of mass $3m$ is at rest at a distance a vertically below O. When P strikes Q the particles join together and the combined particle of mass $4m$ starts to move in a vertical circle with initial speed u.

(*b*) Show that $u = \sqrt{\left(\frac{ga}{8}\right)}$.

(3)

The combined particle comes to instantaneous rest at A.

(*c*) Find

(i) the angle that the string makes with the downward vertical when the combined particle is at A,

(ii) the tension in the string when the combined particle is at A.

(6)

6. A particle P of mass 0.5 kg moves along the positive x-axis. It moves away from the origin O under the action of a single force directed away from O. When $OP = x$ metres, the magnitude of the force is $\frac{3}{(x+1)^3}$ N and the speed of P is v m s^{-1}.

Initially P is at rest at O.

(*a*) Show that $v^2 = 6\left(1 - \frac{1}{(x+1)^2}\right)$.

(6)

(*b*) Show that the speed of P never reaches $\sqrt{6}$ m s^{-1}.

(1)

(*c*) Find x when P has been moving for 2 seconds.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Thursday 29 January 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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The total mark for this paper is 75.

Advice to Candidates

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N29495A

1. A particle P of mass 3 kg is moving in a straight line. At time t seconds, $0 \le t \le 4$, the only force acting on P is a resistance to motion of magnitude $\left(9 + \dfrac{15}{(t+1)^2}\right)$N. At time t seconds the velocity of P is v m s^{-1}. When $t = 4$, $v = 0$.

Find the value of v when $t = 0$.

(7)

2.

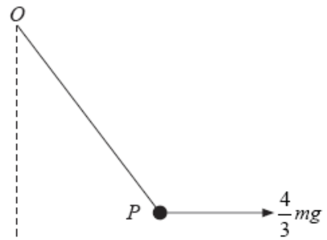

Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P.

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(*a*) the tension in the string,

(5)

(*b*) the elastic energy stored in the string.

(4)

---

3. A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ. Given that P remains at rest relative to the disc, find the least possible value of μ.

(7)

---

4. A small shellfish is attached to a wall in a harbour. The rise and fall of the water level is modelled as simple harmonic motion and the shellfish as a particle. On a particular day the minimum depth of water occurs at 10 00 hours and the next time that this minimum depth occurs is at 22 30 hours. The shellfish is fixed in a position 5 m above the level of the minimum depth of the water and 11 m below the level of the maximum depth of the water. Find

(*a*) the speed, in metres per hour, at which the water level is rising when it reaches the shellfish,

(7)

(*b*) the earliest time after 10 00 hours on this day at which the water reaches the shellfish.

(4)

5.

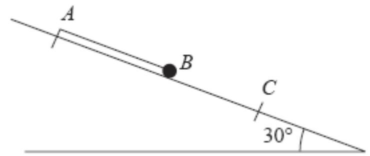

Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2.

Find

(*a*) the length AC,

(5)

(*b*) the greatest speed attained by B as it moves from its initial position to C.

(7)

6.

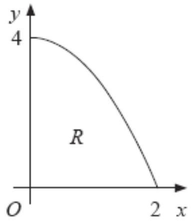

Figure 3

The region R is bounded by part of the curve with equation $y = 4 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 3. The unit of length on both axes is one metre. A uniform solid S is formed by rotating R through 360° about the x-axis.

(*a*) Show that the centre of mass of S is $\frac{5}{8}$ m from O.

(10)

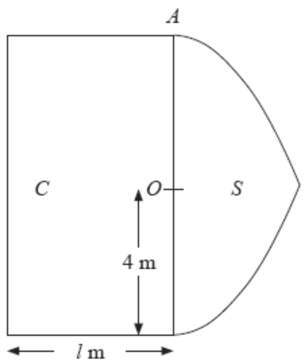

Figure 4

Figure 4 shows a cross section of a uniform solid P consisting of two components, a solid cylinder C and the solid S. The cylinder C has radius 4 m and length l metres. One end of C coincides with the plane circular face of S. The point A is on the circumference of the circular face common to C and S. When the solid P is freely suspended from A, the solid P hangs with its axis of symmetry horizontal.

(*b*) Find the value of l.

(4)

7.

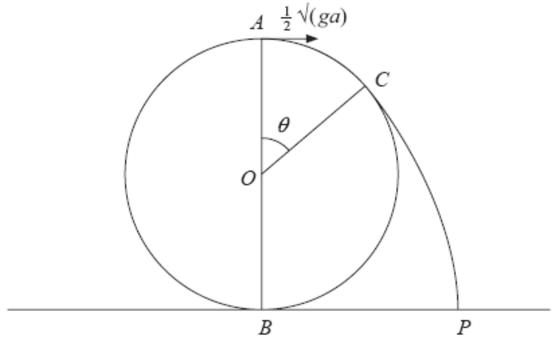

Figure 5

A particle is projected from the highest point A on the outer surface of a fixed smooth sphere of radius a and centre O. The lowest point B of the sphere is fixed to a horizontal plane. The particle is projected horizontally from A with speed $\frac{1}{2}\sqrt{(ga)}$. The particle leaves the surface of the sphere at the point C, where $\angle AOC = \theta$, and strikes the plane at the point P, as shown in Figure 5.

(*a*) Show that $\cos\theta = \frac{3}{4}$.

(7)

(*b*) Find the angle that the velocity of the particle makes with the horizontal as it reaches P.

(8)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Thursday 11 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

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Advice to Candidates

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1. A light elastic string has natural length 8 m and modulus of elasticity 80 N. The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ.

(*a*) Calculate the weight of the particle.
(6)

(*b*) Calculate the elastic energy in the string when the particle is in this position.
(3)

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2. [*The centre of mass of a uniform hollow cone of height h is* $\frac{1}{3}h$ *above the base on the line from the centre of the base to the vertex*.]

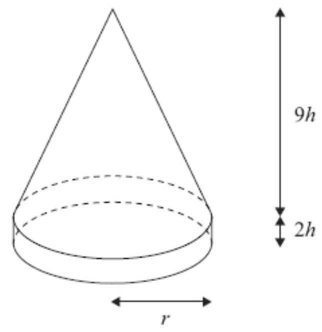

Figure 1

A marker for the route of a charity walk consists of a uniform hollow cone fixed on to a uniform solid cylindrical ring, as shown in Figure 1. The hollow cone has base radius r, height $9h$ and mass m. The solid cylindrical ring has outer radius r, height $2h$ and mass $3m$. The marker stands with its base on a horizontal surface.

(*a*) Find, in terms of h, the distance of the centre of mass of the marker from the horizontal surface.
(5)

When the marker stands on a plane inclined at $\arctan \frac{1}{12}$ to the horizontal it is on the point of toppling over. The coefficient of friction between the marker and the plane is large enough to be certain that the marker will not slip.

(*b*) Find h in terms of r.
(3)

3.

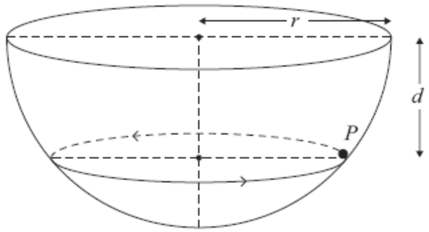

Figure 2

A particle P of mass m moves on the smooth inner surface of a hemispherical bowl of radius r. The bowl is fixed with its rim horizontal as shown in Figure 2. The particle moves with constant angular speed $\sqrt{\left(\frac{3g}{2r}\right)}$ in a horizontal circle at depth d below the centre of the bowl.

(a) Find, in terms of m and g, the magnitude of the normal reaction of the bowl on P. **(4)**

(b) Find d in terms of r. **(4)**

---

4. The finite region bounded by the x-axis, the curve $y = \frac{1}{x^2}$, the line $x = \frac{1}{4}$ and the line $x = 1$, is rotated through one complete revolution about the x-axis to form a uniform solid of revolution.

(a) Show that the volume of the solid is 21π. **(4)**

(b) Find the coordinates of the centre of mass of the solid. **(5)**

---

5. One end of a light inextensible string of length l is attached to a fixed point A. The other end is attached to a particle P of mass m, which is held at a point B with the string taut and AP making an angle arccos $\frac{1}{4}$, with the downward vertical. The particle is released from rest. When AP makes an angle θ with the downward vertical, the string is taut and the tension in the string is T.

(a) Show that

$$T = 3mg\cos\theta - \frac{mg}{2}.$$

(6)

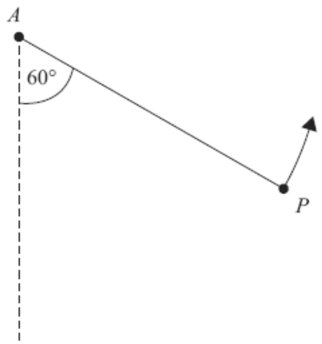

Figure 3

At an instant when AP makes an angle of 60° to the downward vertical, P is moving upwards, as shown in Figure 3. At this instant the string breaks. At the highest point reached in the subsequent motion, P is at a distance d below the horizontal through A.

(b) Find d in terms of l. **(5)**

---

6. A cyclist and her bicycle have a combined mass of 100 kg. She is working at a constant rate of 80 W and is moving in a straight line on a horizontal road. The resistance to motion is proportional to the square of her speed. Her initial speed is 4 m s^{-1} and her maximum possible speed under these conditions is 20 m s^{-1}. When she is at a distance x m from a fixed point O on the road, she is moving with speed v m s^{-1} away from O.

(*a*) Show that

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{8000 - v^3}{10\,000v}$$

(5)

(*b*) Find the distance she travels as her speed increases from 4 m s^{-1} to 8 m s^{-1}.

