AQA Maths FP1

Mark Scheme Pack

2006-2014

Version 1.0: 0106

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

Mark Scheme

2006 examination - January series

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$-x$ EE	deduct x marks for each error	G	graph
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SCA	substantially correct approach	dp	decimal place(s)

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MFP1

Q	Solution	Marks	Totals	Comments
1(a)	f(0.5) = −0.875, f(1) = 1	B1		
	Change of sign, so root between	E1	2	
(b)	Complete line interpolation method	M2,1		M1 for partially correct method
	Estimated root = $\frac{11}{15} \approx 0.73$	A1	3	Allow $\frac{11}{15}$ as answer
	Total		**5**	
2(a)(i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{\frac{1}{2}}$
	$\int_0^9 \frac{1}{\sqrt{x}}\,dx = 6$	A1√	3	ft wrong coeff of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{1}{2}}\,dx = -2x^{-\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{-\frac{1}{2}}$
	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$, so no value	E1	3	'Tending to infinity' clearly implied
(b)	Denominator → 0 as $x \to 0$	E1	1	
	Total		**7**	
3	One solution is $x = 10°$	B1		PI by general formula
	Use of sin 130º = sin 50º	M1		OE
	Second solution is $x = 30°$	A1		OE
	Introduction of $90n$º, or $360n$º or $180n$º	M1		Or $\pi n/2$ or $2\pi n$ or πn
	GS $(10+90n)°, (30+90n)°$	A1√	5	OE; ft one numerical error or omission of 2nd soln
	Total		**5**	
4(a)	Asymptotes $x = 1$, $y = 6$	B1B1	2	
(b)	Curve (correct general shape)	M1		SC Only one branch:
	Curve passing through origin	A1		B1 for origin
	Both branches approaching $x = 1$	A1		B1 for approaching both asymptotes
	Both branches approaching $y = 6$	A1	4	(Max 2/4)
(c)	Correct method	M1		
	Critical values ±1	B1B1		From graph or calculation
	Solution set $-1 < x < 1$	A1√	4	ft one error in CVs; NMS 4/4 after a good graph
	Total		**10**	
5(a)(i)	Full expansion of product	M1		
	Use of $i^2 = -1$	m1		
	$(2+\sqrt{5}i)(\sqrt{5}-i) = 3\sqrt{5}+3i$	A1	3	$\sqrt{5}\sqrt{5} = 5$ must be used − Accept not fully simplified
(ii)	$z^* = x - iy\ (= \sqrt{5} + i)$	M1		
	Hence result	A1	2	Convincingly shown (AG)
(b)(i)	Other root is $\sqrt{5} + i$	B1	1	
(ii)	Sum of roots is $2\sqrt{5}$	B1		
	Product is 6	M1A1	3	
(iii)	$p = -2\sqrt{5}$, $q = 6$	B1		
		B1√	2	ft wrong answers in (ii)
	Total		**11**	

MFP1

Q	Solution	Marks	Totals	Comments
6(a)	X values 1.23, 2.18			
	Y values 0.70, 1.48	B3,2,1	3	−1 for each error
(b)	$\lg y = \lg k + \lg x^n$	M1		
	$\lg x^n = n \lg x$	M1		
	So $Y = nX + \lg k$	A1	3	
(c)	Four points plotted	B2,1√		B1 if one error here; ft wrong values in (a)
	Good straight line drawn	B1√	3	ft incorrect points (approx collinear)
(d)	Method for gradient	M1		
	Estimate for n	A1√	2	Allow AWRT 0.75 - 0.78; ft grad of candidate's graph
	Total		**11**	
7(a)(i)	Reflection ...	M1		
	... in $y = -x$	A1	2	OE
(ii)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1A0 for three correct entries
(iii)	$\mathbf{A}^2 = \mathbf{I}$ or geometrical reasoning	E1	1	
(b)(i)	$\mathbf{B}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	M1A1		M1A0 for three correct entries
	$\mathbf{B}^2 - \mathbf{A}^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$	A1√	3	ft errors, dependent on both M marks
(ii)	$(\mathbf{B}+\mathbf{A})(\mathbf{B}-\mathbf{A}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$	B1		
	$\ldots = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$	M1 A1√	3	ft one error; M1A0 for three correct (ft) entries
	Total		**11**	
8(a)	Good attempt at sketch	M1		
	Correct at origin	A1	2	
(b)(i)	y replaced by $y - 2$	B1		
	Equation is $(y-2)^2 = 12x$	B1√	2	ft $y + 2$ for $y - 2$
(ii)	Equation is $x^2 = 12y$	B1	1	
(c)(i)	$(x+c)^2 = x^2 + 2cx + c^2$	B1		
	$\ldots = 12x$	M1		
	Hence result	A1	3	convincingly shown (AG)
(ii)	Tangent if $(2c-12)^2 - 4c^2 = 0$	M1		
	ie if $-48c + 144 = 0$ so $c = 3$	A1	2	
(iii)	$x^2 - 6x + 9 = 0$	M1		
	$x = 3, y = 6$	A1	2	
(iv)	$c = 4 \Rightarrow$ discriminant $= -48 < 0$	M1A1		OE
	So line does not intersect curve	A1	3	
	Total		**15**	
	TOTAL		**75**	

