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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE
in Further Pure Mathematics FP2
(6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ☐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\text{ where } |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q),\text{ where } |pq| = |c| \text{ and } |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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6668 FP2
Mark Scheme

Question Number	Scheme	Marks
1		
(a)	$(x+2)(x+3)^2-12(x+3)=0$ OR $\dfrac{(x+3)(x+2)-12}{(x+3)}>0$	M1
	$(x+3)(x^2+5x-6)=0 \quad (x+3)(x+6)(x-1)=0$	
	CVs: $-3,\ -6,\ 1$	B1,A1,A1
	$-6<x<-3, \quad x>1$	dM1A1
	OR: $x\in(-6,-3)\cup(1,\infty)$	(6)
(b)	$x>1$	B1 (1) [7]

(a)

M1 Mult through by $(x+3)^2$ and collect on one side or use any other valid method (NOT calculator)

Eg work from $\dfrac{(x+3)(x+2)-12}{(x+3)}>0$

NB: Multiplying by $(x+3)$ is **not** a valid method unless the two cases $x>3$ and $x<3$ are considered separately or -3 stated to be a cv

B1 for -3 seen anywhere

A1A1 other cvs (A1A0 if only one correct)

dM1 obtaining inequalities using their critical values and no other numbers. Award if one correct inequality seen or any valid method eg sketch graph or number line seen

A1 correct inequalities and no extras. Use of … or „ scores A0. May be written in set notation.

No marks for candidates who draw a sketch graph and follow with the cvs without any algebra shown. **Those who use some algebra** after their graph may gain marks as earned (possibly all)

(b) B1 correct answer only shown. Allow $x\ldots1$ if already penalised in (a)

Question Number	Scheme	Marks
2 (a)	$\lvert z\rvert = 4$	B1
	$\arg z = \arctan\left(\frac{-2\sqrt{3}}{2}\right) = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3}$ or 120°	M1A1 (3)
(b)	$z^6 = \left(4\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)\right)^6 = 4^6\left(\cos 4\pi + \mathrm{i}\sin 4\pi\right)$ or $z^6 = \left(4\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}\right)^6$	M1
	$= 4096$ or 4^6 or 2^{12}	A1 cso (2)
	(a) and (b) can be marked together	
(c)	$z^{\frac{3}{4}} = 4^{\frac{3}{4}}\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)^{\frac{3}{4}} = 4^{\frac{3}{4}}\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$	
	$w = \mathrm{i}2\sqrt{2}$ oe or any other correct root	B1
	$4^{\frac{3}{4}}\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + \mathrm{i}\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)^{\frac{3}{4}}$	M1
	$(n = 0$ see above$)$	
	$n = 1$ $w = 2\sqrt{2}$ oe	
	$n = 2$ $w = -\mathrm{i}2\sqrt{2}$ oe	
	$n = 3$ $w = -2\sqrt{2}$ oe	A1A1 (4)
		[9]

(a) B1 Correct modulus seen **Must** be 4

M1 Attempt arg using arctan, nos either way up. Must include minus sign or other consideration of quadrant. (arg = $\frac{\pi}{3}$ scores M0)

A1 $\frac{2\pi}{3}$ or 120^0 Correct answer only seen, award M1A1

(b) M1 apply de Moivre

A1cso 4096 or 4^6 Must have been obtained with the correct argument for z

(c) B1 For $w = \mathrm{i}2\sqrt{2}$ or any single correct root (0 or 0i may be included in all roots) in any Form including polar

M1 Applying de Moivre and use a correct method to attempt 2 or 3 further roots

A1A1 For the other roots (3 correct scores A1A1; 2 correct scores A1)

Accept eg $2\sqrt{2}, \sqrt{8}$, 2.83, $64^{\frac{1}{4}}$, $4^{\frac{3}{4}}$, $4096^{\frac{1}{8}}$ Decimals must be 3 sf min.