(5)

(*c*) Use the trapezium rule, with 2 intervals, to estimate how long it takes for her speed to increase from 4 m s^{-1} to 8 m s^{-1}.

(4)

7.

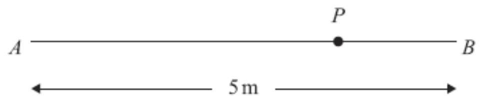

Figure 4

A and B are two points on a smooth horizontal floor, where $AB = 5$ m.

A particle P has mass 0.5 kg. One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N, are attached to P and B, as shown in Figure 4.

(*a*) Find the extensions in the two springs when the particle is at rest in equilibrium.

(5)

Initially P is at rest in equilibrium. It is then set in motion and starts to move towards B. In the subsequent motion P does not reach A or B.

(*b*) Show that P oscillates with simple harmonic motion about the equilibrium position.

(4)

(*c*) Given that the initial speed of P is $\sqrt{10}$ m s^{-1}, find the proportion of time in each complete oscillation for which P stays within 0.25 m of the equilibrium position.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 29 January 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35105A

1. A particle P of mass 0.5 kg is moving along the positive x-axis. At time t seconds, P is moving under the action of a single force of magnitude $[4 + (\cos \pi t)]$ N, directed away from the origin. When $t = 1$, the particle P is moving away from the origin with speed 6 m s^{-1}.

Find the speed of P when $t = 1.5$, giving your answer to 3 significant figures.

(7)

---

2. A particle P moves in a straight line with simple harmonic motion of period 2.4 s about a fixed origin O. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, P is at O. When $t = 0.4$, $v = 4$. Find

(*a*) the greatest speed of P,

(7)

(*b*) the magnitude of the greatest acceleration of P.

(2)

3.

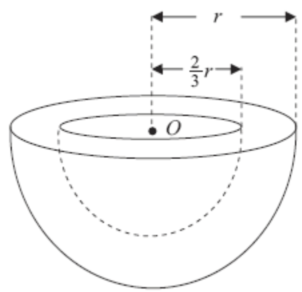

Figure 1

A bowl B consists of a uniform solid hemisphere, of radius r and centre O, from which is removed a solid hemisphere, of radius $\frac{2}{3}r$ and centre O, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of B from O is $\frac{65}{152}r$.

(5)

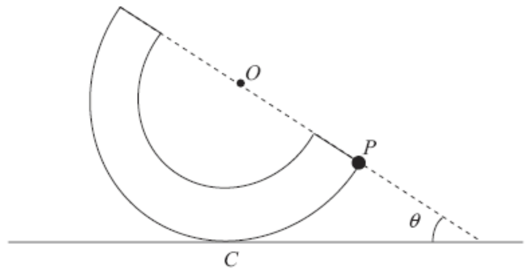

Figure 2

The bowl B has mass M. A particle of mass kM is attached to a point P on the outer rim of B. The system is placed with a point C on its outer curved surface in contact with a horizontal plane. The system is in equilibrium with P, O and C in the same vertical plane. The line OP makes an angle θ with the horizontal as shown in Figure 2. Given that $\tan \theta = \frac{4}{5}$,

(*b*) find the exact value of k.

(5)

4.

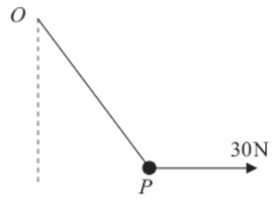

Figure 3

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 30 N is applied to P, as shown in Figure 3. The particle P is in equilibrium and the elastic energy stored in the string is 10 J.

Calculate the length OP.

(10)

5.

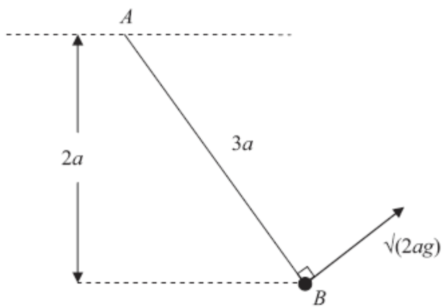

Figure 4

One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The particle is held in equilibrium at a distance $2a$ below the horizontal through A, with the string taut. The particle is then projected with speed $\sqrt{(2ag)}$, in the direction perpendicular to AB, in the vertical plane containing A and B, as shown in Figure 4. In the subsequent motion the string remains taut. When AB is at an angle θ below the horizontal, the speed of the particle is v and the tension in the string is T.

(*a*) Show that $v^2 = 2ag(3 \sin \theta - 1)$.

(5)

(*b*) Find the range of values of T.

(6)

---

6. A bend of a race track is modelled as an arc of a horizontal circle of radius 120 m. The track is not banked at the bend. The maximum speed at which a motorcycle can be ridden round the bend without slipping sideways is 28 m s^{-1}. The motorcycle and its rider are modelled as a particle and air resistance is assumed to be negligible.

(*a*) Show that the coefficient of friction between the motorcycle and the track is $\frac{2}{3}$.

(6)

The bend is now reconstructed so that the track is banked at an angle α to the horizontal. The maximum speed at which the motorcycle can now be ridden round the bend without slipping sideways is 35 m s^{-1}. The radius of the bend and the coefficient of friction between the motorcycle and the track are unchanged.

(*b*) Find the value of $\tan \alpha$.

(8)

---

7. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(*a*) Show that

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}.$$

(4)

(*b*) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D.

(*c*) Find the magnitude of the acceleration of P at D.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6679/01

Edexcel GCE

Mechanics M3

Advanced

Tuesday 15 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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The total mark for this paper is 75.

Advice to Candidates

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1.

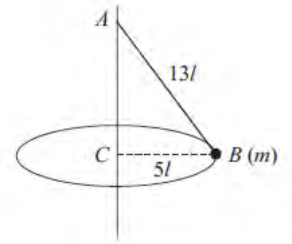

Figure 1

A garden game is played with a small ball B of mass m attached to one end of a light inextensible string of length $13l$. The other end of the string is fixed to a point A on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius $5l$ and centre C, where C is vertically below A. Modelling the ball as a particle, find

(*a*) the tension in the string,

(3)

(*b*) the speed of the ball.

(4)

---

2. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P. The magnitude of this force is inversely proportional to x^2.

At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a sphere of radius R.

(*a*) Prove that the magnitude of the gravitational force on P is $\dfrac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U.

(*b*) Find U in terms of g and R.

(7)

3.

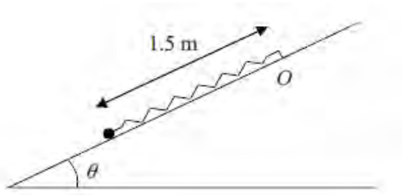

Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(9)

4.

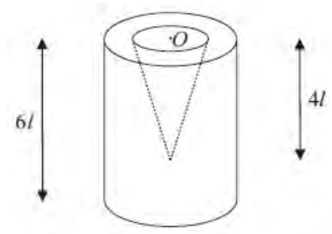

Figure 3

A container is formed by removing a right circular solid cone of height $4l$ from a uniform solid right circular cylinder of height $6l$. The centre O of the plane face of the cone coincides with the centre of a plane face of the cylinder and the axis of the cone coincides with the axis of the cylinder, as shown in Figure 3. The cylinder has radius $2l$ and the base of the cone has radius l.

(a) Find the distance of the centre of mass of the container from O.

(6)

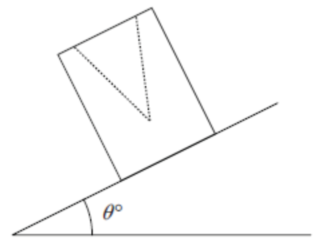

Figure 4

The container is placed on a plane which is inclined at an angle $\theta°$ to the horizontal. The open face is uppermost, as shown in Figure 4. The plane is sufficiently rough to prevent the container from sliding. The container is on the point of toppling.

(b) Find the value of θ.

(4)

5.

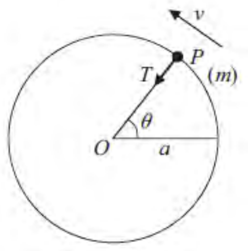

Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at the point O. The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u, where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T, as shown in Figure 5.

(a) Find, in terms of a, g and θ, an expression for v^2.
(3)

(b) Find, in terms of m, g and θ, an expression for T.
(4)

(c) Prove that P moves in a complete circle.
(3)

(d) Find the maximum speed of P.
(2)

6. At time $t = 0$, a particle P is at the origin O moving with speed 2 m s^{-1} along the x-axis in the positive x-direction. At time t seconds ($t > 0$), the acceleration of P has magnitude $\frac{3}{(t+1)^2}$ and is directed towards O.

(a) Show that at time t seconds the velocity of P is $\left(\frac{3}{t+1} - 1\right)$ m s^{-1}.
(5)

(b) Find, to 3 significant figures, the distance of P from O when P is instantaneously at rest.
(7)

7. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A. A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O, vertically below A.

(a) Find the distance AO.
(3)

The particle is now raised to point C vertically below A, where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\frac{a}{g}}$.
(5)

It is given that $1OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.
(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D, moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.
(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 28 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35409A

1. A particle P moves on the positive x-axis. When the distance of P from the origin O is x metres, the acceleration of P is $(7 - 2x)$ m s^{-2}, measured in the positive x-direction. When $t = 0$, P is at O and is moving in the positive x-direction with speed 6 m s^{-1}. Find the distance of P from O when P first comes to instantaneous rest.

(6)

2.

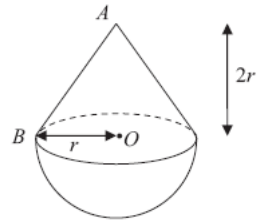

Figure 1

A toy is formed by joining a uniform solid hemisphere, of radius r and mass $4m$, to a uniform right circular solid cone of mass km. The cone has vertex A, base radius r and height $2r$. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the hemisphere is O and OB is a radius of its plane face as shown in Figure 1. The centre of mass of the toy is at O.

