

5.

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

- (a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.1, 1.2]$. (2)
- (b) Find $f'(x)$. (3)
- (c) Using $x_0 = 1.1$ as a first approximation to α , apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α , giving your answer to 3 significant figures. (4)



8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point $Q (aq^2, 2aq)$ lies on the parabola.

(a) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2. \tag{4}$$

This tangent meets the y -axis at the point R .

(b) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q . (3)

(c) Show that l passes through the focus of the parabola. (1)

(d) Find the coordinates of the point where l meets the directrix of the parabola. (2)



9. Given that $z_1 = 3 + 2i$ and $z_2 = \frac{12 - 5i}{z_1}$,

(a) find z_2 in the form $a + ib$, where a and b are real. (2)

(b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2 . (2)

(c) Given that O is the origin, show that $\angle POQ = \frac{\pi}{2}$. (2)

The circle passing through the points O , P and Q has centre C . Find

(d) the complex number represented by C , (2)

(e) the exact value of the radius of the circle. (2)



10. $\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Describe fully the transformations described by each of the matrices **A**, **B** and **C**. (4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(b) Find **D**. (2)

(c) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$. (1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (−15, 15) and (4, 21). This triangle is transformed onto the triangle *OR'S'* by the transformation described by **E**.

(d) Find the coordinates of the vertices of triangle *OR'S'*. (4)

(e) Find the area of triangle *OR'S'* and deduce the area of triangle *ORS*. (3)