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1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{2}{3}$	B1B1	2	**SC** 1/2 for answers 6 and 2
(b)(i)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	B1	1	Accept unsimplified
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		
	Substitution of numerical values	m1		
	$\alpha^3+\beta^3=4$	A1	3	convincingly shown AG
(c)	$\alpha^3\beta^3=\frac{8}{27}$	B1		
	Equation of form $px^2\pm 4px+r=0$	M1		
	Answer $27x^2-108x+8=0$	A1√	3	ft wrong value for $\alpha^3\beta^3$
	Total		**9**	
2	1st increment is 0.2 lg 2 ...	M1		or 0.2 lg 2.1 or 0.2 lg 2.2
	... ≈ 0.06021	A1		PI
	$x=2.2\Rightarrow y\approx 3.06021$	A1√		PI; ft numerical error
	2nd increment is 0.2 lg 2.2	m1		consistent with first one
	... ≈ 0.06848	A1		PI
	$x=2.4\Rightarrow y\approx 3.12869\approx 3.129$	A1√	6	ft numerical error
	Total		**6**	
3	$\Sigma(r^2-r)=\Sigma r^2-\Sigma r$	M1		
	At least one linear factor found	m1		
	$\Sigma(r^2-r)=\frac{1}{6}n(n+1)(2n+1-3)$	m1		OE
	$\ldots=\frac{1}{3}n(n+1)(n-1)$	A1	4	
	Total		**4**	
4	$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ stated or used	B1		Condone decimals and/or degrees until final mark
	Appropriate use of ±	B1		
	Introduction of $2n\pi$	M1		
	Division by 3	M1		Of $\alpha+kn\pi$ or $\pm\alpha+kn\pi$
	$x=\pm\frac{\pi}{18}+\frac{2}{3}n\pi$	A1	5	
	Total		**5**	
5(a)(i)	$\mathbf{M}^2=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	M1 A2,1	3	M1 if 2 entries correct M1A1 if 3 entries correct
(ii)	$\mathbf{M}^4=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	B1√	1	ft error in $\mathbf{M}^2$ provided no surds in $\mathbf{M}^2$
(b)	Rotation (about the origin)	M1		
	... through 45° clockwise	A1	2	
(c)	Awareness of $\mathbf{M}^8=\mathbf{I}$	M1		OE; NMS 2/3
	$\mathbf{M}^{2006}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	m1 A1√	3	complete valid method ft error in $\mathbf{M}^2$ as above
	Total		**9**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$(z+\mathrm{i})^* = x - \mathrm{i}y - \mathrm{i}$	B2	2	
(b)	$\ldots = 2\mathrm{i}x - 2y + 1$	M1		$\mathrm{i}^2 = -1$ used at some stage
	Equating R and I parts	M1		involving at least 5 terms in all
	$x = -2y + 1,\ -y - 1 = 2x$	A1√		ft one sign error in (a)
	$z = -1 + \mathrm{i}$	m1A1√	5	ditto; allow $x = -1$, $y = 1$
	Total		**7**	
7(a)	Stretch parallel to y axis ...	B1		
	... scale-factor $\frac{1}{2}$ parallel to y axis	B1	2	
(b)	$(x-2)^2 - y^2 = 1$	M1A1		
	Translation in x direction ...	A1		
	... 2 units in positive x direction	A1	4	
	Total		**6**	
8(a)(i)	$(1+h)^3 = 1 + 3h + 3h^2 + h^3$	B1		
	$\mathrm{f}(1+h) = 1 + 5h + 4h^2 + h^3$	M1A1√		PI; ft wrong coefficients
	$\mathrm{f}(1+h) - \mathrm{f}(1) = 5h + 4h^2 + h^3$	A1√	4	ft numerical errors
(ii)	Dividing by h	M1		
	$\mathrm{f}'(1) = 5$	A1√	2	ft numerical errors
(b)(i)	$x^2(x+1) = 1$, hence result	B1	1	convincingly shown (AG)
(ii)	$x_2 = 1 - \frac{1}{5} = \frac{4}{5}$	M1A1√		
		A1√	3	ft c's value of f′(1)
(c)	Area $= \int_1^{\infty} x^{-2}\,\mathrm{d}x$	M1		
	$\ldots = \left[-x^{-1}\right]_1^{\infty}$	M1		Ignore limits here
	$\ldots = 0 - -1 = 1$	A1	3	
	Total		**13**	
9(a)(i)	Intersections at $(-1, 0)$, $(3, 0)$	B1B1	2	Allow $x = -1$, $x = 3$
(ii)	Asymptotes $x = 0$, $x = 2$, $y = 1$	B1 × 3	3	
(b)(i)	$y = k \Rightarrow kx^2 - 2kx = x^2 - 2x - 3$	M1A1		M1 for clearing denominator
	$\ldots \Rightarrow (k-1)x^2 + (-2k+2)x + 3 = 0$	A1√		ft numerical error
	$\Delta = 4(k-1)(k-4)$, hence result	m1A1	5	convincingly shown (AG)
(ii)	$y = 4$ at SP	B1		
	$3x^2 - 6x + 3 = 0$, so $x = 1$	M1A1	3	A0 if other point(s) given
(c)	Curve with three branches	B1		approaching vertical asymptotes
	Middle branch correct	B1		Coordinates of SP not needed
	Other two branches correct	B1	3	3 asymptotes shown
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Jan 07