(a) Find the value of k.

(4)

A metal stud of mass λm is attached to the toy at A. The toy is now suspended by a light string attached to B and hangs freely at rest. The angle between OB and the vertical is 30°.

(b) Find the value of λ.

(4)

3.

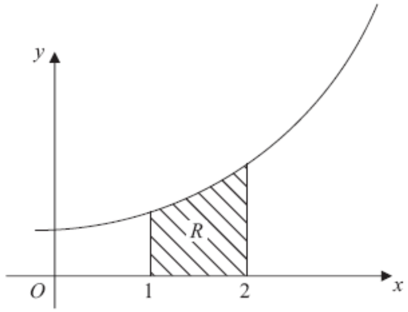

Figure 2

The region R is bounded by the curve with equation $y = e^x$, the line $x = 1$, the line $x = 2$ and the x-axis as shown in Figure 2. A uniform solid S is formed by rotating R through 2π about the x-axis.

(*a*) Show that the volume of S is $\frac{1}{2}\pi(e^4 - e^2)$.

(4)

(*b*) Find, to 3 significant figures, the x-coordinate of the centre of mass of S.

(6)

---

4. A particle P moves along the x-axis. At time t seconds its displacement, x metres, from the origin O is given by $x = 5 \sin(\frac{1}{3}\pi t)$.

(*a*) Prove that P is moving with simple harmonic motion.

(3)

(*b*) Find the period and the amplitude of the motion.

(2)

(*c*) Find the maximum speed of P.

(2)

The points A and B on the positive x-axis are such that $OA = 2$ m and $OB = 3$ m.

(*d*) Find the time taken by P to travel directly from A to B.

(4)

5.

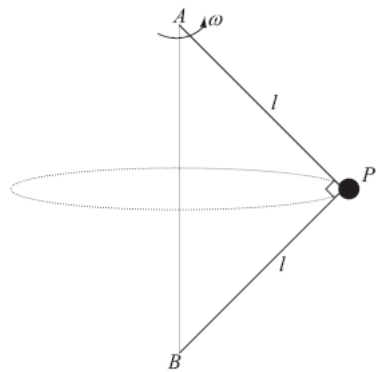

Figure 3

A small ball P of mass m is attached to the ends of two light inextensible strings of length l. The other ends of the strings are attached to fixed points A and B, where A is vertically above B. Both strings are taut and AP is perpendicular to BP as shown in Figure 3. The system rotates about the line AB with constant angular speed ω. The ball moves in a horizontal circle.

(*a*) Find, in terms of m, g, l and ω, the tension in AP and the tension in BP.

(8)

(*b*) Show that $\omega^2 > \dfrac{g\sqrt{2}}{l}$.

(2)

6.

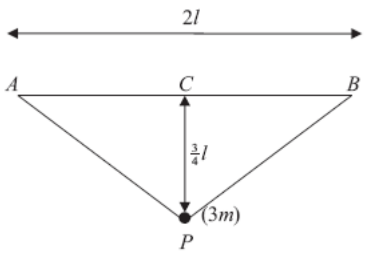

Figure 4

A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C. The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

(*a*) Show that $k = 10$.

(7)

The ball is now pulled vertically downwards until it is at a distance $\frac{12}{5}l$ below C. The ball is released from rest.

(*b*) Find the speed of the ball as it reaches C.

(6)

7.

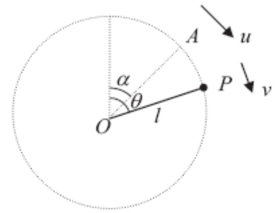

Figure 5

A particle P of mass m is attached to one end of a light rod of length l. The other end of the rod is attached to a fixed point O. The rod can turn freely in a vertical plane about O. The particle is projected with speed u from a point A, where OA makes an angle α with the upward vertical through O and $0 < \alpha < \frac{\pi}{2}$. When OP makes an angle θ with the upward vertical through O the speed of P is v, as shown in Figure 5.

(*a*) Show that $v^2 = u^2 + 2gl(\cos\alpha - \cos\theta)$.

(4)

It is given that $\cos\alpha = \frac{3}{5}$ and that P moves in a complete vertical circle.

(*b*) Show that $u > 2\sqrt{\left(\frac{gl}{5}\right)}$.

(4)

As the rod rotates the least tension in the rod is T and the greatest tension is $5T$.

(*c*) Show that $u^2 = \frac{33}{10}gl$.

(9)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6678/01

Edexcel GCE

Mechanics M3

Advanced Level

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P of mass 0.5 kg moves on the positive x-axis under the action of a single force directed towards the origin O. At time t seconds the distance of P from O is x metres, the magnitude of the force is $0.375x^2$ N and the speed of P is v m s^{-1}.

When $t = 0$, $OP = 8$ m and P is moving towards O with speed 2 m s^{-1}.

(*a*) Show that $v^2 = 260 - \frac{1}{2}x^3$.

(4)

(*b*) Find the distance of P from O at the instant when $v = 5$.

(2)

2.

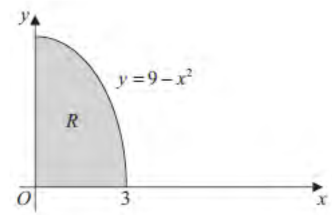

Figure 1

The shaded region R is bounded by the curve with equation $y = 9 - x^2$, the positive x-axis and the positive y-axis, as shown in Figure 1. A uniform solid S is formed by rotating R through 360° about the x-axis.

Find the x-coordinate of the centre of mass of S.

(9)

3.

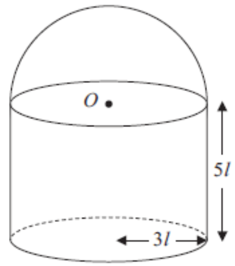

Figure 2

A solid consists of a uniform solid right cylinder of height $5l$ and radius $3l$ joined to a uniform solid hemisphere of radius $3l$. The plane face of the hemisphere coincides with a circular end of the cylinder and has centre O, as shown in Figure 2.

The density of the hemisphere is **twice** the density of the cylinder.

(*a*) Find the distance of the centre of mass of the solid from O.

(5)

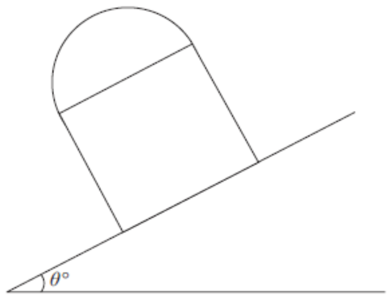

Figure 3

The solid is now placed with its circular face on a plane inclined at an angle $\theta°$ to the horizontal, as shown in Figure 3. The plane is sufficiently rough to prevent the solid slipping. The solid is on the point of toppling.

(*b*) Find the value of θ.

(4)

4.

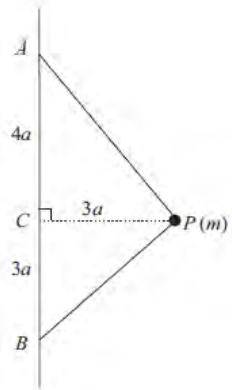

Figure 4

A light inextensible string has its ends attached to two fixed points A and B. The point A is vertically above B and $AB = 7a$. A particle P of mass m is fixed to the string and moves in a horizontal circle of radius $3a$ with angular speed ω. The centre of the circle is C where C lies on AB and $AC = 4a$, as shown in Figure 4. Both parts of the string are taut.

(*a*) Show that the tension in AP is $\frac{5}{7}m(3a\omega^2 + g)$.

(8)

(*b*) Find the tension in BP.

(2)

(*c*) Deduce that $\omega \geq \frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$.

(2)

5. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{7}{6}l$. The coefficient of friction between P and the table is μ.

(*a*) Show that $\mu \geq \frac{1}{2}$.

(4)

The particle is now moved along the table to the point B, where $OB = \frac{3}{2}l$, and released from rest. Given that $\mu = \frac{1}{2}$, find

(*b*) the speed of P at the instant when the string becomes slack,

(5)

(*c*) the total distance moved by P before it comes to rest again.

(3)

6.

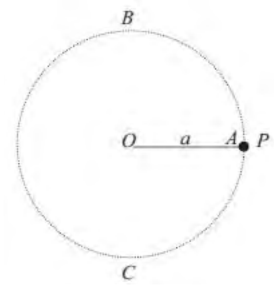

Figure 5

A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal. The point B is vertically above O and the point C is vertically below O, with $OB = OC = a$, as shown in Figure 5. The particle is projected vertically upwards with speed $3\sqrt{(ag)}$.

(*a*) Show that P will pass through B.

(6)

(*b*) Find the speed of P as it reaches C.

(2)

As P passes through C it receives an impulse. Immediately after this, the speed of P is $\frac{5}{12}\sqrt{(11ag)}$ and the direction of motion of P is unchanged.

(*c*) Find the angle between the string and the downward vertical when P comes to instantaneous rest.

(4)

7. A particle P of mass 0.5 kg is attached to the mid-point of a light elastic string of natural length 1.4 m and modulus of elasticity 2 N. The ends of the string are attached to the points A and B on a smooth horizontal table, where AB = 2 m. The mid-point of AB is O and the point C is on the table between O and B where OC = 0.2 m. At time t = 0 the particle is released from rest at C. At time t seconds the length of the string AP is $(1 + x)$ m.

