

General Certificate of Education

Mathematics 6360

MM2B Mechanics 2B

Mark Scheme

2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method			
m or dM	mark is dependent on one or more M marks and is for method			
A	mark is dependent on M or m marks and is for accuracy			
В	mark is independent of M or m marks and is for method and accuracy			
Е	mark is for explanation			
√or ft or F	follow through from previous			
	incorrect result	MC	mis-copy	
CAO	correct answer only	MR	mis-read	
CSO	correct solution only	RA	required accuracy	
AWFW	anything which falls within	FW	further work	
AWRT	anything which rounds to	ISW	ignore subsequent work	
ACF	any correct form	FIW	from incorrect work	
AG	answer given	BOD	given benefit of doubt	
SC	special case	WR	work replaced by candidate	
OE	or equivalent	FB	formulae book	
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme	
–x EE	deduct x marks for each error	G	graph	
NMS	no method shown	c	candidate	
PI	possibly implied	sf	significant figure(s)	
SCA	substantially correct approach	dp	decimal place(s)	

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

Q	Solution	Marks	Total	Comments
1(a)	$\mathbf{v} = (6t^2 - 2t)\mathbf{i} + (1 - 12t^2)\mathbf{j}$	M1		differentiating both components
	, , , , , , , , , , , , , , , , , , , ,	A1		one component correct
		A 1	3	second component correct
(b)(i)	(1) $(6 2)$, (12) , (12)	M1		substituting the value for t into their v
(D)(1)	$\mathbf{v}\left(\frac{1}{3}\right) = \left(\frac{6}{9} - \frac{2}{3}\right)\mathbf{i} + \left(1 - \frac{12}{9}\right)\mathbf{j} = -\frac{1}{3}\mathbf{j}$	A1	2	correct velocity
			_	
(ii)	Travelling due south	A1ft	1	correct description (Follow through from
				$\mathbf{v} = \pm k\mathbf{j}$)
(c)	$\mathbf{a} = (12t - 2)\mathbf{i} - 24t\mathbf{j}$	M1		differentiating their velocity
(c)	•	A1		correct acceleration at time t
	$\mathbf{a}(4) = 46\mathbf{i} - 96\mathbf{j}$	A1	3	correct acceleration at $t = 4$
	F (46) 060 256 556	3.54		
(d)	$\mathbf{F} = 6(46\mathbf{i} - 96\mathbf{j}) = 276\mathbf{i} - 576\mathbf{j}$	M1		apply Newton's second law correctly
	$F = \sqrt{276^2 + 576^2} = 639 \text{ N}$	M1		finding magnitude
	$F = \sqrt{276^2 + 576^2} = 639 \text{ N}$	A1	3	correct magnitude
	or			
	$a = \sqrt{46^2 + 96^2} = 106.45$			
	$F = 6 \times 106.45 = 639 \text{ N}$			
	Total		12	
2(a)	$KE = \frac{1}{2} \times 0.6 \times 14^2 = 58.8 \text{ J}$	M1		use of KE formula
	2	A1	2	correct energy
(b)	$0.6 \times 9.8h = 58.8$	M1		two term energy equation involving PE
(6)		1411		and previous energy
	$h = \frac{58.8}{0.6 \times 9.8} = 10 \text{ m}$	A1		correct equation
	0.0 × 9.8	A 1	3	correct height
				Note: Constant acceleration methods not
				accepted.
(c)(i)	WD against resistance	M1		three term energy equation
(*)(2)	$= 58.8 - 0.6 \times 9.8 \times 8$	A1		correct equation
	=11.76=11.8 J (to 3 sf)	A1	3	correct value
	-11.70-11.03 (to 3 51)			
(ii)	8F = 11.76	M1		using work done = Fd with $d = 8$
	F = 1.47 N	A1ft	2	correct force
				accept 1.48
(d)	The magnitude of the force would <u>vary</u>	B1	1	appropriate explanation
(4)	with the speed of the ball.	21	1	appropriate emplanation
	Total		11	

MM2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	↑ R	B1	1	correct force diagram, with labels and arrows.
(b)	$2T = 0.5 \times 98$ T = 24.5 N AG	M1 A1 A1	3	moment equation correct equation correct positive value for the tension from correct working
(c)(i)	$2 \times 2 \times 24.5 = 3 \times 9.8 \times m + 0.5 \times 98$ $m = \frac{98 - 49}{3 \times 9.8} = \frac{5}{3} = 1.67 \text{ kg (to 3 sf)}$ Or $2 \times 2.45 = 3 \times 9.8m$ $m = \frac{49}{29.4} = \frac{5}{3} = 1.67 \text{ kg}$	B1 M1 A1 A1 (M1A1)	4	tension doubled moment equation correct equation correct mass for equation
(ii)	$R = 24.5 \times 2 + 98 + \frac{5}{3} \times 9.8 = 163 \text{ N}$	M1 A1 A1	3	considering vertical equilibrium with 3 terms correct equation correct reaction must be consistent with 3(c)(i) if awarding accuracy marks
(d)	This allows the centre of mass to be placed at the centre of the rod for the moment calculations. Total	B1	1 12	correct explanation

MM2B (cont)