ALT 1 for (c): $z^3 = 64 = w^4 \Rightarrow w = (\pm)2\sqrt{2}$ ($\pm$ not needed) B1

Use rotational symmetry to find other 2/3 roots M1

Remaining roots as above A1A1

ALT 2: $z^4 = 64$ $z^2 = \pm 8$

$z = \pm 2\sqrt{2}$ $z = \pm\sqrt{-8} = \pm \mathrm{i}2\sqrt{2}$

B1 any one correct, M1 attempt remaining 2/3 roots; A1A1 as above

Question Number	Scheme		Marks
3	$\frac{dy}{dx}+\frac{y}{\tan x}=3\cos 2x$		
	$\int \cot x\,dx=\ln\lvert\sin x\rvert,\quad \text{IF}=\sin x$		M1
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$		
	$y\sin x=\int 3\cos 2x\sin x\,dx$		M1A1
	$y\sin x=\int 3\left(2\cos^2 x-1\right)\sin x\,dx$	$y\sin x=\frac{3}{2}\int \left(\sin 3x-\sin x\right)dx$	
	$y\sin x=3\left[-\frac{2}{3}\cos^3 x+\cos x\right](+c)$	$y\sin x=\frac{3}{2}\left[-\frac{1}{3}\cos 3x+\cos x\right](+c)$	dM1A1
	$y=\frac{3\cos x-2\cos^3 x+c'}{\sin x}$ oe	$y=\frac{-3\cos 3x+3\cos x+c'}{2\sin x}$	B1ft [6] (A1 on e-PEN)

M1 Divide by tan and attempt IF $e^{\int \cot x\,dx}$ or equivalent needed

M1 Multiply through by IF and integrate LHS

A1 correct so far

dM1 dep (on previous M mark) integrate RHS using double angle or factor formula

$$k\cos^2 x\sin x\to\pm\cos^3 x,\ k\sin^2 x\cos x\to k\sin^3 x,\ \cos 3x\to\pm\frac{1}{3}\sin 3x,\ \sin 3x\to\pm\frac{1}{3}\cos 3x$$

A1 All correct so far constant not needed

B1ft obtain answer in form $y=$ **...** any equivalent form Constant must be included and dealt with correctly. Award if correctly obtained from the previous line

Alternatives for integrating the RHS:

(i) By parts: Needs 2 applications of parts or one application followed by a trig method. Give M1 only if method is complete and A1 for a correct result.

(ii) $y\sin x=\int 3\left(1-2\sin^2 x\right)\sin x\,dx=\int 3\sin x-6\sin^3 x\,dx$

Then use $\sin 3x=3\sin x-4\sin^3 x$ to get $y\sin x=\int \frac{3}{2}\left(\sin 3x-\sin x\right)dx$ and integration shown above - both steps needed for M1

	ALTERNATIVE: Mult through by $\cos x$	
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$	M1
	$y\sin x=\int 3\cos 2x\sin x\,dx$	M1A1
	Rest as main scheme	

Question Number	Scheme	Marks
4 **(a)**	$r^2\left(r^2+2r+1\right)-\left(r^2-2r+1\right)r^2$	M1 A1
	$\equiv r^4+2r^3+r^2-r^4+2r^3-r^2$ or $r^2\left(r^2+2r+1-r^2+2r-1\right)$	
	$\equiv 4r^3$ *	A1 (3)
(b)	$\left(\sum_1^n 4r^3=\right)$ $\left(1\times2^2-0\right)+\left(2^2\times3^2-1^2\times2^2\right)+\left(3^2\times4^2-2^2\times3^2\right)\ldots$	M1
	$+\left(n^2\times(n+1)^2-(n-1)^2\times n^2\right)$	
	$=n^2\left(n+1\right)^2$	A1
	$\sum_1^n r^3=\frac{1}{4}n^2\left(n+1\right)^2$	A1
	$\therefore \sum_1^n r^3=\left(\frac{1}{2}n\left(n+1\right)\right)^2=\left(\sum_1^n r\right)^2$	
	So $\left(1^3+2^3+3^3+\ldots.+n^3\right)=\left(1+2+3\ldots+n\right)^2$ *	A1cso (4) [7] (B1 on e-PEN)