(*a*) Show that the tension in BP is $\frac{2}{7}(3 - 10x)$ N.
(2)

(*b*) Find, in terms of x, the tension in AP.
(1)

(*c*) Show that P performs simple harmonic motion with period $2\pi\sqrt{\left(\frac{7}{80}\right)}$ s.
(6)

(*d*) Find the greatest speed of P during the motion.
(2)

The point D lies between O and A, where OD = 0.1 m.

(*e*) Find the time taken by P to move directly from C to D.
(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 27 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40094A

1. A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to a fixed point A. The particle is released from rest at A and comes to instantaneous rest 1.1 m below A.

Find the modulus of elasticity of the string.

(4)

---

2. A particle P is moving in a straight line with simple harmonic motion. The centre of the oscillation is the fixed point C, the amplitude of the oscillation is 0.5 m and the time to complete one oscillation is $\frac{2\pi}{3}$ seconds. The point A is on the path of P and 0.2 m from C.

Find

(*a*) the magnitude and direction of the acceleration of P when it passes through A,

(3)

(*b*) the speed of P when it passes through A,

(2)

(*c*) the time P takes to move directly from C to A.

(3)

---

3. A particle P is moving in a straight line. At time t seconds, P is at a distance x metres from a fixed point O on the line and is moving away from O with speed $\frac{10}{x+6}$ m s^{-1}.

(*a*) Find the acceleration of P when $x = 14$.

(4)

Given that $x = 2$ when $t = 1$,

(*b*) find the value of t when $x = 14$.

(6)

---

4. A light elastic string AB has natural length 0.8 m and modulus of elasticity 19.6 N. The end A is attached to a fixed point. A particle of mass 0.5 kg is attached to the end B. The particle is moving with constant angular speed ω rad s^{-1} in a horizontal circle whose centre is vertically below A. The string is inclined at 60° to the vertical.

(*a*) Show that the extension of the string is 0.4 m.

(5)

(*b*) Find the value of ω.

(5)

---

5. Above the Earth's surface, the magnitude of the gravitational force on a particle due to the Earth is inversely proportional to the square of the distance of the particle from the centre of the Earth. The Earth is modelled as a sphere of radius R and the acceleration due to gravity at the Earth's surface is g. A particle P of mass m is at a height x above the surface of the Earth.

(*a*) Show that the magnitude of the gravitational force acting on P is

$$\frac{mgR^2}{(R+x)^2}.$$

(3)

A rocket is fired vertically upwards from the surface of the Earth. When the rocket is at height $2R$ above the surface of the Earth its speed is $\sqrt{\left(\frac{gR}{2}\right)}$. You may assume that air resistance can be ignored and that the engine of the rocket is switched off before the rocket reaches height R.

Modelling the rocket as a particle,

(*b*) find the speed of the rocket when it was at height R above the surface of the Earth.

(9)

---

6. A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium at the point A, vertically below O, when it is set in motion with a horizontal speed $\frac{1}{2}\sqrt{(11gl)}$. When the string has turned through an angle θ and the string is still taut, the tension in the string is T.

(*a*) Show that $T = 3mg\left(\cos\theta + \frac{1}{4}\right)$.

(8)

At the instant when P reaches the point B, the string becomes slack.

Find

(*b*) the speed of P at B,

(3)

(*c*) the maximum height above B reached by P before it starts to fall.

(4)

7.

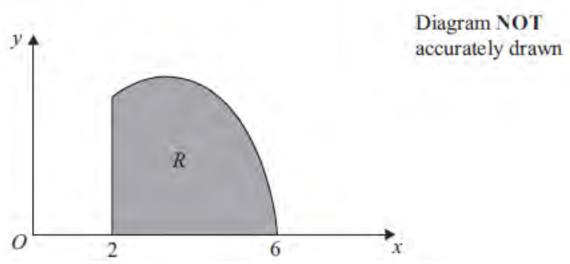

Figure 1

The shaded region R is bounded by the curve with equation $y = \frac{1}{2}x(6 - x)$, the x-axis and the line $x = 2$, as shown in Figure 1. The unit of length on both axes is 1 cm. A uniform solid P is formed by rotating R through 360° about the x-axis.

(*a*) Show that the centre of mass of P is, to 3 significant figures, 1.42 cm from its plane face. **(9)**

The uniform solid P is placed with its plane face on an inclined plane which makes an angle θ with the horizontal. Given that the plane is sufficiently rough to prevent P from sliding and that P is on the point of toppling when $\theta = \alpha$,

(*b*) find the angle α. **(4)**

Given instead that P is on the point of sliding down the plane when $\theta = \beta$ and that the coefficient of friction between P and the plane is 0.3,

(*c*) find the angle β. **(3)**

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6678/01

Edexcel GCE

Mechanics M3

Advanced Level

Thursday 14 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40691A

1. A particle P is moving along the positive x-axis. At time $t = 0$, P is at the origin O. At time t seconds, P is x metres from O and has velocity $v = 2e^{-x}$ m s^{-1} in the direction of x increasing.

(a) Find the acceleration of P in terms of x.

(3)

(b) Find x in terms of t.

(6)

2. A particle P moves in a straight line with simple harmonic motion about a fixed centre O. The period of the motion is $\frac{\pi}{2}$ seconds. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, P is at O and $v = 6$. Find

(a) the greatest distance of P from O during the motion,

(3)

(b) the greatest magnitude of the acceleration of P during the motion,

(2)

(c) the smallest positive value of t for which P is 1 m from O.

(3)

3.

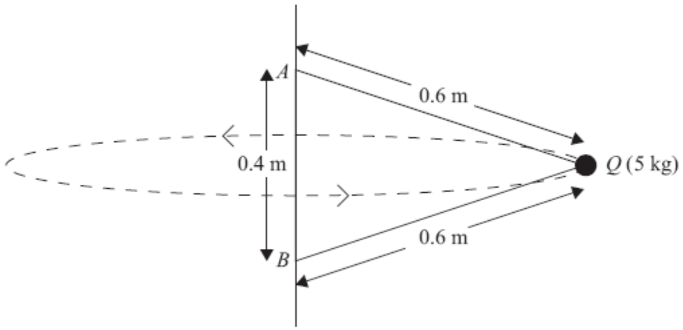

Figure 1

A particle Q of mass 5 kg is attached by two light inextensible strings to two fixed points A and B on a vertical pole. Each string has length 0.6 m and A is 0.4 m vertically above B, as shown in Figure 1.

Both strings are taut and Q is moving in a horizontal circle with constant angular speed 10 rad s^{-1}.

Find the tension in

(i) AQ,

(ii) BQ.

(10)

4.

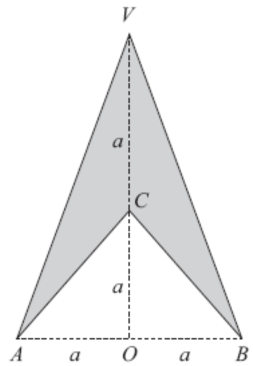

Figure 2

Figure 2 shows the cross-section $AVBC$ of the solid S formed when a uniform right circular cone of base radius a and height a, is removed from a uniform right circular cone of base radius a and height $2a$. Both cones have the same axis VCO, where O is the centre of the base of each cone.

(a) Show that the distance of the centre of mass of S from the vertex V is $\frac{5}{4}a$.

(5)

The mass of S is M. A particle of mass kM is attached to S at B. The system is suspended by a string attached to the vertex V, and hangs freely in equilibrium. Given that VA is at an angle 45° to the vertical through V,

(b) find the value of k.

(5)

5. A fixed smooth sphere has centre O and radius a. A particle P is placed on the surface of the sphere at the point A, where OA makes an angle α with the upward vertical through O. The particle is released from rest at A. When OP makes an angle θ to the upward vertical through O, P is on the surface of the sphere and the speed of P is v.

Given that $\cos \alpha = \frac{3}{5}$,

(a) show that

$$v^2 = \frac{2ga}{5}(3 - 5\cos\theta),$$

(4)

(b) find the speed of P at the instant when it loses contact with the sphere.

(8)

6.

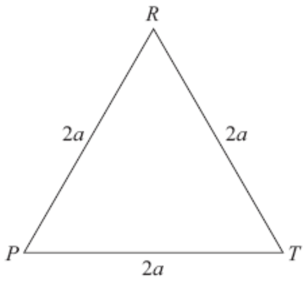

Figure 3

Figure 3 shows a uniform equilateral triangular lamina *PRT* with sides of length $2a$.

(*a*) Using calculus, prove that the centre of mass of *PRT* is at a distance $\frac{2\sqrt{3}}{3}a$ from *R*.

(6)

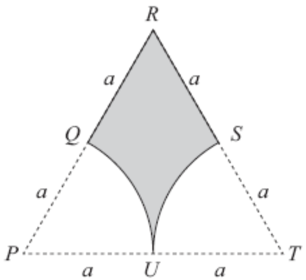

Figure 4

The circular sector *PQU*, of radius *a* and centre *P*, and the circular sector *TUS*, of radius *a* and centre *T*, are removed from *PRT* to form the uniform lamina *QRSU* shown in Figure 4.

(*b*) Show that the distance of the centre of mass of *QRSU* from *U* is $\frac{2a}{3\sqrt{3}-\pi}$.

(6)

7. A particle *B* of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point *A*. The particle is hanging in equilibrium at the point *E*, vertically below *A*.

(*a*) Show that *AE* = 0 9 m.

(3)

The particle is held at *A* and released from rest. The particle first comes to instantaneous rest at the point *C*.

(*b*) Find the distance *AC*.

(5)

(c) Show that while the string is taut, *B* is moving with simple harmonic motion with centre *E*.

(4)

(*d*) Calculate the maximum speed of *B*.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

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You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41479A

1. A particle P is moving along the positive x-axis. When the displacement of P from the origin is x metres, the velocity of P is v m s^{-1} and the acceleration of P is $9x$ m s^{-2}.

