OCR Maths M2

Topic Questions from Papers

Energy, Work and Power

1 A car of mass 700 kg is travelling up a hill which is inclined at a constant angle of $5°$ to the horizontal. At a certain point P on the hill the car's speed is $20\,\text{m s}^{-1}$. The point Q is 400 m further up the hill from P, and at Q the car's speed is $15\,\text{m s}^{-1}$.

(i) Calculate the work done by the car's engine as the car moves from P to Q, assuming that any resistances to the car's motion may be neglected. [4]

Assume instead that the resistance to the car's motion between P and Q is a constant force of magnitude 200 N.

(ii) Given that the acceleration of the car at Q is zero, show that the power of the engine as the car passes through Q is 12.0 kW, correct to 3 significant figures. [3]

(iii) Given that the power of the car's engine at P is the same as at Q, calculate the car's retardation at P. [3]

(Q6, June 2005)

2 A box of mass 50 kg is dragged along a horizontal floor by a constant force of magnitude 400 N acting at an angle of α above the horizontal. The total resistance to the motion of the box has magnitude 300 N. The box starts from rest at the point O, and passes the point P, 25 m from O, with a speed of $2\,\text{m s}^{-1}$.

(i) For the box's motion from O to P, find

(a) the increase in kinetic energy of the box, [1]

(b) the work done against the resistance to motion of the box. [1]

(ii) Hence calculate α. [3]

(Q3, Jan 2006)

3 Marco is riding his bicycle at a constant speed of $12\,\text{m s}^{-1}$ along a horizontal road, working at a constant rate of 300 W. Marco and his bicycle have a combined mass of 75 kg.

(i) Calculate the wind resistance acting on Marco and his bicycle. [2]

Nicolas is riding his bicycle at the same speed as Marco and directly behind him. Nicolas experiences 30% less wind resistance than Marco.

(ii) Calculate the power output of Nicolas. [2]

The two cyclists arrive at the bottom of a hill which is at an angle of $1°$ to the horizontal. Marco increases his power output to 500 W.

(iii) Assuming Marco's wind resistance is unchanged, calculate his instantaneous acceleration immediately after starting to climb the hill. [5]

Marco reaches the top of the hill at a speed of $13\,\text{m s}^{-1}$. He then freewheels down a hill of length 200 m which is at a constant angle of $10°$ to the horizontal. He experiences a constant wind resistance of 120 N.

(iv) Calculate Marco's speed at the bottom of this hill. [5]

(Q7, Jan 2006)

4 A child of mass 35 kg runs up a flight of stairs in 10 seconds. The vertical distance climbed is 4 m. Assuming that the child's speed is constant, calculate the power output. [4]

(Q1, June 2006)

5 A car of mass 900 kg is travelling at a constant speed of $30\,\text{m s}^{-1}$ on a level road. The total resistance to motion is 450 N.

(i) Calculate the power output of the car's engine. [1]

A roof box of mass 50 kg is mounted on the roof of the car. The total resistance to motion of the vehicle increases to 500 N.

(ii) The car's engine continues to work at the same rate. Calculate the maximum speed of the car on the level road. [2]

The power output of the car's engine increases to 15 000 W. The resistance to motion of the car, with roof box, remains 500 N.

(iii) Calculate the instantaneous acceleration of the car on the level road when its speed is $25\,\text{m s}^{-1}$. [3]

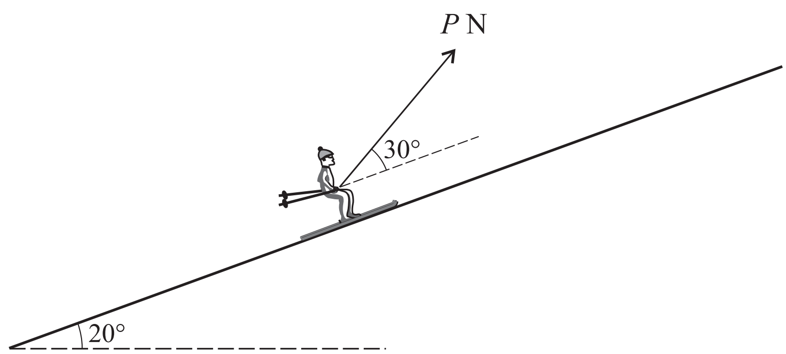
(iv) The car climbs a hill which is at an angle of 5° to the horizontal. Calculate the instantaneous retardation of the car when its speed is $26\,\text{m s}^{-1}$. [3]