MM2B (cont	Solution	Marks	Total	Comments
4(a)	$\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mgl(1 - \cos 60^\circ)$	M1		three/four term energy equation with a trig
	$U^2 = v^2 + gl$	A1		term correct equation
	$U^2 = v^2 + gl$ $v = \sqrt{U^2 - gl}$	dM1		solving for v or v^2
	$v = \sqrt{U^2 - gl}$	A1	4	correct v in a simplified form
(b)	$T - mg\cos 60^\circ = m\frac{v^2}{l}$	M1		resolving towards the centre of the circle
	•			with three terms
	$T = m\left(\frac{U^2 - gl}{l} + \frac{g}{2}\right) = m\left(\frac{U^2}{l} - \frac{g}{2}\right)$	dM1		substituting for v^2
	$l = m \left(\frac{l}{l} + \frac{1}{2} \right) = m \left(\frac{l}{l} - \frac{2}{2} \right)$	A1		correct equation
		dM1 A1	5	making T the subject correct expression for T. Simplification
		Ai	3	not necessary.
				130 130 330 130 1
(c)	$T - m\alpha - m\frac{U^2}{2}$	3.54		
	$l - mg - m \frac{l}{l}$	M1		considering the vertical forces and using
	$T - mg = m\frac{U^2}{l}$ $T = m\left(\frac{U^2}{l} + g\right)$			Newton's second law with $\frac{U^2}{I}$
	$l = m \left(\frac{l}{l} + g \right)$	A1	2	correct T
	Total		11	
5(a)	$F = 800 + \frac{1200}{20}t = 800 + 60t$	3.51		
		M1 A1		finding the gradient of the line correct gradient
	1200a = 800 + 60t	B1		correct intercept
	$a = \frac{800}{1200} + \frac{60}{1200}t = \frac{2}{3} + \frac{t}{20}$	dM1		using Newton's second law on two terms
	AG	A1	5	correct result from correct working
(b)	c^2 t c^2 c^2			
(0)	$v = \int \frac{1}{3} + \frac{1}{20} dt = \frac{1}{3} + \frac{1}{40} + c$	M1		integrating
	$v = \int \frac{2}{3} + \frac{t}{20} dt = \frac{2t}{3} + \frac{t^2}{40} + c$ $v = 0, t = 0 \Rightarrow c = 0$	A1		correct integral with or without c
	$v = \frac{2t}{3} + \frac{t^2}{40}$	A1	3	showing $c = 0$
	$s = \int_0^{20} \frac{2t}{3} + \frac{t^2}{40} dt$			
(c)	$s = \int_{0}^{20} \frac{2t}{t} + \frac{t^2}{t} dt$	M1		integrating
	$3 - J_0 = \frac{3}{3} + \frac{40}{40}$	A 1		correct integral, with or without c .
	$\begin{bmatrix} t^2 & t^3 \end{bmatrix}^{20}$	dM1		use of both limits or finding <i>c</i>
	$= \left[\frac{t^2}{3} + \frac{t^3}{120}\right]_0^{20}$	W1/11		and or oom minute or mining o
	= 200 m	A1	4	correct distance
	- • • · · ·		•	
(d)	The $\frac{2t}{3}$ term would change, because only	D.1		
	3	B1		correct term
	the constant term in the force would			
		1		I .
	change. When integrated this becomes the <i>t</i> term in the velocity.	B1	2	correct explanation

MM2B (cont)

Q Q	Solution	Marks	Total	Comments
		IVIALKS	TULAL	Comments
6(a)	$a = \frac{14^2}{50} = 3.92$	M1		finding acceleration
	50	A1		correct acceleration
	$F = 1200 \times 3.92 \text{ AG}$	dM1		use of $F = ma$
	= 4704 N	A 1	4	correct force from correct working
	170111			_
(b)	$R = 1200 \times 9.8 = 11760$	B1		normal reaction
	$4704 \le \mu \times 11760$	M1		applying $F \le \mu R$ or $F = \mu R$
	-			
	$\mu \ge \frac{4704}{11760}$ AG			
		A 1	2	a a manat manualt from a commant recording
	$\mu \ge 0.4$	A1	3	correct result from correct working
	Total		7	
7(a)	$20\frac{\mathrm{d}v}{\mathrm{d}t} = -10\sqrt{v}$	M1		applying Newton's second law with $\frac{dv}{dt}$
		A1		αι ·
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\sqrt{v}}{2}$			correct differential equation
		dM1		separating variables
	$\int \frac{1}{\sqrt{v}} dv = \int -\frac{1}{2} dt$ $2\sqrt{v} = -\frac{t}{2} + c$ $t = 0, v = 25 \Rightarrow c = 10$			
	VV 2 AU			
	$2\sqrt{v} = -\frac{l}{2} + c$	13.41		. , , , .
	2	dM1 A1		integrating
	$t = 0, v = 25 \Rightarrow c = 10$	dM1		correct integrals with or without c
		QIVI I		finding the constant of integration
	$v = \left(\frac{20-t}{4}\right)^2$	A 1	7	
		A1	7	correct final result from correct working
(b)	t = 20	B1	1	correct time
(2)	Total	<i>D</i> 1	8	
	TOTAL		75	
	TOTAL			