MFP1

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1(a)(i)	Roots are $\pm 4\text{i}$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1 \pm 4\text{i}$	M1A1	2	M1 for correct method
(b)(i)	$(1+x)^3 = 1 + 3x + 3x^2 + x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1+\text{i})^3 = 1 + 3\text{i} - 3 - \text{i} = -2 + 2\text{i}$	M1A1	2	M1 if $\text{i}^2 = -1$ used
(iii)	$(1+\text{i})^3 + 2(1+\text{i}) - 4\text{i}$	M1		with attempt to evaluate
	$\ldots = (-2+2\text{i}) + (2-2\text{i}) = 0$	A1	2	convincingly shown (**AG**)
	Total		**10**	
2(a)(i)	$\mathbf{A}+\mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct; Condone $\frac{2\sqrt{3}}{2}$ for $\sqrt{3}$
(ii)	$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	B3,2,1	3	Deduct one for each error; **SC** B2,1 for **AB**
(b)(i)	Rotation 30° anticlockwise (abt O)	M1A1	2	M1 for rotation
(ii)	Reflection in $y = (\tan 15^\circ)x$	M1A1	2	M1 for reflection
(iii)	Reflection in x-axis	B2F	2	1/2 for reflection in y-axis ft (M1A1) only for the **SC**
	Alt: Answer to (i) followed by answer to (ii)	M1A1F	(2)	M1A0 if in wrong order or if order not made clear
	Total		**11**	
3(a)	$\alpha + \beta = -2,\ \alpha\beta = \frac{3}{2}$	B1B1	2	
(b)	Use of expansion of $(\alpha+\beta)^2$	M1		
	$\alpha^2 + \beta^2 = (-2)^2 - 2\left(\frac{3}{2}\right) = 1$	m1A1	3	convincingly shown (**AG**); m1A0 if $\alpha + \beta = 2$ used
(c)	$\alpha^4 + \beta^4$ given in terms of $\alpha+\beta, \alpha\beta$ and/or $\alpha^2+\beta^2$	M1A1		M1A0 if num error made
	$\alpha^4 + \beta^4 = -\frac{7}{2}$	A1	3	OE
	Total		**8**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\lg y = \lg a + b \lg x$	M1A1	2	M1 for use of one log law
(b)	Use of above result	M1		
	$a = 10$	A1		
	b = gradient	m1		OE; PI by answer $\pm\frac{1}{2}$
	$\ldots = -\frac{1}{2}$	A1	4	
	Total		**6**	
5(a)	Asymptotes $y = 0$, $x = -1$, $x = 1$	B1 × 3	3	
(b)	Three branches approaching two vertical asymptotes	B1		Asymptotes not necessarily drawn
	Middle branch passing through O	B1		with no stationary points
	Curve approaching $y = 0$ as $x \to \pm\infty$	B1		
	All correct	B1	4	with asymptotes shown and curve approaching all asymptotes correctly
(c)	Critical values $x = -1$, 0 and 1	B1		
	Solution set $-1 < x < 0$, $x > 1$	M1A1	3	M1 if one part correct or consistent with c's graph
	Total		**10**	
6(a)(i)	$(2r-1)^2 = 4r^2 - 4r + 1$	B1	1	
(ii)	$\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$	M1		
	$\ldots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	m1A1		
	$\sum 1 = n$	B1		
	Result convincingly shown	A1	5	**AG**
(b)	Sum = f (100) – f (50)	M1A1		M1 for 100 ± 1 and 50 ± 1
	$\ldots = 1\,166\,650$	A2	4	**SC** f(100) – f(51) = 1 156 449**:** 3/4
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Particular solution, eg $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	B1		Degrees or decimals penalised in 3rd mark only
	Introduction of $n\pi$ or $2n\pi$	M1		
	GS $x = -\frac{\pi}{6} + n\pi$	A1F	3	OE(accept unsimplified); ft incorrect first solution
(b)(i)	$\mathrm{f}(0.05) \approx 0.542\ 66$ $\mathrm{g}(0.05) \approx 0.542\ 68$	B1 B1	2	either value AWRT 0.5427 both values correct to 4DP
(ii)	$\frac{\mathrm{g}(h)-\mathrm{g}(0)}{h} = \frac{\sqrt{3}}{2} - \frac{1}{4}h$	M1A1	2	M1A0 if num error made
(iii)	As $h \to 0$ this gives $\mathrm{g}'(0) = \frac{\sqrt{3}}{2}$	A1F	1	AWRT 0.866; ft num error
	Total		**8**	
8(a)	$x = 10 \Rightarrow 4 - \frac{y^2}{9} = 1$	M1		
	$\Rightarrow y^2 = 27$	A1		PI
	$\Rightarrow y = \pm 3\sqrt{3}$	A1	3	
(b)	One branch generally correct Both branches correct Intersections at (± 5, 0)	B1 B1 B1	3	Asymptotes not needed With implied asymptotes
(c)	Required tangent is $x = 5$	B1F	1	ft wrong value in (b)
(d)(i)	y correctly eliminated Fractions correctly cleared $16x^2 - 200x + 625 = 0$	M1 m1 A1	3	convincingly shown (**AG**)
(ii)	$x = \frac{25}{4}$ Equal roots $\Rightarrow$ tangency	B1 E1	2	No need to mention repeated root, but B0 if other values given as well Accept 'It's a tangent'
	Total		**12**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

Mark Scheme

2007 examination - June series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

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M	mark is for method
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A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
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A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

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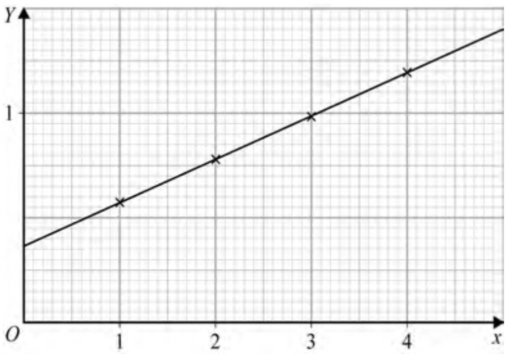
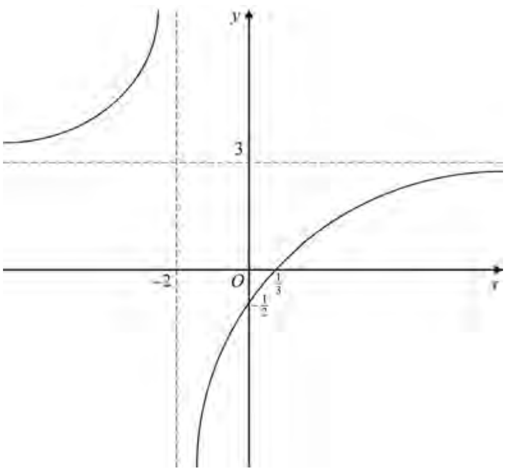
Otherwise we require evidence of a correct method for any marks to be awarded.

