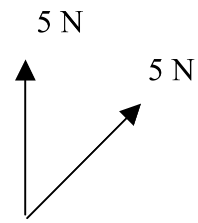
MECHANICS (C) UNIT 1 TEST PAPER 5

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A stone is projected vertically **downwards** with an initial speed of 0·6 ms^{-1} and reaches the ground after 2·9 seconds. Find the height above ground level from which it was projected. [3]

2. Two forces, both of magnitude 5 N, act on a particle in the directions with bearings 000^0 and 070^0, as shown.
Calculate
(i) the magnitude of the resultant force on the particle, [3]
(ii) the bearing on which this resultant force acts. [2]



3. The acceleration of a particle P is $(8t - 18)$ ms^{-2}, where t seconds is the time that has elapsed since P passed through a fixed point O on the straight line on which it is moving.
At time $t = 3$, P has speed 2 ms^{-1}. Find
(i) the velocity of P at time t, [3]
(ii) the values of t when P is instantaneously at rest. [3]

4. A car, of mass 1800 kg, pulls a trailer of mass 350 kg along a straight horizontal road. When the car is accelerating at 0·2 ms^{-2}, the resistances to the motion of the car and trailer have magnitudes 300 N and 100 N respectively. Find, at this time,
(i) the driving force produced by the engine of the car, [3]
(ii) the tension in the tow-bar between the car and the trailer. [4]

5. A train starts from rest at a station S and accelerates at a constant rate for $2x$ seconds to a speed of $5x$ ms^{-1}. It maintains this speed until 126 seconds after it left S and then decelerates at a constant rate until it comes to rest at another station T, $20x$ seconds after it left S.
(i) Sketch a velocity-time graph for this journey. [4]
Given that the distance between S and T is 5·4 km,
(ii) show that $x^2 + 7x = 120$. [4]
(iii) Find the value of x. [3]

6. *A, B* and *C* are three small spheres of equal radius and masses $2m$, m and $5m$ respectively. They are placed in a straight line on a smooth horizontal surface. *A* is projected with speed 6 ms^{-1} towards *B*, which is at rest. After they collide, *B* starts to move with speed 8 ms^{-1}.

(i) Find the speed of *A* after it collides with *B*. [3]

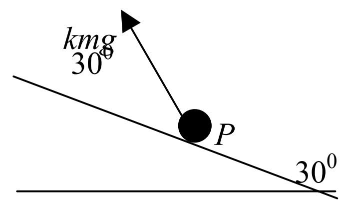
After travelling 3 m, *B* hits *C*, which is then travelling towards *B* at 2·2 ms^{-1}. *C* is brought to rest by this impact.

(ii) Show that the direction of *B*'s motion is reversed and find its new speed. [3]

(iii) Find how far *B* now travels before it collides with *A* again. [6]

(iv) State a modelling assumption that you have made about the spheres. [1]

7. A particle *P*, of mass m, is in contact with a rough plane inclined at 30Ý to the horizontal as shown. A light string is attached to *P* and makes an angle of 30Ý with the plane. When the tension in this string has magnitude kmg, *P* is just on the point of moving up the plane.



(i) Show that μ, the coefficient of friction between *P* and the plane, is $\dfrac{k\sqrt{3}-1}{\sqrt{3}-k}$. [6]

(ii) Given further that $k = \dfrac{3\sqrt{3}}{7}$, deduce that $\mu = \dfrac{\sqrt{3}}{6}$. [3]

The string is now removed.

(iii) Determine whether *P* will move down the plane and, if it does, find its acceleration. [5]

(iv) Give a reason why the way in which *P* is shown in the diagram might not be consistent with the modelling assumptions that have been made. [1]

MECHANICS 1 (C) TEST PAPER 5 : ANSWERS AND MARK SCHEME

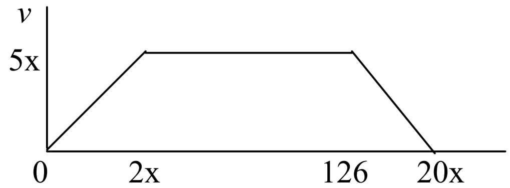
1. ~~$s = ut + \frac{1}{2}at^2$: $h = 0{\cdot}6(2{\cdot}9) + 4{\cdot}9(2{\cdot}9)^2 = 42{\cdot}9$ m~~ M1 A1 A1 3

2. (i) $R = 2(5 \sin 55^0) = 8{\cdot}19$ N M1 A1 A1
(ii) Bearing = $70 \div 2 = 035^0$ M1 A1 5

3. (i) $v = \int \text{adt} = 4t^2 - 18t + c$ $v(3) = 2 : c = 20$ $v = 4t^2 - 18t + 20$ M1 A1 A1
(ii) $v = 0 : 2(t-2)(2t-5) = 0$ $t = 2, t = 2{\cdot}5$ M1 A1 A1 6

4. (i) $F - 400 = 2150 \times 0{\cdot}2$ $F = 400 + 430 = 830$ N M1 A1 A1
(ii) $F - 300 - T = 1800 \times 0{\cdot}2$ $T = 530 - 360 = 170$ N M1 A1 M1 A1 7

5. (i)



B2 graph
B2 labelling

(ii) Area = $\frac{1}{2}\times 5x\,(20x + 126 - 2x) = 45x^2 + 315x = 5400$ (given) — M1 M1 A1

$\div 45 : x^2 + 7x = 120$ — A1

(iii) $x^2 + 7x - 120 = 0$ $\quad (x - 8)(x + 15) = 0$ $\quad x = 8$ — M1 A1 A1 — 11

6. (i) Momentum : $2m(6) = 2mv_A + 8m$ $\quad v_A = 2 \text{ ms}^{-1}$ — M1 A1 A1

(ii) $8m - 11m = mv_B$ $\quad v_B = -3$, i.e. 3 ms^{-1} in reverse direction — M1 A1 A1

(iii) B has moved 3 m in s, during which time A has moved 0·75 m — M1 A1

so A and B are 2·25 m apart. Let d = required distance : — A1

$d \div 3 = (2{\cdot}25 - d) \div 2$ $\quad 2d = 6{\cdot}75 - 3d$ $\quad d = 1{\cdot}35$ m — M1 A1 A1

(iv) Modelled as particles, so width of spheres is negligible — B1 — 13

7. (i) Resolve // to plane : $\mu R + mg \sin 30^0 = kmg \cos 30^0$ — M1 A1

Resolve perp. to plane : $R + kmg \sin 30^0 = mg \cos 30^0$ — M1 A1

$\mu mg(\frac{\sqrt{3}}{2} - \frac{1}{2}k) = mg(\frac{k\sqrt{3}}{2} - \frac{1}{2})$ $\quad \mu = \frac{k\sqrt{3} - 1}{\sqrt{3} - k}$ — M1 A1

(ii) With $k = \frac{3\sqrt{3}}{7}$ $\quad \mu = (\,{}^{9}/_{7} - 1) \div \frac{4\sqrt{3}}{7} = \frac{\sqrt{3}}{6}$ — M1 A1 A1

(iii) Force down plane = $\frac{1}{2}mg$ $\quad$ Max. friction = $\frac{\sqrt{3}}{6} \times mg\frac{\sqrt{3}}{2} = \frac{1}{4}mg$ — B1 M1 A1

so moves down with acceleration $\quad g = 2{\cdot}45 \text{ ms}^{-2}$ — M1 A1

(iv) P is shown as a ball, in which case it would roll — B1 — 15