(a) M1 Multiply out brackets May remove common factor r^2 first

A1 a correct statement

A1 fully correct solution which must include at least one intermediate line

ALT: Use difference of 2 squares:

M1 remove common factor and apply diff of 2 squares to rest

A1 $r^2\left(r+1+r-1\right)\left(r+1-\left(r-1\right)\right)$

$=r^2\left(2r\times2\right)$

A1 $=4r^3$

(b) M1 Use result to write out a list of terms; sufficient to show cancelling needed

Minimum 2 at start and 1 at end $\sum_1^n 4r^3$ or $\sum_1^n r^3$ need not be shown here or for next mark

A1 Correctly extracting $n^2\left(n+1\right)^2$ as the only remaining non-zero term.

A1 Obtaining $\sum_1^n r^3=\frac{1}{4}n^2\left(n+1\right)^2$

A1cso (Shown B1 on e-PEN) for deducing the required result.

Working from **either** side can gain full marks

Working from **both** sides can gain full marks provided the working joins correctly in the middle.

If ***r* used instead of *n***, penalise the final A mark.

Question Number	Scheme	Marks
5 (a)	$w = \dfrac{z}{z+3\mathrm{i}}$	
	$w(z+3\mathrm{i}) = z \quad z = \dfrac{3\mathrm{i}w}{1-w}$ or $\dfrac{-3\mathrm{i}w}{w-1}$	M1A1
	$\lvert z\rvert = 2 \quad \left\lvert\dfrac{3\mathrm{i}w}{1-w}\right\rvert = 2$	dM1
	$\lvert 3\mathrm{i}w\rvert = 2\lvert 1-w\rvert$	
	$w = u + \mathrm{i}v \quad 9(u^2+v^2) = 4\left((1-u)^2+v^2\right)$	ddM1A1
	$9u^2+9v^2 = 4(1-2u+u^2+v^2)$	
(i)	$5u^2+5v^2+8u-4=0$	dddM1
	$\left(u+\dfrac{4}{5}\right)^2 + v^2 = \dfrac{36}{25}$	
(ii)	So a circle, Centre $\left(-\dfrac{4}{5}, 0\right)$ Radius $\dfrac{6}{5}$ (oe fractions or decimals)	A1A1 (8)
(b)	Circle drawn on an Argand diagram in correct position ft their centre and radius	B1ft
	Region inside correct circle shaded no ft	B1 (2) **[10]**

(a) M1 re-arrange to $z = ...$

A1 correct result

dM1 dep (on first M1) using $\lvert z\rvert = 2$ with their previous result

ddM1 dep (on both previous M marks) use $w = u + \mathrm{i}v$ (or eg $w = x + \mathrm{i}y$) and find the moduli. Moduli to contain no is and must be +. Allow 9 or 3 and 4 or 2

A1 for a correct equation in u and v or any other pair of variables

dddM1 dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms

A1A1 deduce circle and give correct centre and radius. Completion of square may not be shown. Deduct 1 for each error or omission. (Enter A1A0 on e-PEN)

Special Case: If $z = \dfrac{3\mathrm{i}w}{w-1}$ obtained, give M1A0 but all other marks can be awarded.

(b) Mark diagram only - ignore any working shown.

B1ft No numbers needed but circle must be in the correct region (or on the correct axis) for *their* centre and the centre and radius must be consistent (ie check how the circle crosses the axes) B0 if the equation in (a) is not an equation of a circle.

