FP3 Vectors

1. June 2010 qu.1

The line l_1 passes through the points (0, 0, 10) and (7, 0, 0) and the line l_2 passes through the points (4, 6, 0) and (3, 3, 1). Find the shortest distance between l_1 and l_2. [7]

2. June 2010 qu.7

A line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. A plane Π passes through the points (1, 3, 5) and (5, 2, 5), and is parallel to l.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r.n} = p$. [4]

(ii) Find the distance between l and Π. [4]

(iii) Find an equation of the line which is the reflection of l in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [4]

3. Jan 2010 qu. 1

Determine whether the lines $\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2}$ and $\frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$

intersect or are skew. [5]

4. Jan 2010 qu. 5

A regular tetrahedron has vertices at the points

$$A(0, 0, \tfrac{2}{3}\sqrt{6}), \quad B(\tfrac{2}{3}\sqrt{3}, 0, 0), \quad C(-\tfrac{1}{3}\sqrt{3}, 1, 0), \quad D(-\tfrac{1}{3}\sqrt{3}, -1, 0).$$

(i) Obtain the equation of the face ABC in the form $x + \sqrt{3}y + \left(\frac{1}{2}\sqrt{2}\right)z = \frac{2}{3}\sqrt{3}$. [5]

(ii) Give a geometrical reason why the equation of the face ABD can be expressed as

$$x - \sqrt{3}y + \left(\frac{1}{2}\sqrt{2}\right)z = \frac{2}{3}\sqrt{3}.$$ [2]

(iii) Hence find the cosine of the angle between two faces of the tetrahedron. [4]

5. June 2009 qu.3

A line l has equation $\frac{x-6}{-4} = \frac{y+7}{8} = \frac{z+10}{7}$ and a plane p has equation $3x - 4y - 2z = 8$.

(i) Find the point of intersection of l and p. [3]

(ii) Find the equation of the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = d$. [5]

6. June 2009 qu.6

The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$.

(i) Express the equation of Π_1 in the form $\mathbf{r.n} = p$. [4]

The plane Π_2 has equation $\mathbf{r}.\begin{pmatrix} 7 \\ 17 \\ -3 \end{pmatrix} = 21$.

(ii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [5]

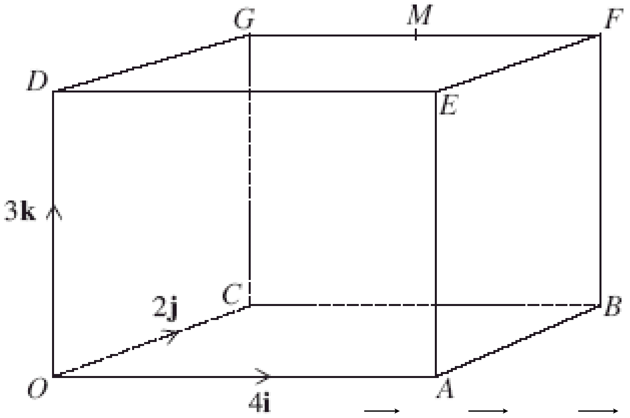
7. Jan 2009 qu. 3

Two skew lines have equations $\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3}$ and $\frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}$.

(i) Find the direction of the common perpendicular to the lines. [2]

(ii) Find the shortest distance between the lines. [4]

8. Jan 2009 qu. 6



The cuboid *OABCDEFG* shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}, \overrightarrow{OC} = 2\mathbf{j}, \overrightarrow{OD} = 3\mathbf{k}$, and *M* is the mid-point of *GF*.

(i) Find the equation of the plane *ACGE*, giving your answer in the form $\mathbf{r.n} = p$. [4]

(ii) The plane *OEFC* has equation $\mathbf{r}.(3\mathbf{i} - 4\mathbf{k}) = 0$. Find the acute angle between the planes *OEFC* and *ACGE*. [4]

(iii) The line *AM* meets the plane *OEFC* at the point *W*. Find the ratio *AW* : *WM*. [5]

9. June 2008 qu.2

Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$. [7]

10. June 2008 qu.5

Two lines have equations $\frac{x-k}{2} = \frac{y+1}{-5} = \frac{z-1}{-3}$ and $\frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2}$, where k is a constant.

(i) Show that, for all values of k, the lines intersect, and find their point of intersection in terms of k. [6]

(ii) For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$. [4]

11. Jan 2008 qu. 3

Two fixed points, A and B, have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to the origin O, and a variable point P has position vector $\mathbf{r}$.