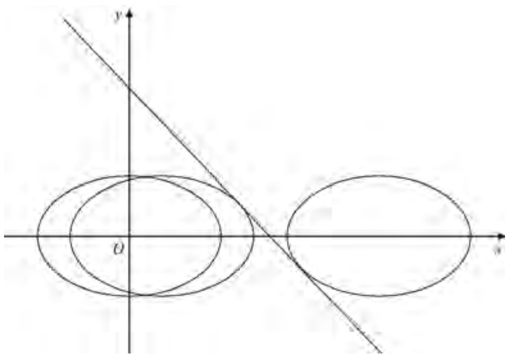
MFP1

Q	Solution	Mark	Total	Comments
1(a)	$\mathbf{M} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(b)	$p = 3$	B1F		ft after B1 in (a)
	L is $y = -x$	B1	2	Allow $p = -3$, L is $y = x$
(c)	$\mathbf{M}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$	B1F		Or by geometrical reasoning; ft as before
	$\ldots = 9\mathbf{I}$	B1F	2	ft as before
	Total		**6**	
2(a)	f(1.6) = −1.304, f(1.8) = 0.632	B1,B1		Allow 1 dp throughout
	Sign change, so root between	E1	3	
(b)	f(1.7) considered first	M1		
	f(1.7) = −0.387, so root > 1.7	A1		
	f(1.75) = 0.109375, so root ≈ 1.7	m1A1	4	m1 for f(1.65) after error
	Total		**7**	
3(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	$z - 3\mathrm{i}z^* = x + \mathrm{i}y - 3\mathrm{i}x - 3y$	m1		Condone sign error here
	R = $x - 3y$, I = $-3x + y$	A1	3	Condone inclusion of i in I Allow if correct in (b)
(b)	$x - 3y = 16$, $-3x + y = 0$	M1		
	Elimination of x or y	m1		
	$z = -2 - 6\mathrm{i}$	A1F	3	Accept $x = -2$, $y = -6$; ft $x + 3y$ for $x - 3y$
	Total		**6**	
4(a)	$\alpha + \beta = \frac{1}{2}$, $\alpha\beta = 2$	B1B1	2	
(b)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$	M1		
	$\ldots = \frac{\frac{1}{2}}{2} = \frac{1}{4}$	A1	2	Convincingly shown (AG)
(c)	Sum of roots = 1	B1F		PI by term $\pm x$; ft error(s) in (a)
	Product of roots = $\frac{16}{\alpha\beta} = 8$	B1F		ft wrong value of $\alpha\beta$
	Equation is $x^2 - x + 8 = 0$	B1F	3	ft wrong sum/product; "= 0" needed
	Total		**7**	

MFP1 (cont)

Q	Solution	Mark	Total	Comments
5(a)	Values 0.788, 0.992, 1.196 in table	B2,1	2	B1 if one correct (or if wrong number of dp given)
(b)	$\lg ab^x = \lg a + \lg b^x$	M1		
	$\lg b^x = x \lg b$	M1		
	So $Y = (\lg b)\,x + \lg a$	A1	3	Allow NMS
(c)		B1F		Four points plotted; ft wrong values in (a)
		B1F	2	Good straight line drawn; ft incorrect points
(d)	a = antilog of y-intercept	M1A1		Accept 2.23 to 2.52
	b = antilog of gradient	M1A1	4	Accept 1.58 to 1.62
	Total		**11**	
6	One value of $2x - \frac{\pi}{2}$ is $\frac{\pi}{3}$	B1		OE (PI); degrees/decimals penalised in 6th mark only
	Another value is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$	B1F		OE (PI); ft wrong first value
	Introduction of $2n\pi$ or $n\pi$	M1		
	General solution for x	m1		
	GS $x = \frac{5\pi}{12} + n\pi$ or $x = \frac{7\pi}{12} + n\pi$	A2,1	6	OE; A1 if one part correct
	Total		**6**	
7(a)	Asymptotes $x = -2$, $y = 3$	B1,B1	2	
(b)		B1		Curve approaching asymptotes
		B1,B1		Passing through $\left(\frac{1}{3}, 0\right)$ and $\left(0, -\frac{1}{2}\right)$
		B1,B1	5	Both branches generally correct B1 if two branches shown
(c)	Solution set is $x > \frac{1}{3}$	B2,1F	2	B1 for good attempt; ft wrong point of intersection
	Total		**9**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
8(a)	$\int\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)\mathrm{d}x=\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}\;(+c)$	M1A1		M1 for adding 1 to index at least once
	$\int_0^1 \ldots=\left(\frac{3}{4}+\frac{3}{2}\right)-0=\frac{9}{4}$	m1A1	4	Condone no mention of limiting process; m1 if "− 0" stated or implied
(b)	Second term is $x^{-\frac{4}{3}}$	B1		
	Integral of this is $-3x^{-\frac{1}{3}}$	M1A1		M1 for correct index
	$x^{-\frac{1}{3}}\to\infty$ as $x\to0$, so no value	E1	4	
	Total		**8**	
9(a)	Intersections $\left(\pm\sqrt{2},0\right)$, $(0,\pm1)$	B1B1	2	Allow B1 for $\left(\sqrt{2},0\right)$, $(0,1)$
(b)	Equation is $\frac{(x-k)^2}{2}+y^2=1$	M1A1	2	M1 if only one small error, eg $x+k$ for $x-k$
(c)	Correct elimination of y Correct expansion of squares Correct removal of denominator Answer convincingly established	M1 M1 M1 A1	4	AG
(d)	Tgt $\Rightarrow 4(k+4)^2-12(k^2+6)=0$	M1		
	$\ldots\Rightarrow k^2-4k+1=0$	m1A1		OE
	$\ldots\Rightarrow k=2\pm\sqrt{3}$	A1	4	
(e)		B1 B2	3	Curve to left of line Curve to right of line Curves must touch the line in approx correct positions SC 1/3 if both curves are incomplete but touch the line correctly
	Total		**15**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

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2008 examination - January series

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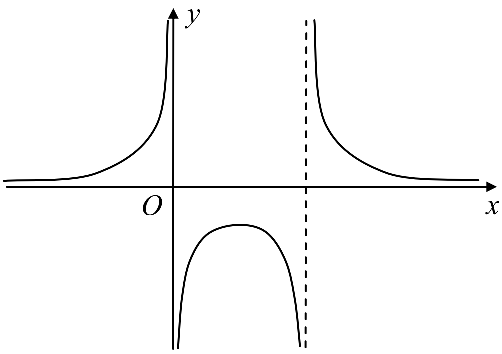
MFP1