B1 Region inside the **correct** circle shaded. (no ft here)

Question Number	Scheme	Marks
	ALTERNATIVE for 5(a):	
	Let $z = x + \mathrm{i}y$	
	$w = \dfrac{x+\mathrm{i}y}{x+\mathrm{i}(y+3)}$	
	$= \dfrac{(x+\mathrm{i}y)(x-\mathrm{i}(y+3))}{(x+\mathrm{i}(y+3))(x-\mathrm{i}(y+3))}$	M1
	$= \dfrac{x^2+y^2+3y-3\mathrm{i}x}{x^2+y^2+6y+9}$	A1
	$\dfrac{3y+4-3\mathrm{i}x}{6y+13}$ as $\lvert z\rvert = 2 \Rightarrow x^2+y^2=4$	dM1
	$w = u+\mathrm{i}v$ so $u = \dfrac{3y+4}{6y+13}$ $v = \dfrac{-3x}{6y+13}$	ddM1 A1
	Using $u = \dfrac{\frac{1}{2}(6y+13)}{6y+13} - \dfrac{\frac{5}{2}}{6y+13}$	
	$u^2+v^2 = \dfrac{9y^2+24y+16+9x^2}{(6y+13)^2} = \dfrac{24y+52}{(6y+13)^2} = \dfrac{4}{6y+13}$	
	$= \dfrac{8}{5}\left(\dfrac{1}{2}-u\right)$	dddM1
	$\therefore 5u^2+5v^2+8u=4$ Then as main scheme: Circle, centre, radius	A1A1 (8)

M1 Rationalise the denominator - must use conjugate of the denominator
A1 Expand brackets and obtain correct numerator and denominator
dM1 Use $x^2+y^2=4$ in their expression to remove the squares
ddM1 Equating real and imaginary parts
A1 Correct expressions for u and v in terms of x and y
dddM1 Uses $u^2+v^2 = \ldots$ to eliminate x and y and obtain an equation of the circle
A1A1 As main scheme

Question Number	Scheme	Marks
6 (a)	$r\sin\theta = 3a\sin\theta + 3a\sin\theta\cos\theta$ OR $3a\sin\theta + \frac{3}{2}a\sin 2\theta$	M1
	$\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 3a\cos\theta + 3a\cos^2\theta - 3a\sin^2\theta$ $\quad$ $3a\cos\theta + 3a\cos 2\theta$	dM1
	$2\cos^2\theta + \cos\theta - 1 = 0$ terms in any order	A1
	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	
	$\cos\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{3}$ $\quad (\theta = \pi$ need not be seen$)$	ddM1A1
	$r = 3a \times \frac{3}{2} = \frac{9}{2}a$	A1 (6)
(b)	Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{3}} 9a^2(1+\cos\theta)^2 \mathrm{d}\theta$	
	$= \frac{9a^2}{2}\int_0^{\frac{\pi}{3}}(1 + 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$	M1
	$= \frac{9a^2}{2}\int_0^{\frac{\pi}{3}}\left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right)\mathrm{d}\theta$	M1
	$= \frac{9a^2}{2}\left[\theta + 2\sin\theta + \frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$\frac{9a^2}{2}\left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} \quad (-0)\right]$	
	$\frac{9a^2}{2}\left[\frac{\pi}{2} + \frac{9\sqrt{3}}{8}\right] = \left(\frac{9\pi}{4} + \frac{81\sqrt{3}}{16}\right)a^2$	A1 (5) [11]

(a)M1 using $r\sin\theta$ $\quad r\cos\theta$ scores M0

dM1 Attempt the differentiation of $r\sin\theta$, inc use of product rule or $\sin 2\theta = 2\sin\theta\cos\theta$

A1 Correct 3 term quadratic in $\cos\theta$

ddM1 dep on both M marks. Solve their quadratic (usual rules) giving one or two roots

A1 Correct quadratic solved to give $\theta = \frac{\pi}{3}$

A1 Correct r obtained $\quad$ No need to see coordinates together in brackets

Special Case: If $r\cos\theta$ used, score M0M1A0M0A0A0to

(b)M1 Use of correct area formula, $\frac{1}{2}$ may be seen later, inc squaring the bracket to obtain 3 terms - limits need not be shown.