When $x = 2$, $v = 6$.

Show that $v^2 = 9x^2$.

(4)

2.

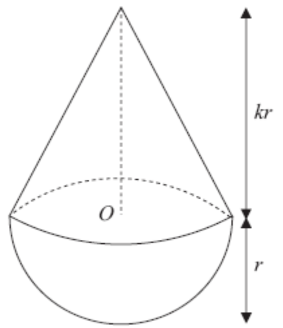

Figure 1

A uniform solid consists of a right circular cone of radius r and height kr, where $k > \sqrt{3}$, fixed to a hemisphere of radius r. The centre of the plane face of the hemisphere is O and this plane face coincides with the base of the cone, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of the solid from O is

$$\frac{(k^2 - 3)r}{4(k + 2)}.$$

(5)

The point A lies on the circumference of the base of the cone. The solid is suspended by a string attached at A and hangs freely in equilibrium. The angle between AO and the vertical is θ, where $\tan\theta = \frac{11}{14}$.

(*b*) Find the value of k.

(4)

---

3. A particle P of mass 0.6 kg is moving along the x-axis in the positive direction. At time $t = 0$, P passes through the origin O with speed 15 m s^{-1}. At time t seconds the distance OP is x metres, the speed of P is v m s^{-1} and the resultant force acting on P has magnitude $\frac{12}{(t+2)^2}$ newtons. The resultant force is directed towards O.

(a) Show that $v = 5\left(\frac{4}{t+2} + 1\right)$.

(5)

(b) Find the value of x when $t = 5$.

(5)

4.

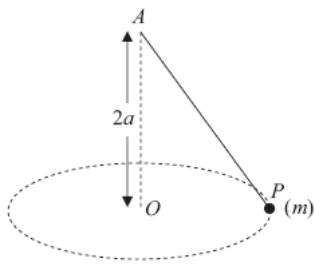

Figure 2

A particle P of mass m is attached to one end of a light elastic string, of natural length $2a$ and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point A. The particle moves with constant speed v in a horizontal circle with centre O, where O is vertically below A and $OA = 2a$, as shown in Figure 2.

(a) Show that the extension in the string is $\frac{2}{5}a$.

(6)

(b) Find v^2 in terms of a and g.

(5)

---

5. A particle P is moving in a straight line with simple harmonic motion on a smooth horizontal floor. The particle comes to instantaneous rest at points A and B where AB is 0.5 m. The mid-point of AB is O. The mid-point of OA is C. The mid-point of OB is D. The particle takes 0.2 s to travel directly from C to D. At time $t = 0$, P is moving through O towards A.

(a) Show that the period of the motion is $\frac{6}{5}$ s.

(5)

(b) Find the distance of P from B when $t = 2$ s.

(3)

(c) Find the maximum magnitude of the acceleration of P.

(2)

(d) Find the maximum speed of P.

(2)

6.

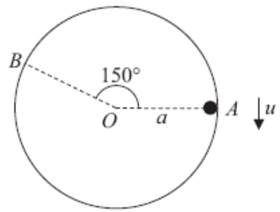

Figure 3

A smooth hollow cylinder of internal radius a is fixed with its axis horizontal. A particle P moves on the inner surface of the cylinder in a vertical circle with radius a and centre O, where O lies on the axis of the cylinder. The particle is projected vertically downwards with speed u from point A on the circle, where OA is horizontal. The particle first loses contact with the cylinder at the point B, where $\angle AOB = 150°$, as shown in Figure 3. Given that air resistance can be ignored,

(a) show that the speed of P at B is $\sqrt{\left(\frac{ag}{2}\right)}$,

(3)

(b) find u in terms of a and g.

(4)

After losing contact with the cylinder, P crosses the diameter through A at the point D. At D the velocity of P makes an angle $\theta°$ with the horizontal.

(c) Find the value of θ.

(7)

---

7. A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B, where AB is horizontal and AB = 0.48 m. Initially P is held at rest at the mid-point, M, of the line AB and the tension in the string is 240 N.

(*a*) Show that $\lambda = 400$.

(3)

The particle is now held at rest at the point C, where C is 0.07 m vertically below M. The particle is released from rest at C.

(*b*) Find the magnitude of the initial acceleration of P.

(6)

(*c*) Find the speed of P as it passes through M.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679/01R

Edexcel GCE

Mechanics M3 (R)

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

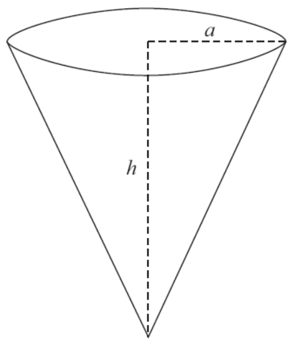

Figure 1

A hollow right circular cone, of base radius a and height h, is fixed with its axis vertical and vertex downwards, as shown in Figure 1. A particle moves with constant speed v in a horizontal circle of radius $\frac{1}{3}a$ on the smooth inner surface of the cone.

Show that $v = \sqrt{\left(\frac{1}{3}hg\right)}$.

(7)

---

2. A particle of mass 4 kg is moving along the horizontal x-axis under the action of a single force which acts in the positive x-direction. At time t seconds the force has magnitude $\left(1 + 3t^{\frac{1}{2}}\right)$ N.

When $t = 0$ the particle has speed 2 m s^{-1} in the positive x-direction. Find the work done by the force in the interval $0 \leq t \leq 4$.

(7)

---

3. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point A. The particle P is held at rest at the point B, which is 1 m vertically below A, and then released.

(a) Find the acceleration of P immediately after it is released from rest.

(4)

The particle comes to instantaneous rest for the first time at the point C.

(b) Find the distance BC.

(6)

---

4. A particle P is moving along the positive x-axis. At time t seconds, $t \geq 0$, P is x metres from the origin O and is moving away from O with velocity v m s^{-1}, where $v = \frac{4}{(x+2)}$.

When $t = 0$, P is at O. Find

(a) the distance of P from O when $t = 2$,

(5)

(b) the magnitude and direction of the acceleration of P when $t = 2$.

(5)

5.

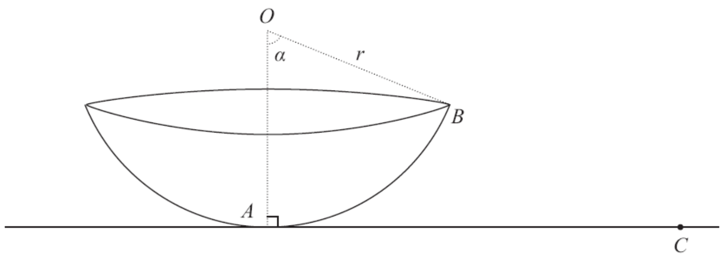

Figure 2

Part of a hollow spherical shell, centre O and radius r, forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at A with the rim uppermost and horizontal. The point A is the lowest point of the bowl. The point B, where $\angle AOB = \alpha$ and $\tan \alpha = \frac{3}{4}$, is on the rim of the bowl, as shown in Figure 2. A small smooth marble M is placed inside the bowl at A, and given an initial horizontal speed $\sqrt{(gr)}$. The motion of M takes place in the vertical plane OAB.

(*a*) Show that the speed of M as it reaches B is $\sqrt{\left(\frac{3}{5}gr\right)}$.

(4)

After leaving the surface of the bowl at B, M moves freely under gravity and first strikes the horizontal surface at the point C. Given that $r = 0.4$m,

(*b*) find the distance AC.

(8)

6. (*a*) A uniform lamina is in the shape of a quadrant of a circle of radius a. Show, by integration, that the centre of mass of the lamina is at a distance of $\frac{4a}{3\pi}$ from each of its straight edges.

(7)

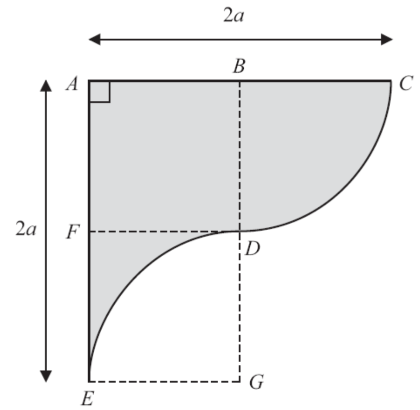

Figure 3

A second uniform lamina $ABCDEFA$ is shown shaded in Figure 3. The straight sides AC and AE are perpendicular and $AC = AE = 2a$. In the figure, the midpoint of AC is B, the midpoint of AE is F, and $ABDF$ and $DGEF$ are squares of side a. BCD is a quadrant of a circle with centre B. DGE is a quadrant of a circle with centre G.

(*b*) Find the distance of the centre of mass of the lamina from the side AE.

(5)

The lamina is smoothly hinged to a horizontal axis which passes through E and is perpendicular to the plane of the lamina. The lamina has weight W newtons. The lamina is held in equilibrium in a vertical plane, with A vertically above E, by a horizontal force of magnitude X newtons applied at C.

(*c*) Find X in terms of W.

(3)

7. Two points A and B are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point A. A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point B. A particle P of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point C on the straight line between A and B.

(*a*) Show that the length of AC is 1.76 m.

(4)

The particle P is now held at the point D on the line AB such that $AD = 2.16$ m. The particle is then released from rest and in the subsequent motion both strings remain taut.

(*b*) Show that P moves with simple harmonic motion.

(4)

(*c*) Find the speed of P as it passes through the point C.

(2)

(*d*) Find the time from the instant when P is released from D until the instant when P is first moving with speed 2 m s^{-1}.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6679/01

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43021A

1.

