





**Question 1 continued**

Lined writing area for the answer to Question 1.





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Question 2 continued

Lined area for writing answers to Question 2 continued.



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3. Find, in the form  $y = f(x)$ , the general solution of the differential equation

$\tan x \frac{dy}{dx} + y = 3 \cos 2x \tan x, \quad 0 < x < \frac{\pi}{2}$  (6)

Handwritten solution area with horizontal lines.



4. (a) Show that

$$r^2(r+1)^2 - (r-1)^2 r^2 \equiv 4r^3 \tag{3}$$

Given that  $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$

(b) use the identity in (a) and the method of differences to show that

$$(1^3 + 2^3 + 3^3 + \dots + n^3) = (1 + 2 + 3 + \dots + n)^2 \tag{4}$$

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5. A transformation  $T$  from the  $z$ -plane to the  $w$ -plane is given by

$$w = \frac{z}{z + 3i}, \quad z \neq -3i$$

The circle with equation  $|z| = 2$  is mapped by  $T$  onto the curve  $C$ .

(a) (i) Show that  $C$  is a circle.

(ii) Find the centre and radius of  $C$ .

**(8)**

The region  $|z| \leq 2$  in the  $z$ -plane is mapped by  $T$  onto the region  $R$  in the  $w$ -plane.

(b) Shade the region  $R$  on an Argand diagram.

**(2)**

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6.

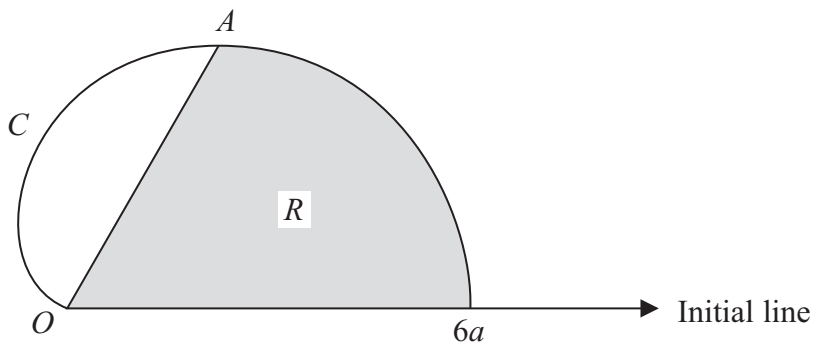


Figure 1

The curve  $C$ , shown in Figure 1, has polar equation

$$r = 3a(1 + \cos\theta), \quad 0 \leq \theta < \pi$$

The tangent to  $C$  at the point  $A$  is parallel to the initial line.

(a) Find the polar coordinates of  $A$ .

(6)

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve  $C$ , the initial line and the line  $OA$ .

(b) Use calculus to find the area of the shaded region  $R$ , giving your answer in the form  $a^2(p\pi + q\sqrt{3})$ , where  $p$  and  $q$  are rational numbers.

(5)

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7.

$$y = \tan^2 x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Show that  $\frac{d^2 y}{dx^2} = 6 \sec^4 x - 4 \sec^2 x$  (4)

(b) Hence show that  $\frac{d^3 y}{dx^3} = 8 \sec^2 x \tan x (A \sec^2 x + B)$ , where  $A$  and  $B$  are constants to be found. (3)

(c) Find the Taylor series expansion of  $\tan^2 x$ , in ascending powers of  $\left(x - \frac{\pi}{3}\right)$ , up to and including the term in  $\left(x - \frac{\pi}{3}\right)^3$  (4)

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**Question 7 continued**

Lined area for writing the answer to Question 7.



8. (a) Show that the transformation  $x = e^u$  transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 7x \frac{dy}{dx} + 16y = 2 \ln x, \quad x > 0 \tag{I}$$

into the differential equation

$$\frac{d^2y}{du^2} - 8 \frac{dy}{du} + 16y = 2u \tag{II}$$

**(6)**

(b) Find the general solution of the differential equation (II), expressing  $y$  as a function of  $u$ .

**(7)**

(c) Hence obtain the general solution of the differential equation (I).

**(1)**

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