M1 Use double angle formula (formula to be of form $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$) to obtain an integrable function - limits need not be shown, $\frac{1}{2}$ from area formula may be missing,

dM1 attempt the integration - limits not needed – dep on 2nd M mark but not the first

A1 correct integration – substitution of limits not required

A1 correct final answer any equivalent provided in the demanded form.

Question Number	Scheme	Marks
7 (a)	$\frac{dy}{dx} = 2\tan x \sec^2 x$	B1
	OR $\frac{dy}{dx} = 2\tan x\left(1+\tan^2 x\right)$	
	$\frac{d^2y}{dx^2} = 2\sec^4 x + 4\tan^2 x \sec^2 x$; OR $\frac{d^2y}{dx^2} = 2\sec^2 x + 2\times 3\tan^2 x \sec^2 x$	M1 A1
	$= 2\sec^4 x + 4\left(\sec^2 x - 1\right)\sec^2 x$; OR $= 2\sec^2 x + 6\left(\sec^2 x - 1\right)\sec^2 x$	
	$= 6\sec^4 x - 4\sec^2 x$ *	A1cso (4)
(b)	$\frac{d^3y}{dx^3} = 24\sec^3 x \sec x \tan x - 8\sec^2 x \tan x$	M1A1
	$= 8\sec^2 x \tan x\left(3\sec^2 x - 1\right)$	A1cso (3)
(c)	$y_{\frac{\pi}{3}} = \left(\sqrt{3}\right)^2 \ (=3)$ $\quad \left(\frac{dy}{dx}\right)_{\frac{\pi}{3}} = 2\sqrt{3}\times\left(\frac{2}{1}\right)^2 \ \left(=8\sqrt{3}\right)$	B1(both)
	$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{3}} = 6\times 2^4 - 4\times 2^2 = 80$	
	$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{3}} = 8\times 4\times\sqrt{3}\left(3\times 2^2 - 1\right) = 352\sqrt{3}$	M1(attempt both)
	$\tan^2 x = y_{\frac{\pi}{3}} + \left(x-\frac{\pi}{3}\right)\left(\frac{dy}{dx}\right)_{\frac{\pi}{3}} + \frac{1}{2!}\left(x-\frac{\pi}{3}\right)^2\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{3}} + \frac{1}{3!}\left(x-\frac{\pi}{3}\right)^3\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{3}}$	
	$= 3 + 8\sqrt{3}\left(x-\frac{\pi}{3}\right) + 40\left(x-\frac{\pi}{3}\right)^2 + \frac{176}{3}\sqrt{3}\left(x-\frac{\pi}{3}\right)^3$	M1A1 (4)[11]

(a)B1 $\frac{dy}{dx} = 2\tan x \sec^2 x$

M1 attempting the second derivative, inc using the product rule or $\sec^2\theta = \tan^2\theta + 1$ **Must** start from the result given in (a)

A1 a correct second derivative in any form

A1cso for a correct result following completely correct working $\sec^2\theta = \tan^2\theta + 1$ must be d seen or used

Question Number	Scheme	Marks
(b) M1	attempting the third derivative, inc using the chain rule	
A1	a correct derivative	
A1	a completely correct final result	
(c) B1	$y_{\frac{\pi}{3}} = \left(\sqrt{3}\right)^2$ or 3 **and** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\frac{\pi}{3}} = 2\sqrt{3}\times\left(\frac{2}{1}\right)^2$ or $8\sqrt{3}$	
M1	obtaining values for second and third derivatives at $\frac{\pi}{3}$ (need not be correct but must be obtained from their derivatives)	
M1	using a correct Taylor's expansion using $\left(x-\frac{\pi}{3}\right)$ and their derivatives. (2! or 2, 3! or 6 must be seen or implied by the work shown) This mark is not dependent.	
A1	for a correct final answer **Must** start $\tan^2 x = ...$ or $y = ...$ f(x) scores A0 unless defined as $\tan^2 x$or y here or earlier. Accept equivalents eg awrt 610 (609.6…) $\sqrt{371712}$ But no factorials in this final answer.	