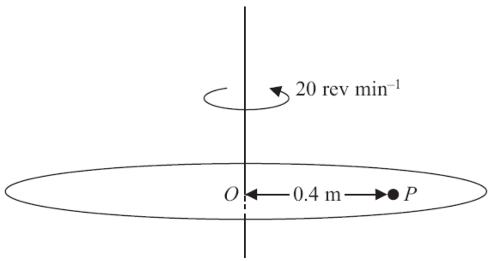

Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre O. A particle P rests on the disc at a distance 0.4 m from O, as shown in Figure 1. The coefficient of friction between P and the disc is μ. The particle P is on the point of slipping.

Find the value of μ.

(6)

---

2. A particle P of mass 0.5 kg is moving along the positive x-axis in the positive x-direction. The only force on P is a force of magnitude $\left(2t + \frac{1}{2}\right)$N acting in the direction of x increasing, where t seconds is the time after P leaves the origin O. When $t = 0$, P is at rest at O.

(*a*) Find an expression, in terms of t, for the velocity of P at time t seconds.

(3)

The particle passes through the point A with speed 6 m s^{-1}.

(*b*) Find the distance OA.

(6)

3.

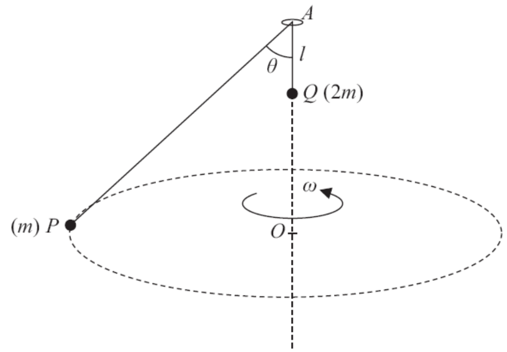

Figure 2

Two particles P and Q, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string of length $6l$. The string passes through a small smooth fixed ring at the point A. The particle Q is hanging freely at a distance l vertically below A. The particle P is moving in a horizontal circle with constant angular speed ω. The centre O of the circle is vertically below A. The particle Q does not move and AP makes a constant angle θ with the downward vertical, as shown in Figure 2.

Show that

(i) $\theta = 60°$,

(ii) $\omega = \sqrt{\left(\frac{2g}{5l}\right)}$.

(8)

---

4. A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point O on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{5}$. The particle is held at rest at a point B on the plane, where OB = 1.5 m. When P is at B, the tension in the string is 20 N. The particle is released from rest.

(*a*) Find the speed of P when OP = 1.2 m.

(7)

The particle comes to rest at the point C.

(*b*) Find the distance BC.

(2)

5.

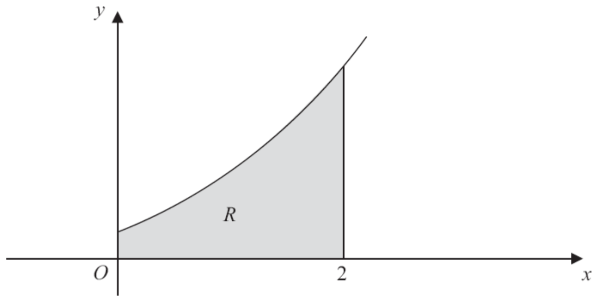

Figure 3

The shaded region R is bounded by the curve with equation $y = (x + 1)^2$, the x-axis, the y-axis and the line with equation $x = 2$, as shown in Figure 3. The region R is rotated through 2π radians about the x-axis to form a uniform solid S.

(*a*) Use algebraic integration to find the x coordinate of the centre of mass of S.

(8)

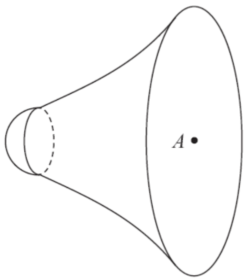

Figure 4

A uniform solid hemisphere is fixed to S to form a solid T. The hemisphere has the same radius as the smaller plane face of S and its plane face coincides with the smaller plane face of S, as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of S. The centre of the circular plane face of T is A. All lengths are measured in centimetres.

(*b*) Find the distance of the centre of mass of T from A.

(5)

6.

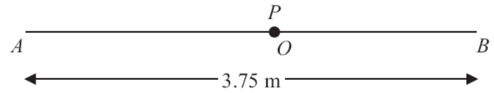

Figure 5

The points A and B are 3.75 m apart on a smooth horizontal floor. A particle P has mass 0.8 kg. One end of a light elastic spring, of natural length 1.5 m and modulus of elasticity 24 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 0.75 m and modulus of elasticity 18 N, are attached to P and B. The particle P rests in equilibrium at the point O, where AOB is a straight line, as shown in Figure 5.

(*a*) Show that $AO = 2.4$ m.

(4)

The point C lies on the straight line AOB between O and B. The particle P is held at C and released from rest.

(*b*) Show that P moves with simple harmonic motion.

(5)

The maximum speed of P is $\sqrt{2}$ m s^{-1}.

(*c*) Find the time taken by P to travel 0.3 m from C.

(5)

7.

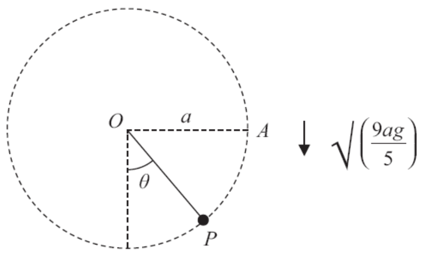

Figure 6

A particle P of mass $5m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\left(\frac{9ag}{5}\right)}$. When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is T.

(*a*) Show that $T = 3mg\,(5\cos\theta + 3)$.

(6)

At the instant when the particle reaches the point B the string becomes slack.

(*b*) Find the speed of P at B.

(3)

At time $t = 0$, P is at B.

At time t, before the string becomes taut once more, the coordinates of P are (x, y) referred to horizontal and vertical axes with origin O. The x-axis is directed along OA produced and the y-axis is vertically upward.

(*c*) Find

(i) x in terms of t, a and g,

(ii) y in terms of t, a and g.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

WME03/01

Pearson Edexcel International Advanced Level

Mechanics M3

Advanced/Advanced Subsidiary

Monday 27 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\ \text{m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P43139A

1. A particle P of mass 0.5 kg moves along the positive x-axis under the action of a single force of magnitude F newtons. The force acts along the x-axis in the direction of x increasing. When P is x metres from the origin O, it is moving away from O with speed $\sqrt{\left(8x^{\frac{3}{2}} - 4\right)}$ m s^{-1}.

Find F when P is 4 m from O.

(5)

---

2. A particle P of mass m is attached to one end of a light elastic spring, of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to a fixed point A on a rough horizontal plane. The particle is held at rest on the plane at a point B, where $AB = \frac{1}{2}l$, and released from rest. The coefficient of friction between P and the plane is $\frac{1}{4}$.

Find the distance of P from B when P first comes to rest.

(9)

---

3. A light rod AB of length $2a$ has a particle P of mass m attached to B. The rod is rotating in a vertical plane about a fixed smooth horizontal axis through A. Given that the greatest tension in the rod is $\frac{9mg}{8}$, find, to the nearest degree, the angle between the rod and the downward vertical when the speed of P is $\sqrt{\left(\frac{ag}{20}\right)}$.

(8)

4.

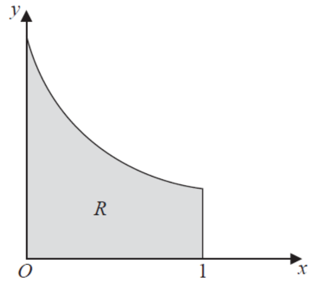

Figure 1

Figure 1 shows a sketch of the region R bounded by the curve with equation $y = e^{-x}$, the line $x = 1$, the x-axis and the y-axis. A uniform solid S is formed by rotating R through 2π radians about the x-axis.

(a) Show that the volume of S is $\frac{\pi}{2}\left(1 - e^{-2}\right)$.

(3)

(b) Find, in terms of e, the distance of the centre of mass of S from O.

(7)

---

5. A solid S consists of a uniform solid hemisphere of radius r and a uniform solid circular cylinder of radius r and height $3r$. The circular face of the hemisphere is joined to one of the circular faces of the cylinder, so that the centres of the two faces coincide. The other circular face of the cylinder has centre O. The mass per unit volume of the hemisphere is $3k$ and the mass per unit volume of the cylinder is k.

(a) Show that the distance of the centre of mass of S from O is $\frac{9r}{4}$.

(5)

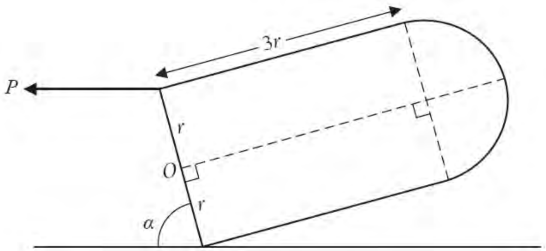

Figure 2

The solid S is held in equilibrium by a horizontal force of magnitude P. The circular face of S has one point in contact with a fixed rough horizontal plane and is inclined at an angle α to the horizontal. The force acts through the highest point of the circular face of S and in the vertical plane through the axis of the cylinder, as shown in Figure 2. The coefficient of friction between S and the plane is μ. Given that S is on the point of slipping along the plane in the same direction as P,

(b) show that $\mu = \frac{1}{8}(9 - 4\cot\alpha)$.

(7)

6.