Q	Solution	Marks	Totals	Comments
1	$z_1 + 4\mathrm{i}\, z_1{}^* = (2 + \mathrm{i}) + 4\mathrm{i}\,(2 - \mathrm{i})$	M1		Use of conjugate
	$\ldots = (2 + \mathrm{i}) + (8\mathrm{i} + 4)$	M1		Use of $\mathrm{i}^2 = -1$
	$\ldots = 6 + 9\mathrm{i}$, so $x = 6$ and $y = 3$	M1A1	4	M1 for equating Real and imaginary parts
	Total		**4**	
2	$0.01(2^1)$ added to value of y	M1		Variations possible here
	So $y(1.01) \approx 4.02$	A1		PI
	Second increment is $0.01(2^{1.01})$	m1		
	$\ldots \approx 0.020\,139$	A1		
	So $y(1.02) \approx 4.040\,14$	A1	5	
	Total		**5**	
3	Use of $\tan\frac{\pi}{4} = 1$	B1		Degrees or decimals penalised in last mark only
	Introduction of $n\pi$	M1		or kn at any stage
	Division of all terms by 4	m1		
	Addition of $\pi/8$	m1		OE
	GS $x = \frac{3\pi}{16} + \frac{n\pi}{4}$	A1	5	OE
	Total		**5**	
4(a)	Use of formula for $\sum r^3$ or $\sum r$	M1		
	n is a factor of the expression	m1		clearly shown
	So is $(n + 1)$	m1		ditto
	$S_n = \frac{1}{4}n(n+1)(n^2+n-12)$	A1		
	$\ldots = \frac{1}{4}n(n+1)(n+4)(n-3)$	A1F	5	ft wrong value for k
(b)	$n = 1000$ substituted into expression	m1		The factor 1004, or 1000 + 4, seen
	Conclusion convincingly shown	A1	2	not '2008 × 124749625'
	Need $\frac{1000}{4}$ is even, hence conclusion			OE
	Total		**7**	
5(a)	Asymptotes are $y = \pm\frac{1}{2}x$	M1A1	2	OE; M1 for $y = \pm\, mx$
(b)	$x = 4$ substituted into equation	M1		
	$y^2 = 3$ so $y = \pm\sqrt{3}$	A1	2	Allow NMS
(c)(i)	y-coords are $2 \pm \sqrt{3}$	B1F	1	ft wrong answer to (b)
(ii)	Hyperbola is $\frac{x^2}{4} - (y-2)^2 = 1$	M1A1		M1A0 if $y + 2$ used
	Asymptotes are $y = 2 \pm \frac{1}{2}x$	B1F	3	ft wrong gradients in (a)
	Total		**8**	
6(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$	M1A1		M1 if zeroes appear in the right places
	$= 12\mathbf{I}$	A1F	3	ft provided of right form
(ii)	$q\cos 60° = \frac{1}{2}q = \sqrt{3} \Rightarrow q = 2\sqrt{3}$	M1A1		OE **SC** $q = 2\sqrt{3}$ NMS 1/3
	Other entries verified	E1	3	surd for sin 60º needed
(b)(i)	SF = $q = 2\sqrt{3}$	B1F	1	ft wrong value for q
(ii)	Equation is $y = x \tan 30°$	B1	1	
(c)	$\mathbf{M}^4 = 144\mathbf{I}$	B1F		PI; ft wrong value in (a)(i)
	$\mathbf{M}^4$ gives enlargement SF 144	B1F	2	ft if c's $\mathbf{M}^4 = k\mathbf{I}$
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
7(a)(i)	$(-1+h)^3 = -1 + 3h - 3h^2 + h^3$	B1		PI
	$y_B = (-1 + 3h - 3h^2 + h^3) + 1 - h + 1$	B1F		ft numerical error
	$... = 1 + 2h - 3h^2 + h^3$	B1	3	convincingly shown (AG)
(ii)	Subtraction of 1 and division by h	M1M1		
	Gradient of chord $= 2 - 3h + h^2$	A1	3	
(iii)	As $h \to 0$, gr(chord) $\to$ gr(tgt) $= 2$	E1B1F	2	E0 if '$h = 0$' used; ft wrong value of p
(b)(i)	$x_2 = -1 - \frac{1}{2} = -1.5$	M1		
		A1F	2	ft wrong gradient
(ii)	Tangent at A drawn	M1		
	α and x_2 shown correctly	A1	2	dep't only on the last M1
	Total		**12**	
8(a)(i)	$\alpha + \beta = 2,\ \alpha\beta = 4$	B1B1		
	$\alpha^3 + \beta^3 = (2)^3 - 3(4)(2) = -16$	M1A1		
	$\alpha^3\beta^3 = (4)^3 = 64$, hence result	M1A1	6	convincingly shown (AG)
(ii)	Discriminant 0, so roots equal	B1E1	2	or by factorisation
(b)	$x = \dfrac{2 \pm \sqrt{4-16}}{2}$	M1		or by completing square
	$... = 1 \pm \frac{1}{2}\mathrm{i}\sqrt{12}$	A1	2	
(c)	$\alpha, \beta = 1 \pm \mathrm{i}\sqrt{3}$			
	and $\alpha^3 = \beta^3$, hence result	E2	2	
	Total		**12**	
9(a)	Asymptotes $x = 0,\ x = 4,\ y = 0$	B1 × 3	3	
(b)	$y = k \Rightarrow 2 = kx(x-4)$	M1		
	$... \Rightarrow 0 = kx^2 - 4kx - 2$	A1		
	Discriminant $= (4k)^2 + 8k$	m1		
	At SP $y = -\frac{1}{2}$	A1		not just $k = -\frac{1}{2}$
	$... \Rightarrow 0 = -\frac{1}{2}x^2 + 2x - 2$	m1		
	So $x = 2$	A1	6	
(c)		B1 B1 B1	3	Curve with three branches approaching vertical asymptotes correctly Outer branches correct Middle branch correct
	Total		**12**	
	TOTAL		**75**	