Question Number	Scheme	Marks
8 (a)	$x = e^u \quad \frac{dx}{du} = e^u$ or $\frac{du}{dx} = e^{-u}$ or $\frac{dx}{du} = x$ or $\frac{du}{dx} = \frac{1}{x}$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -e^{-u}\frac{du}{dx}\frac{dy}{du} + e^{-u}\frac{d^2y}{du^2}\frac{du}{dx} = e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right)$	M1A1
	$x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 16y = 2\ln x$	
	$e^{2u} \times e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right) - 7e^u \times e^{-u}\frac{dy}{du} + 16y = 2\ln\left(e^u\right)$	dM1
	$\frac{d^2y}{du^2} - 8\frac{dy}{du} + 16y = 2u$ *	A1cso (6)

(a) B1 for $\frac{dx}{du} = e^u$ oe as shown seen explicitly or used

M1 obtaining $\frac{dy}{dx}$ using chain rule here or seen later

M1 obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark)

A1 a correct expression for $\frac{d^2y}{dx^2}$ any equivalent form

dM1 substituting in the equation to eliminate x **Only** u and y now Depends on the 2nd M mark

A1cso obtaining the given result from completely correct work

	ALTERNATIVE 1	
	$x = e^u \quad \frac{dx}{du} = e^u = x$	B1
	$\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du} = x\frac{dy}{dx}$	M1
	$\frac{d^2y}{du^2} = 1\frac{dx}{du} \times \frac{dy}{dx} + x\frac{d^2y}{dx^2} \times \frac{dx}{du} = x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2}$	M1A1
	$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$	
	$\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) - 7x \times \frac{1}{x}\frac{dy}{du} + 16y = 2\ln\left(e^u\right)$	
	$\frac{d^2y}{du^2} - 8\frac{dy}{du} + 16y = 2u$ *	dM1A1cso (6)

B1 As above

M1 obtaining $\frac{dy}{du}$ using chain rule here or seen later

M1 obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)

Question Number	Scheme	Marks
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1A1cso	As main scheme	
	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}\right) - 7x \times \frac{1}{x}\frac{dy}{du} + 16y = 2u$	
	$\frac{d^2y}{du^2} - 8\frac{dy}{du} + 16y = 2u$ *	dM1A1cso

See the notes for the main scheme.

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2 - 8m + 16 = 0$	
	$(m-4)^2 = 0 \quad m = 4$	M1A1
	(CF =) $(A + Bu)e^{4u}$	A1
	PI: try $y = au + b$ (or $y = cu^2 + au + b$ different derivatives, c = 0)	
	$\frac{dy}{du} = a \quad \frac{d^2y}{du^2} = 0$	M1
	$0 - 8a + 16(au + b) = 2u$	
	$a = \frac{1}{8} \quad b = \frac{1}{16}$ oe (decimals must be 0.125 and 0.0625)	dM1A1
	$\therefore y = (A + Bu)e^{4u} + \frac{1}{8}u + \frac{1}{16}$	B1ft (7)
(c)	$y = (A + B\ln x)x^4 + \frac{1}{8}\ln x + \frac{1}{16}$	B1 (1)
		[14]

(b) M1 writing down the correct aux equation and solving to m = … (usual rules)
A1 the correct solution ($m = 4$)
A1 the correct CF – can use any (single) variable
M1 using an appropriate PI and finding $\frac{dy}{du}$ **and** $\frac{d^2y}{du^2}$ Use of $y = \lambda u$ scores M0
dM1 substitute in the equation to obtain values for the unknowns Dependent on the second M1
A1 correct unknowns two or three (c = 0)
B1ft a complete solution, follow through their CF and PI. Must have y = a function of u
Allow recovery of incorrect variables.

(c) B1 reverse the substitution to obtain a correct expression for y in terms of x No ft here
x^4 or $e^{4\ln x}$ allowed. Must start y = …

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