(Q4, June 2006)

6 A skier of mass 80 kg is pulled up a slope which makes an angle of 20° with the horizontal. The skier is subject to a constant frictional force of magnitude 70 N. The speed of the skier increases from $2\,\text{m s}^{-1}$ at the point A to $5\,\text{m s}^{-1}$ at the point B, and the distance AB is 25 m.

(i) By modelling the skier as a small object, calculate the work done by the pulling force as the skier moves from A to B. [5]

(ii)



It is given that the pulling force has constant magnitude P N, and that it acts at a constant angle of 30° above the slope (see diagram). Calculate P. [3]

(Q4, Jan 2007)

7 A model train has mass 100 kg. When the train is moving with speed $v\,\text{m s}^{-1}$ the resistance to its motion is $3v^2$ N and the power output of the train is $\frac{3000}{v}$ W.

(i) Show that the driving force acting on the train is 120 N at an instant when the train is moving with speed $5\,\text{m s}^{-1}$. [2]

(ii) Find the acceleration of the train at an instant when it is moving horizontally with speed $5\,\text{m s}^{-1}$. [2]

The train moves with constant speed up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{98}$.

(iii) Calculate the speed of the train. *(Q5, Jan 2007)* [5]

8 A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of 35° with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

(Q1, June 2007)

9 A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100\,\text{m s}^{-1}$ to $150\,\text{m s}^{-1}$ in a time t seconds.

(i) Calculate the value of t. [4]

(ii) Calculate the acceleration of the rocket at the instant when its speed is $120\,\text{m s}^{-1}$. [4]

(Q3, June 2007)

10 A cyclist and her bicycle have a combined mass of 70 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of $4\,\text{m s}^{-1}$ at B. The level of B is 6 m above the level of A. For the cyclist's motion from A to B, find

(i) the increase in kinetic energy, [2]

(ii) the increase in gravitational potential energy. [2]

During the ascent the resistance to motion is constant and has magnitude 60 N. The work done by the cyclist in moving from A to B is 8000 J.

(iii) Calculate the distance AB. [4]

(Q5, June 2007)

11 A particle of mass m kg is projected directly up a rough plane with a speed of $5\,\text{m s}^{-1}$. The plane makes an angle of 30° with the horizontal and the coefficient of friction is 0.2. Calculate the distance the particle travels up the plane before coming instantaneously to rest. [6]

(Q2, Jan 2008)

12 A car of mass 1200 kg has a maximum speed of $30\,\text{m s}^{-1}$ when travelling on a horizontal road. The car experiences a resistance of kv N, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant. The maximum power of the car's engine is 45 000 W.

(i) Show that $k = 50$. [2]

(ii) Find the maximum possible acceleration of the car when it is travelling at $20\,\text{m s}^{-1}$ on a horizontal road. [3]

(iii) The car climbs a hill, which is inclined at an angle of 10° to the horizontal, at a constant speed of $15\,\text{m s}^{-1}$. Calculate the power of the car's engine. [3]

(Q4, Jan 2008)

13 A car is pulled at constant speed along a horizontal straight road by a force of 200 N inclined at 35° to the horizontal. Given that the work done by the force is 5000 J, calculate the distance moved by the car. [3]

(Q1, June 2008)

14 A bullet of mass 9 grams passes horizontally through a fixed vertical board of thickness 3 cm. The speed of the bullet is reduced from $250\,\text{m s}^{-1}$ to $150\,\text{m s}^{-1}$ as it passes through the board. The board exerts a constant resistive force on the bullet. Calculate the magnitude of this resistive force. [4]

(Q2, June 2008)

15 The resistance to the motion of a car of mass 600 kg is kv N, where $v\,\text{m s}^{-1}$ is the car's speed and k is a constant. The car ascends a hill of inclination α, where $\sin\alpha = \frac{1}{10}$. The power exerted by the car's engine is 12 000 W and the car has constant speed $20\,\text{m s}^{-1}$.