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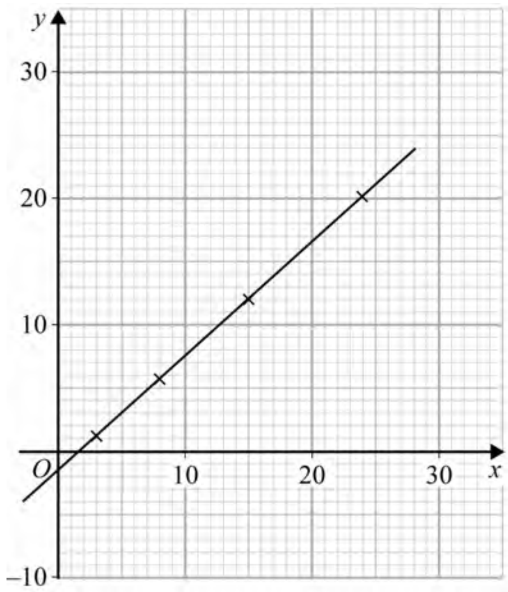
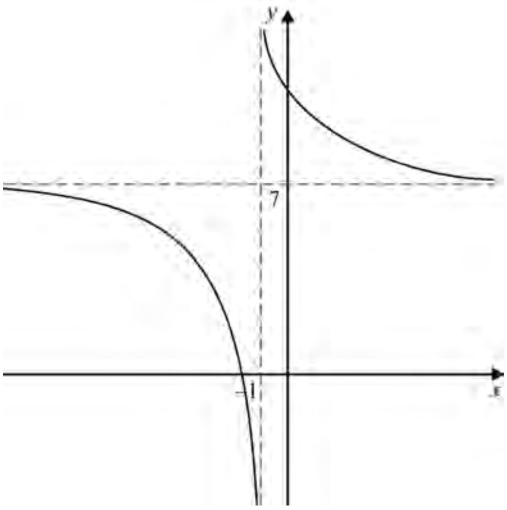
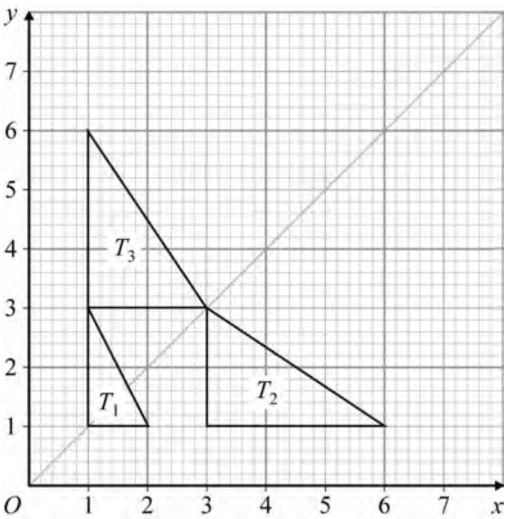
MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha + \beta = -1,\ \alpha\beta = 5$	B1B1	2	
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\ldots = 1 - 10 = -9$	M1 A1F	 2	with numbers substituted ft sign error(s) in (a)
(c)	$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$ $\ldots = -\dfrac{9}{5}$	M1 A1	 2	 AG: A0 if $\alpha + \beta = 1$ used
(d)	Product of new roots is 1 Eqn is $5x^2 + 9x + 5 = 0$	B1 B1F	 2	PI by constant term 1 or 5 ft wrong value for product
	Total		**8**	
2(a)	Use of $z^* = x - \mathrm{i}y$ Use of $\mathrm{i}^2 = -1$ $3\mathrm{i}z + 2z^* = (2x - 3y) + \mathrm{i}\,(3x - 2y)$	M1 M1 A1	 3	 Condone inclusion of i in I part
(b)	Equating R and I parts $2x - 3y = 7,\ \ 3x - 2y = 8$ $z = 2 - \mathrm{i}$	M1 m1 A1	 3	 with attempt to solve Allow $x = 2,\ y = -1$
	Total		**6**	
3(a)	$\int x^{-1/2}\,\mathrm{d}x = 2x^{1/2}\,(+c)$ $x^{1/2} \to \infty$ as $x \to \infty$, so no value	M1A1 E1	 3	M1 for correct power in integral
(b)	$\int x^{-3/2}\,\mathrm{d}x = -2x^{-1/2}\,(+c)$ $x^{-1/2} \to 0$ as $x \to \infty$ $\int_9^\infty x^{-3/2}\,\mathrm{d}x = -2(0 - \frac{1}{3}) = \frac{2}{3}$	M1A1 E1 A1	 4	M1 for correct power in integral PI Allow A1 for correct answer even if not fully explained
	Total		**7**	
4(a)	Multiplication by $x + 2$ $Y = aX + b$ convincingly shown	M1 A1	 2	applied to all 3 terms AG
(b)(i)	$X = 8, 15, 24$ in table $Y = 5.72, 12, 20.1$ in table	B1 B1	 2	 Allow correct to 2SF

MFP1 (cont)

Q	Solution	Marks	Total	Comments
4(b)(ii)				
	Four points plotted	B1F		ft incorrect values in table
	Reasonable line drawn	B1F	2	ft incorrect points
(iii)	Method for gradient	M1		or algebraic method for a or b
	a = gradient ≈ 0.9	A1		Allow from 0.88 to 0.93 incl
	b = Y-intercept ≈ -1.5	B1F	3	Allow from -2 to -1 inclusive; ft incorrect points/line **NMS** B1 for a, B1 for b
	Total		**9**	
5(a)	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ stated or used	B1		Degrees or decimals penalised in 5th mark only
	Appropriate use of $\pm$	B1		OE
	Introduction of $2n\pi$	M1		OE
	Subtraction of $\frac{\pi}{3}$ and multiplication by 2	m1		All terms multiplied by 2
	$x = -\frac{2\pi}{3} \pm \frac{\pi}{2} + 4n\pi$	A1	5	OE
5(b)	$n = 1$ gives min pos $x = \frac{17\pi}{6}$	M1A1	2	**NMS** 1/2 provided (a) correct
	Total		**7**	
6(a)	$\mathbf{AB} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(b)	$\mathbf{A}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	B1		
	$\ldots = 4\mathbf{I}$	B1	2	
(c)	$(\mathbf{AB})^2 = -16\mathbf{I}$	B1		
	$\mathbf{B}^2 = 4\mathbf{I}$	B1		PI
	so $\mathbf{A}^2\,\mathbf{B}^2 = 16\mathbf{I}$ (hence result)	B1	3	Condone absence of conclusion
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Curve translated 7 in y direction ... and 1 in negative x direction	B1 B1	2	or answer in vector form
(b)(i)	Asymptotes $x = -1$ and $y = 7$	B1B1	2	
(ii)	Intersections at (0, 8) ... ... and $(-\frac{8}{7}, 0)$	B1 M1A1	3	Allow AWRT –1.14; NMS 1/2
(c)	 At least one branch Complete graph All correct including asymptotes	B1 B1 B1	3	of correct shape translation of $y = 1/x$ in roughly correct positions
	Total		**10**	
8(a)	Matrix is $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1 if zeros in correct positions; allow NMS
(b)	 Third triangle shown correctly	M1A1	2	M1A0 if one point wrong