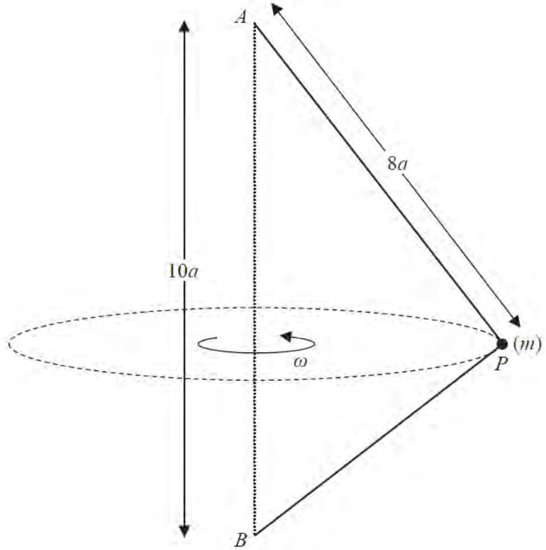

Figure 3

A light inextensible string of length $14a$ has its ends attached to two fixed points A and B, where A is vertically above B and $AB = 10a$. A particle of mass m is attached to the string at the point P, where $AP = 8a$. The particle moves in a horizontal circle with constant angular speed ω and with both parts of the string taut, as shown in Figure 3.

(a) Show that angle $APB = 90°$.

(2)

(b) Show that the time for the particle to make one complete revolution is less than $2\pi\sqrt{\left(\frac{32a}{5g}\right)}$.

(13)

7.

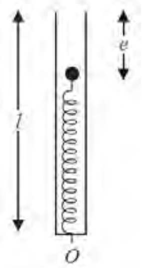

Figure 4

A smooth hollow narrow tube of length l has one open end and one closed end. The tube is fixed in a vertical position with the closed end at the bottom. A light elastic spring has natural length l and modulus of elasticity $8mg$. The spring is inside the tube and has one end attached to a fixed point O on the closed end of the tube. The other end of the spring is attached to a particle P of mass m. The particle rests in equilibrium at a distance e below the top of the tube, as shown in Figure 4.

(*a*) Find e in terms of l.

(2)

The particle P is now held inside the tube at a distance $\frac{1}{2}l$ below the top of the tube and released from rest at time $t = 0$.

(*b*) Prove that P moves with simple harmonic motion of period $2\pi\sqrt{\left(\frac{l}{8g}\right)}$.

(6)

The particle P passes through the open top of the tube with speed u.

(*c*) Find u in terms of g and l.

(4)

(*d*) Find the time taken for P to first attain a speed of $\sqrt{\left(\frac{9gl}{32}\right)}$.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
WME03/01

Pearson Edexcel International Advanced Level

Mechanics M3

Advanced/Advanced Subsidiary

Monday 19 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P44521A

1. A particle P moves in a straight line with simple harmonic motion. The period of the motion is $\frac{\pi}{4}$ seconds. At time $t = 0$, P is at rest at the point A and the acceleration of P has magnitude $20\ \text{m s}^{-2}$.

Find

(*a*) the amplitude of the motion,

(3)

(*b*) the greatest speed of P during the motion,

(1)

(*c*) the time P takes to travel a total distance of 1.5 m after it has first left A.

(4)

2.

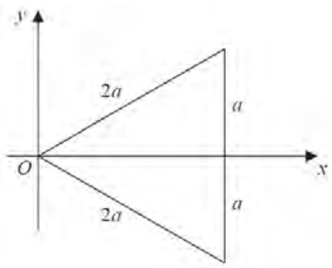

Figure 1

A uniform lamina L is in the shape of an equilateral triangle of side $2a$. The lamina is placed in the xy-plane with one vertex at the origin O and an axis of symmetry along the x-axis, as shown in Figure 1.

Use algebraic integration to find the x coordinate of the centre of mass of L.

(6)

3.

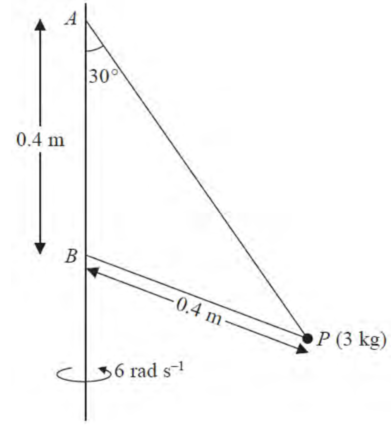

Figure 2

A particle P of mass 3 kg is attached by two light inextensible strings to two fixed points A and B on a fixed vertical pole. Both strings are taut and P is moving in a horizontal circle with constant angular speed $6\ \text{rad s}^{-1}$. String AP is inclined at 30° to the vertical. String BP has length 0.4 m and A is 0.4 m vertically above B, as shown in Figure 2.

Find the tension in

(i) AP,

(ii) BP.

(9)

4. At time $t = 0$, a particle P of mass 0.4 kg is at the origin O moving with speed $4\ \text{m s}^{-1}$ along the x-axis in the positive x direction. At time t seconds, $t \geq 0$, the resultant force acting on P has magnitude $\frac{4}{(t+5)^2}$ N and is directed away from O.

(*a*) Show that the speed of P cannot exceed $6\ \text{m s}^{-1}$.

(5)

The particle passes through the point A when $t = 2$ and passes through the point B when $t = 7$.

(*b*) Find the distance AB.

(4)

(*c*) Find the gain in kinetic energy of P as it moves from A to B.

(3)

5.

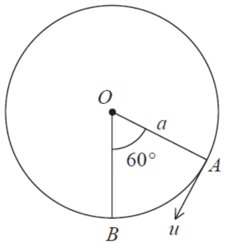

Figure 3

A particle P of mass $2m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Initially the particle is at the point A where $OA = a$ and OA makes an angle 60° with the downward vertical. The particle is projected downwards from A with speed u in a direction perpendicular to the string, as shown in Figure 3. The point B is vertically below O and $OB = a$. As P passes through B it strikes and adheres to another particle Q of mass m which is at rest at B.

(*a*) Show that the speed of the combined particle immediately after the impact is $\frac{2}{3}\sqrt{u^2 + ag}$.

(6)

(*b*) Find, in terms of a, g, m and u, the tension in the string immediately after the impact.

(3)

The combined particle moves in a complete circle.

(*c*) Show that $u^2 \geq \frac{41ag}{4}$.

(6)

---

6. A particle of mass m is attached to one end of a light elastic string, of natural length $6a$ and modulus of elasticity $9mg$. The other end of the string is attached to a fixed point A on a ceiling. The particle hangs in equilibrium at the point B, where B is vertically below A and $AB = (6 + p)a$.

(*a*) Show that $p = \frac{2}{3}$.

(2)

The particle is now released from rest at a point C vertically below B, where $AC < \frac{22}{3}a$.

(*b*) Show that the particle moves with simple harmonic motion.

(4)

(*c*) Find the period of this motion.

(2)

(*d*) Explain briefly the significance of the condition $AC < \frac{22}{3}a$.

(1)

The point D is vertically below A and $AD = 8a$. The particle is now released from rest at D. The particle first comes to instantaneous rest at the point E.

(*e*) Find, in terms of a, the distance AE.

(4)

7.

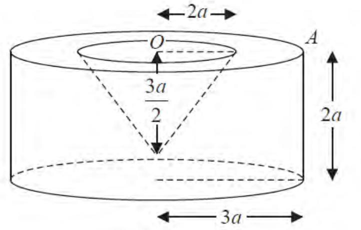

Diagram not drawn to scale

Figure 4

A uniform right circular solid cylinder has radius $3a$ and height $2a$. A right circular cone of height $\frac{3a}{2}$ and base radius $2a$ is removed from the cylinder to form a solid S, as shown in Figure 4. The plane face of the cone coincides with the upper plane face of the cylinder and the centre O of the plane face of the cone is also the centre of the upper plane face of the cylinder.

(*a*) Show that the distance of the centre of mass of S from O is $\frac{69a}{64}$.

(5)

The point A is on the open face of S such that $OA = 3a$, as shown in Figure 4. The solid is now suspended from A and hangs freely in equilibrium.

(*b*) Find the angle between OA and the horizontal.

(3)

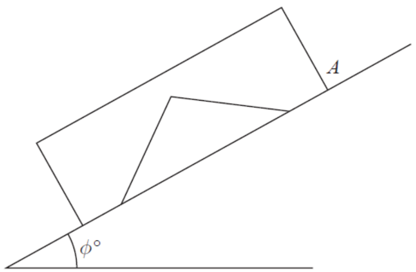

Figure 5

The solid is now placed on a rough inclined plane with the face through A in contact with the inclined plane, as shown in Figure 5. The solid rests in equilibrium on this plane. The coefficient of friction between the plane and S is 0.6 and the plane is inclined at an angle $\varphi°$ to the horizontal. Given that S is on the point of sliding down the plane, (c)

(*c*) show that $\varphi = 31$ to 2 significant figures.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679/01R

Edexcel GCE

Mechanics M3 (R)

Advanced Subsidiary

Monday 19 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

This paper is strictly for students outside the UK.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679R), your surname, initials and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43176A

1. A particle P of mass 0.25 kg is moving along the positive x-axis under the action of a single force. At time t seconds P is x metres from the origin O and is moving away from O with speed v m s^{-1} where $\frac{dv}{dx} = 3$. It is given that $x = 2$ and $v = 3$ when $t = 0$.

(*a*) Find the magnitude of the force acting on P when $x = 5$.
(4)

(*b*) Find the value of t when $x = 5$.
(4)

2.