(i) Show that $k = 0.6$. [3]

The power exerted by the car's engine is increased to 16 000 W.

(ii) Calculate the maximum speed of the car while ascending the hill. [3]

The car now travels on horizontal ground and the power remains 16 000 W.

(iii) Calculate the acceleration of the car at an instant when its speed is $32\,\text{m s}^{-1}$. [3]

(Q3, June 2008)

16 A car of mass 800 kg experiences a resistance of magnitude kv^2 N, where k is a constant and $v\,\text{m s}^{-1}$ is the car's speed. The car's engine is working at a constant rate of P W. At an instant when the car is travelling on a horizontal road with speed $20\,\text{m s}^{-1}$ its acceleration is $0.75\,\text{m s}^{-2}$. At an instant when the car is ascending a hill of constant slope 12° to the horizontal with speed $10\,\text{m s}^{-1}$ its acceleration is $0.25\,\text{m s}^{-2}$.

(i) Show that $k = 0.900$, correct to 3 decimal places, and find P. **[7]**

The power is increased to $1.5P$ W.

(ii) Calculate the maximum steady speed of the car on a horizontal road. **[3]**

(Q4, Jan 2009)

17 A boy on a sledge slides down a straight track of length 180 m which descends a vertical distance of 40 m. The combined mass of the boy and the sledge is 75 kg. The initial speed is $3\,\text{m s}^{-1}$ and the final speed is $12\,\text{m s}^{-1}$. The magnitude, R N, of the resistance to motion is constant. By considering the change in energy, calculate R. **[5]**

(Q1, June 2009)

18 A car of mass 1100 kg has maximum power of 44 000 W. The resistive forces have constant magnitude 1400 N.

(i) Calculate the maximum steady speed of the car on the level. **[2]**

The car is moving on a hill of constant inclination α to the horizontal, where $\sin\alpha = 0.05$.

(ii) Calculate the maximum steady speed of the car when ascending the hill. **[3]**

(iii) Calculate the acceleration of the car when it is descending the hill at a speed of $10\,\text{m s}^{-1}$ working at half the maximum power. **[3]**

(Q2, June 2009)

19 Find the average power exerted by a climber of mass 75 kg when climbing a vertical distance of 40 m in 2 minutes. **[3]**

(Q1, Jan 2010)

20 A car of mass 700 kg is moving along a horizontal road against a constant resistance to motion of 400 N. At an instant when the car is travelling at $12\,\text{m}\,\text{s}^{-1}$ its acceleration is $0.5\,\text{m}\,\text{s}^{-2}$.

(i) Find the driving force of the car at this instant. **[2]**

(ii) Find the power at this instant. **[2]**

The maximum steady speed of the car on a horizontal road is $35\,\text{m}\,\text{s}^{-1}$.

(iii) Find the maximum power of the car. **[2]**

The car now moves at maximum power against the same resistance up a slope of constant angle θ° to the horizontal. The maximum steady speed up the slope is $12\,\text{m}\,\text{s}^{-1}$.

(iv) Find θ. **[4]**

(Q4, Jan 2010)

21 The maximum power produced by the engine of a small aeroplane of mass 2 tonnes is 128 kW. Air resistance opposes the motion directly and the lift force is perpendicular to the direction of motion. The magnitude of the air resistance is proportional to the square of the speed and the maximum steady speed in level flight is $80\,\text{m}\,\text{s}^{-1}$.

(i) Calculate the magnitude of the air resistance when the speed is $60\,\text{m}\,\text{s}^{-1}$. **[5]**

The aeroplane is climbing at a constant angle of 2° to the horizontal.