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(c)	Matrix of reflection is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	B1		**Alt**: calculating matrix from the coordinates: M1 A2,1
	Multiplication of above matrices	M1		in correct order
	Answer is $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$	A1F	3	ft wrong answer to (a); **NMS** 1/3
	Total		**7**	
9(a)	Equation is $y - 4 = m(x - 3)$	M1A1	2	OE; M1A0 if one small error
(b)	Elimination of x	M1		
	$4y - 16 = m(y^2 - 12)$	A1		OE (no fractions)
	Hence result	A1	3	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$(3m - 1)(m - 1) = 0$	m1A1		OE; m1 for attempt at solving
	Tangents $y = x + 1$, $y = \frac{1}{3}x + 3$	A1A1	5	OE
(d)	$m = 1 \Rightarrow y^2 - 4y + 4 = 0$	M1		OE; $m = 1$ needed for this
	so point of contact is (1, 2)	A1		
	$m = \frac{1}{3} \Rightarrow \frac{1}{3}y^2 - 4y + 12 = 0$	M1		OE; $m = \frac{1}{3}$ needed for this
	so point of contact is (9, 6)	A1	4	
	Total		**14**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

Mark Scheme

2009 examination – January series

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MFP1

Q	Solution	Marks	Total	Comments
1	First increment is 0.2, so $y \approx 1.2$	B1B1		PI; variations possible here
	Second increment is $0.2\sqrt{1+0.2^2}$	M1		
	... $\approx 0.203\ 961$, so $y \approx 1.403\ 96$	A2,1F	5	A1 if accuracy lost; ft num error
	Total		**5**	
2(a)	Other root is $2 - 3\mathrm{i}$	B1	1	
(b)	Sum of roots = 4	B1F		ft error in (a)
	So $b = -4$	B1F		ft wrong value for sum
	Product is 13	B1		
	So $c = 13$	B1F	4	ft wrong value for product
	Alternative:			
	Substituting $2 + 3\mathrm{i}$ into equation	M1		
	Equating R and I parts	m1		
	$12 + 3b = 0$, so $b = -4$	A1		
	$-5 + 2b + c = 0$, so $c = 13$	A1F	(4)	ft wrong value for b
	Total		**5**	
3	$\tan\frac{\pi}{3} = \sqrt{3}$	B1		Decimals/degrees penalised at 5^{th} mark
	Introduction of $n\pi$	M1		(or $2n\pi$) at any stage
	Going from $\frac{\pi}{2} - 3x$ to x	m1		Including dividing all terms by 3
	$x = \frac{\pi}{18} + \frac{1}{3}n\pi$	A2,1F	5	Allow +, − or ±; A1 with dec/deg; ft wrong first solution
	Total		**5**	
4(a)	$S_n = 3\Sigma r^2 - 3\Sigma r + \Sigma 1$	M1		
	Correct expressions substituted	m1		At least for first two terms
	Correct expansions	A1		
	$\Sigma 1 = n$	B1		
	Answer convincingly obtained	A1	5	AG
(b)	$S_{2n} - S_n$ attempted	M1		Condone $S_{2n} - S_{n+1}$ here
	Answer $7n^3$	A1	2	
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 2k\\ 2k & 0\end{bmatrix}$	B1	1	
(ii)	$\mathbf{A}^2=\begin{bmatrix}2k^2 & 0\\ 0 & 2k^2\end{bmatrix}$	B2,1	2	B1 if three entries correct
(b)	$(\mathbf{A}+\mathbf{B})^2 = =\begin{bmatrix}4k^2 & 0\\ 0 & 4k^2\end{bmatrix}$	B2,1		B1 if three entries correct
	$\mathbf{B}^2=\mathbf{A}^2$, hence result	B1B1	4	
(c)(i)	$\mathbf{A}^2$ is an enlargement (centre O)	M1		
	with SF 2	A1	2	Condone $2k^2$
(ii)	Scale factor is now $\sqrt{2}$	B1		Condone $\sqrt{2}k$
	Mirror line is $y = x\tan 22\frac{1}{2}°$	M1A1	3	
	Total		**12**	
6(a)(i)	Asymptotes $x=0$, $x=2$, $y=1$	B1×3	3	
(ii)	Intersections at (1, 0) and (3, 0)	B1	1	
(iii)	At least one branch approaching asymptotes	B1		
	Each branch	B1×3	4	
(b)	$0 < x < 1$, $2 < x < 3$	B1,B1	2	Allow B1 if one repeated error occurs, eg ≤ for <
	Alternative: Complete correct algebraic method	M1A1	(2)	
	Total		**10**	
7(a)	Use of similar triangles or algebra	M1		Some progress needed
	Correct relationship established	m1A1		eg $\frac{r-a}{c}=\frac{b-a}{c-d}$
	Hence result convincingly shown	A1	4	AG
(b)(i)	$c = \mathrm{f}(a) = 24$, $d = \mathrm{f}(b) = -21$	B1,B1		
	$r=\frac{38}{15}$ (≈ 2.5333)	B1F	3	Allow AWRT 2.53; ft small error
(ii)	$\beta = 20^{\frac{1}{3}} \approx 2.714(4)$	M1A1		Allow AWRT 2.71
	So $\beta - r \approx 0.181 \approx 0.18$ (AG)	A1	3	Allow only 2dp if earlier values to 3dp
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int x^{-\frac{3}{4}}\,\mathrm{d}x = 4x^{\frac{1}{4}}\ (+\,c)$	M1A1		M1 if index correct
	This tends to ∞ as $x \to \infty$, so no value	A1F	3	ft wrong coefficient
(b)	$\int x^{-\frac{5}{4}}\,\mathrm{d}x = -4x^{-\frac{1}{4}}\ (+\,c)$	M1A1		M1 if index correct
	$\int_1^\infty x^{-\frac{5}{4}}\,\mathrm{d}x = 0-(-4) = 4$	A1F	3	ft wrong coefficient
(c)	Subtracting 4 leaves ∞, so no value	B1F	1	ft if c has 'no value' in (a) but has a finite answer in (b)
	Total		**7**	
9(a)	Asymptotes are $y = \pm\sqrt{2}x$	M1A1	2	M1A0 if correct but not in required form
(b)	Asymptotes correct on sketch	B1F		With gradients steeper than 1; ft from $y = \pm\, mx$ with $m > 1$
	Two branches in roughly correct positions	B1		
	Approaching asymptotes correctly	B1	3	Asymptotes $y = \pm\, mx$ needed here
(c)(i)	Elimination of y	M1		
	Clearing denominator correctly	M1		
	$x^2 - 2cx - (c^2+2) = 0$	m1A1	4	Convincingly found (AG)
(ii)	Discriminant = $8c^2 + 8$	B1		Accept unsimplified
	… > 0 for all c, hence result	E1	2	OE
(iii)	Solving gives $x = c \pm\sqrt{2(c^2+1)}$	M1A1		
	$y = x + c = 2c \pm\sqrt{2(c^2+1)}$	A1	3	Accept $y = c + \dfrac{2c \pm\sqrt{8c^2+8}}{2}$
	Total		**14**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP1 Further Pure 1

