

4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

- (a) show that $2.2 < \alpha < 2.3$ (2)
- (b) Taking 2.2 as a first approximation to α , apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α , giving your answer to 3 decimal places. (5)
- (c) Use linear interpolation once on the interval $[2.2, 2.3]$ to find another approximation to α , giving your answer to 3 decimal places. (3)



6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C . (1)

(b) Write down the coordinates of the focus S of C . (1)

(c) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3 \quad (5)$$

The normal to C at P meets the x -axis at the point N .

(d) Find the area of triangle PSN in terms of t , giving your answer in its simplest form. (4)



7. $\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}$, where a is a constant.

(a) Find the value of a for which the matrix \mathbf{A} is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find \mathbf{B}^{-1} .

(3)

The transformation represented by \mathbf{B} maps the point P onto the point Q .

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.

(3)



8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(7)