(ii) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} = \lambda\, \mathbf{a}$, where $0 \le \lambda \le 1$. [2]

(ii) Given that P is a point on the line AB, use a property of the vector product to explain why $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \mathbf{0}$. [2]

(iii) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} \times (\mathbf{a} - \mathbf{b}) = \mathbf{0}$. [3]

12. Jan 2008 qu. 3

A tetrahedron $ABCD$ is such that AB is perpendicular to the base BCD. The coordinates of the points A, C and D are $(-1, -7, 2)$, $(5, 0, 3)$ and $(-1, 3, 3)$ respectively, and the equation of the plane BCD is $x + 2y - 2z = -1$.

(i) Find, in either order, the coordinates of B and the length of AB. [5]

(ii) Find the acute angle between the planes ACD and BCD. [6]

13. June 2007 qu.2

A line l has equation $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and a plane Π has equation $8x - 7y + 10z = 7$. Determine whether l lies in Π, is parallel to Π without intersecting it, or intersects Π at one point. [5]

14. June 2007 qu.6

Lines l_1 and l_2 have equations $\dfrac{x-3}{2} = \dfrac{y-4}{-1} = \dfrac{z+1}{1}$ and $\dfrac{x-5}{4} = \dfrac{y-1}{3} = \dfrac{z-1}{2}$ respectively.

(i) Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r}.\ \mathbf{n} = p$. [5]

(ii) Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r}.\ \mathbf{n} = p$. [2]

(iii) Find the distance between the planes Π_1 and Π_2. [2]

(iv) State the relationship between the answer to part (iii) and the lines l_1 and l_2. [1]

15. Jan 2007 qu. 2

Find the equation of the line of intersection of the planes with equations

$\mathbf{r}.(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 4$ and $\mathbf{r}.(\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}) = 6$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [5]

16. Jan 2007 qu. 7

The position vectors of the points A, B, C, D, G are given by

$\mathbf{a} = 6\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = \mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, $\mathbf{d} = 3\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$, $\mathbf{g} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ respectively.

(i) The line through A and G meets the plane BCD at M. Write down the vector equation of the line through A and G and hence show that the position vector of M is $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$. [6]

(ii) Find the value of the ratio $AG : AM$. [1]

(iii) Find the position vector of the point P on the line through C and G, such that $\overrightarrow{CP} = \frac{4}{3}\overrightarrow{CG}$. [2]

(iv) Verify that P lies in the plane ABD. [4]

17. June 2006 qu.3

Find the perpendicular distance from the point with position vector $12\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ to the line with equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + t(8\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$. [6]

18. June 2006 qu.5

A line l_1 has equation $\frac{x}{2} = \frac{y+4}{3} = \frac{z+9}{5}$

(i) Find the cartesian equation of the plane which is parallel to l_1 and which contains the points (2, 1, 5) and (0, −1, 5). [5]

(ii) Write down the position vector of a point on l_1 with parameter t. [1]

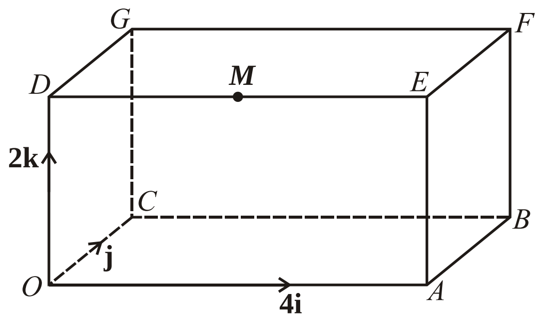
(iii) Hence, or otherwise, find an equation of the line l_2 which intersects l_1 at right angles and which passes through the point (−5, 3, 4).

Give your answer in the form $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$. [4]

19. Jan 2006 qu. 1

Find the acute angle between the skew lines $\frac{x+3}{1} = \frac{y-2}{1} = \frac{z-4}{-1}$ and $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z+3}{1}$. [4]

20. Jan 2006 qu. 6



The cuboid *OABCDEFG* shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}, \overrightarrow{OC} = \mathbf{j}, \overrightarrow{OD} = 2\mathbf{k}$, an M is the mid-point of DE.

(i) Find a vector perpendicular to $\overrightarrow{MB}$ and $\overrightarrow{OF}$ [3]

(ii) Find the cartesian equations of the planes *CMG* and *OEG*. [5]

(iii) Find an equation of the line of intersection of the planes *CMG* and *OEG*, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ [3]