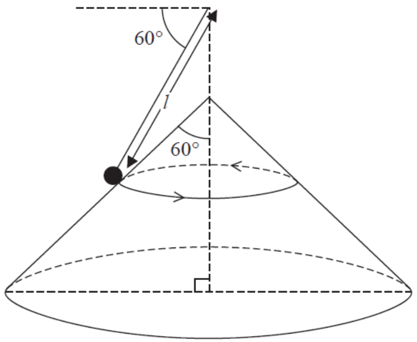

Figure 1

A cone of semi-vertical angle 60° is fixed with its axis vertical and vertex upwards. A particle of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point vertically above the vertex of the cone. The particle moves in a horizontal circle on the smooth outer surface of the cone with constant angular speed ω, with the string making a constant angle 60° with the horizontal, as shown in Figure 1.

(*a*) Find the tension in the string, in terms of m, l, ω and g.
(7)

The particle remains on the surface of the cone.

(*b*) Show that the time for the particle to make one complete revolution is greater than

$$2\pi\sqrt{\frac{l\sqrt{3}}{2g}}$$

(6)

---

3. One end A of a light elastic string AB, of modulus of elasticity mg and natural length a, is fixed to a point on a rough plane inclined at an angle θ to the horizontal. The other end B of the string is attached to a particle of mass m which is held at rest on the plane. The string AB lies along a line of greatest slope of the plane, with B lower than A and $AB = a$. The coefficient of friction between the particle and the plane is μ, where $\mu < \tan\theta$. The particle is released from rest.

(*a*) Show that when the particle comes to rest it has moved a distance $2a(\sin\theta - \mu\cos\theta)$ down the plane.
(6)

(*b*) Given that there is no further motion, show that $\mu \geq \frac{1}{3}\tan\theta$.
(5)

4.

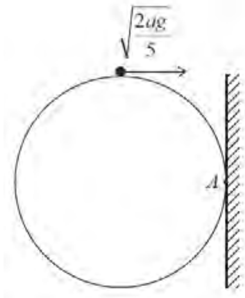

Figure 2

A smooth sphere of radius a is fixed with a point A of its surface in contact with a fixed vertical wall. A particle is placed on the highest point of the sphere and is projected towards the wall and perpendicular to the wall with horizontal speed $\sqrt{\frac{2ag}{5}}$, as shown in Figure 2.

The particle leaves the surface of the sphere with speed V.

(*a*) Show that $V = \sqrt{\frac{4ag}{5}}$
(7)

The particle strikes the wall at the point X.

(*b*) Find the distance AX.
(9)

5.

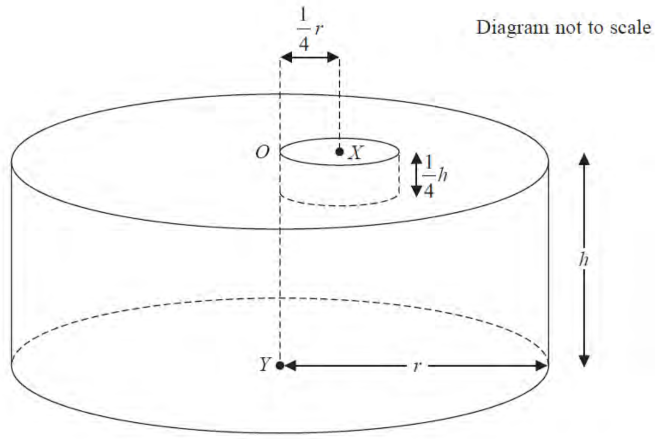

Figure 3

A uniform solid right circular cylinder has height h and radius r. The centre of one plane face is O and the centre of the other plane face is Y. A cylindrical hole is made by removing a solid cylinder of radius $\frac{1}{4}r$ and height $\frac{1}{4}h$ from the end with centre O. The axis of the cylinder removed is parallel to OY and meets the end with centre O at X, where $OX = \frac{1}{4}r$. One plane face of the cylinder removed coincides with the plane face through O of the original cylinder. The resulting solid S is shown in Figure 3.

(*a*) Show that the centre of mass of S is at a distance $\frac{85h}{168}$ from the plane face containing O.

(7)

The solid S is freely suspended from O. In equilibrium the line OY is inclined at an angle arctan(17) to the horizontal.

(*b*) Find r in terms of h.

(6)

6. A light elastic string, of natural length l and modulus of elasticity $4mg$, has one end attached to a fixed point A. The other end is attached to a particle P of mass m. The particle hangs freely at rest in equilibrium at the point E. The distance of E below A is $(l + e)$.

(*a*) Find e in terms of l.

(2)

At time $t = 0$, the particle is projected vertically downwards from E with speed $\sqrt{gl}$.

(*b*) Prove that, while the string is taut, P moves with simple harmonic motion.

(5)

(*c*) Find the amplitude of the simple harmonic motion.

(3)

(*d*) Find the time at which the string first goes slack.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6679/01

Edexcel GCE

Mechanics M3

Advanced Level

Monday 19 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1.

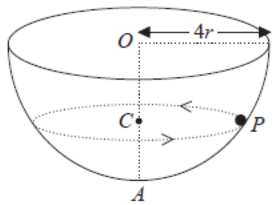

Figure 1

A hemispherical bowl of internal radius $4r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O. A particle P moves in a horizontal circle, with centre C, on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\frac{3g}{8r}}$.

The point C lies on OA, as shown in Figure 1.

Find, in terms of r, the distance OC.

(9)

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2. A particle P of mass m is fired vertically upwards from a point on the surface of the Earth and initially moves in a straight line directly away from the centre of the Earth. When P is at a distance x from the centre of the Earth, the gravitational force exerted by the Earth on P is directed towards the centre of the Earth and has magnitude $\frac{k}{x^2}$, where k is a constant.

At the surface of the Earth the acceleration due to gravity is g. The Earth is modelled as a fixed sphere of radius R.

(*a*) Show that $k = mgR^2$.

(2)

When P is at a height $\frac{R}{4}$ above the surface of the Earth, the speed of P is $\sqrt{\frac{gR}{2}}$.

Given that air resistance can be ignored,

(*b*) find, in terms of R, the greatest distance from the centre of the Earth reached by P.

(7)

3.

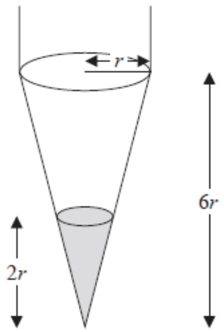

Figure 2

Figure 2 shows a container in the shape of a uniform right circular conical shell of height $6r$. The radius of the open circular face is r. The container is suspended by two vertical strings attached to two points at opposite ends of a diameter of the open circular face. It hangs with the open circular face uppermost and axis vertical.

Molten wax is poured into the container. The wax solidifies and adheres to the container, forming a uniform solid right circular cone. The depth of the wax in the container is $2r$. The container together with the wax forms a solid S.

The mass of the container when empty is m and the mass of the wax in the container is $3m$.

(*a*) Find the distance of the centre of mass of the solid S from the vertex of the container.

(4)

One of the strings is now removed and the solid S hangs freely in equilibrium suspended by the remaining vertical string.

(*b*) Find the size of the angle between the axis of the container and the downward vertical.

(3)

4.

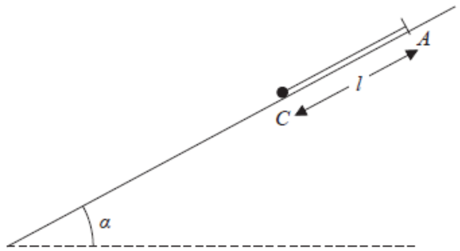

Figure 3

One end of a light elastic string, of natural length l and modulus of elasticity $3mg$, is fixed to a point A on a fixed plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$.

A small ball of mass $2m$ is attached to the free end of the string. The ball is held at a point C on the plane, where C is below A and $AC = l$ as shown in Figure 3. The string is parallel to a line of greatest slope of the plane. The ball is released from rest. In an initial model the plane is assumed to be smooth.

(*a*) Find the distance that the ball moves before first coming to instantaneous rest.

(5)

In a refined model the plane is assumed to be rough. The coefficient of friction between the ball and the plane is μ. The ball first comes to instantaneous rest after moving a distance $\frac{2}{5}l$.

(*b*) Find the value of μ.

(6)

5.

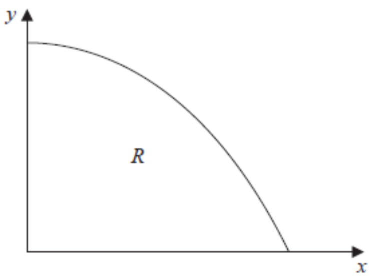

Figure 4

Figure 4 shows the region R bounded by part of the curve with equation $y = \cos x$, the x-axis and the y-axis. A uniform solid S is formed by rotating R through 2π radians about the x-axis.

(a) Show that the volume of S is $\frac{\pi^2}{4}$.

(4)

(b) Find, using algebraic integration, the x-coordinate of the centre of mass of S.

(7)

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6. A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed U. The particle moves in a complete vertical circle.

(a) Show that $U \geq \sqrt{5ag}$.

(8)

As P moves in the circle the least tension in the string is T and the greatest tension is kT.

Given that $U = 3\sqrt{ag}$,

(b) find the value of k.

(5)

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7. A particle P of mass m is attached to one end of a light elastic spring of natural length l. The other end of the spring is attached to a fixed point A. The particle is hanging freely in equilibrium at the point B, where $AB = 1.5l$.

(a) Show that the modulus of elasticity of the spring is $2mg$.

(3)

The particle is pulled vertically downwards from B to the point C, where $AC = 1.8l$, and released from rest.

(b) Show that P moves in simple harmonic motion with centre B.

(6)

(c) Find the greatest magnitude of the acceleration of P.

(2)

The midpoint of BC is D. The point E lies vertically below A and $AE = 1.2l$.

(d) Find the time taken by P to move directly from D to E.

(4)

TOTAL FOR PAPER: 75 MARKS

END