(ii) Find the maximum acceleration at an instant when the speed of the aeroplane is $60\,\text{m}\,\text{s}^{-1}$. **[4]**

(Q3, June 2010)

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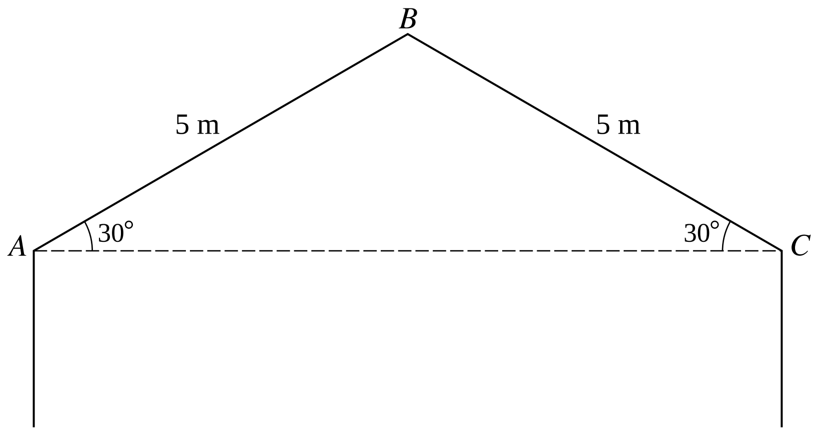

A small ball of mass 0.2 kg is projected with speed $11\,\text{m}\,\text{s}^{-1}$ up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B. The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is 5 m and AB is inclined at 30° to the horizontal (see diagram).

(i) Show that the speed of the ball when it reaches B is $5.44\,\text{m}\,\text{s}^{-1}$, correct to 2 decimal places. **[6]**

(Q7, June 2010)

23 The resistance to the motion of a car is $kv^{\frac{3}{2}}$ N, where v m s^{-1} is the car's speed and k is a constant. The power exerted by the car's engine is 15 000 W, and the car has constant speed 25 m s^{-1} along a horizontal road.

(i) Show that $k = 4.8$. **[3]**

With the engine operating at a much lower power, the car descends a hill of inclination α, where $\sin\alpha = \frac{1}{15}$. At an instant when the speed of the car is 16 m s^{-1}, its acceleration is 0.3 m s^{-2}.

(ii) Given that the mass of the car is 700 kg, calculate the power of the engine. **[5]**

(Q2, Jan 2011)

24 A block of mass 25 kg is dragged 30 m up a slope inclined at 5° to the horizontal by a rope inclined at 20° to the slope. The tension in the rope is 100 N and the resistance to the motion of the block is 70 N. The block is initially at rest. Calculate

(i) the work done by the tension in the rope, **[2]**

(ii) the change in the potential energy of the block, **[2]**

(iii) the speed of the block after it has moved 30 m up the slope. **[4]**

(Q4, Jan 2011)

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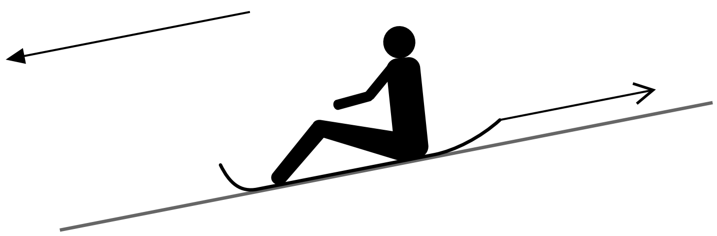

A sledge with its load has mass 70 kg. It moves down a slope and the resistance to the motion of the sledge is 90 N. The speed of the sledge is controlled by the constant tension in a light rope, which is attached to the sledge and parallel to the slope (see diagram). While travelling 20 m down the slope, the speed of the sledge decreases from 2.1 m s^{-1} to 1.4 m s^{-1} and it descends a vertical distance of 3 m.

(i) Calculate the change in energy of the sledge and its load. **[4]**

(ii) Calculate the tension in the rope. **[3]**

(Q1, June 2011)

26 A car of mass 1250 kg travels along a straight road inclined at 2° to the horizontal. The resistance to the motion of the car is kv N, where v m s^{-1} is the speed of the car and k is a constant. The car travels at a constant speed of 25 m s^{-1} up the slope and the engine of the car works at a constant rate of 21 kW.

(i) Calculate the value of k. **[4]**

(ii) Calculate the constant speed of the car on a horizontal road. **[3]**

(Q2, June 2011)

27 A car of mass 1500 kg travels up a line of greatest slope of a straight road inclined at 5° to the horizontal. The power of the car's engine is constant and equal to 25 kW and the resistance to the motion of the car is constant and equal to 750 N. The car passes through point *A* with speed $10\,\mathrm{m\,s^{-1}}$.

(i) Find the acceleration of the car at *A*. **[5]**

The car later passes through a point *B* with speed $20\,\mathrm{m\,s^{-1}}$. The car takes 28 s to travel from *A* to *B*.

(ii) Find the distance *AB*. **[7]**

(Q5, Jan 2012)

28 A car of mass 1600 kg moves along a straight horizontal road. The resistance to the motion of the car has constant magnitude 800 N and the car's engine is working at a constant rate of 20 kW.

(i) Find the acceleration of the car at an instant when the car's speed is $20\,\mathrm{m\,s^{-1}}$. **[4]**

The car now moves up a hill inclined at 4° to the horizontal. The car's engine continues to work at 20 kW and the magnitude of the resistance to motion remains at 800 N.

(ii) Find the greatest steady speed at which the car can move up the hill. **[4]**

(Q2, June 2012)

29 A block is being pushed in a straight line along horizontal ground by a force of 18 N inclined at 15° below the horizontal. The block moves a distance of 6 m in 5 s with constant speed. Find

(i) the work done by the force, **[3]**

(ii) the power with which the force is working. **[2]**

(Q1, Jan 2013)

30 A car of mass 1500 kg travels along a straight horizontal road. The resistance to the motion of the car is $kv^{\frac{1}{2}}$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the car and k is a constant. At the instant when the engine produces a power of 15 000 W, the car has speed $15\,\mathrm{m\,s^{-1}}$ and is accelerating at $0.4\,\mathrm{m\,s^{-2}}$.

(i) Find the value of k. **[4]**

It is given that the greatest steady speed of the car on this road is $30\,\mathrm{m\,s^{-1}}$.

(ii) Find the greatest power that the engine can produce. **[3]**

(Q2, Jan 2013)

31 A particle of mass 0.5 kg is held at rest at a point *P*, which is at the bottom of an inclined plane. The particle is given an impulse of 1.8 N s directed up a line of greatest slope of the plane.

(i) Find the speed at which the particle starts to move. **[2]**

The particle subsequently moves up the plane to a point *Q*, which is 0.3 m above the level of *P*.

(ii) Given that the plane is smooth, find the speed of the particle at *Q*. **[4]**

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It is given instead that the plane is rough. The particle is now projected up the plane from P with initial speed $3\,\text{m}\,\text{s}^{-1}$, and comes to rest at a point R which is 0.2 m above the level of P.

(iii) Given that the plane is inclined at 30° to the horizontal, find the magnitude of the frictional force on the particle. **[4]**

(Q6, Jan 2013)

32 A and B are two points on a line of greatest slope of a smooth inclined plane, with B a vertical distance of 8 m below the level of A. A particle of mass 0.75 kg is projected down the plane from A with a speed of $2\,\text{m}\,\text{s}^{-1}$. Find

(i) the loss in potential energy of the particle as it moves from A to B, **[2]**

(ii) the speed of the particle when it reaches B. **[4]**

(Q1, June 2013)

33 The power developed by the engine of a car as it travels at a constant speed of $32\,\text{m}\,\text{s}^{-1}$ on a horizontal road is 20 kW.

(i) Calculate the resistance to the motion of the car. **[3]**

The car, of mass 1500 kg, now travels down a straight road inclined at 2° to the horizontal. The resistance to the motion of the car is unchanged.

(ii) Find the power produced by the engine of the car when the car has speed $32\,\text{m}\,\text{s}^{-1}$ and is accelerating at $0.1\,\text{m}\,\text{s}^{-2}$. **[4]**

(Q2, June 2013)