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2009 examination - June series

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MFP1

Q	Solution	Marks	Totals	Comments
1(a)	$\alpha+\beta=-\frac{1}{2}$, $\alpha\beta=-4$	B1B1	2	
(b)	$\alpha^2+\beta^2=(-\frac{1}{2})^2-2(-4)=8\frac{1}{4}$	M1A1F	2	M1 for substituting in correct formula; ft wrong answer(s) in (a)
(c)	Sum of roots = $4(8\frac{1}{4})=33$ Product = $16(\alpha\beta)^2=256$ Equation is $x^2-33x+256=0$	B1F B1F B1F	 3	ft wrong answer in (b) ft wrong answer in (a) ft wrong sum and/or product; allow '$p=-33$, $q=256$'; condone omission of '= 0'
	Total		**7**	
2(a)	When $x=2$, $y=-3$ Use of $(2+h)^2=4+4h+h^2$ Correct method for gradient Gradient = $\dfrac{-3-2h+h^2+3}{h}=-2+h$	B1 M1 M1 A2,1	 5	PI A1 if only one small error made
(b)	As h tends to 0, ... the gradient tends to -2	E2,1 B1F	 3	E1 for '$h=0$' dependent on at least E1 ft small error in (a)
	Total		**8**	
3(a)(i)	$z^2=(x^2-4)+\mathrm{i}(4x)$ R and I parts clearly indicated	M1A1 A1F	 3	M1 for use of $\mathrm{i}^2=-1$ Condone inclusion of i in I part ft one numerical error
(ii)	$z^2+2z^*=\left(x^2+2x-4\right)+\mathrm{i}\left(4x-4\right)$	M1A1F	2	M1 for correct use of conjugate ft numerical error in (i)
(b)	z^2+2z^* real if imaginary part zero ... ie if $x=1$	M1 A1F	 2	 ft provided imaginary part linear
	Total		**7**	
4(a)	$\lg(ab^x)=\lg a+\lg(b^x)$ ... $=\lg a+x\lg b$ Correct relationship established [SC After M0M0, B2 for correct form]	M1 M1 A1	 3	Use of one log law Use of another log law
(b)(i)	When $x=2.3$, $Y\approx 1.1$, so $y\approx 12.6$	M1A1		Allow 12.7; allow NMS
(ii)	When $y=80$, $Y\approx 1.90$, so $x\approx 1.1$	M1A1	4	M1 for $Y\approx 1.9$, allow NMS
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
5(a)	$\cos\frac{\pi}{3}=\frac{1}{2}$	B1		Decimals/degrees penalised at 6th mark only
	Appropriate use of ±	B1		OE
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $3x-\pi$ to x	m1		including dividing all terms by 3
	$x=\frac{\pi}{3}\pm\frac{\pi}{9}+\frac{2}{3}n\pi$	A2,1F	6	OE; A1 with decimals and/or degrees; ft wrong first solution
(b)	At least one value in given range	M1		compatible with c's GS
	Correct values $\frac{92}{9}\pi,\ \frac{94}{9}\pi,\ \frac{98}{9}\pi$	A2,1	3	A1 if one omitted or wrong values included; A0 if only one correct value given
	Total		**9**	
6(a)	Ellipse with centre of origin	B1		Allow unequal scales on axes
	$(\pm\sqrt{3},0)$ and (0 ± 2) shown on diagram	B2,1	3	Condone AWRT 1.7 for $\sqrt{3}$; B1 for incomplete attempt
(b)	y replaced by $\frac{1}{2}y$	M1A1		M1A0 for $2y$ instead of $\frac{1}{2}y$
	Equation is now $\frac{x^2}{3}+\frac{y^2}{16}=1$	A1	3	
(c)	Attempt at completing the square	M1		
	$4(x-1)^2+3(y+1)^2\ \ldots$	A1A1		
	[Alt: replace x by $x-a$ and y by $y-b$	(M1)		M1 if one replacement correct
	$4x^2-8ax+3y^2-6by\ \ldots$]	(m1A1)		Condone errors in constant terms
	$a=1$ and $b=-1$	A1A1	5	
	Total		**11**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
7(a)(i)	Matrix is $\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix} \cos 30° & \sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix}$ (PI)
(ii)	Matrix is $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$ (PI)
(b)	SF 2, line $y = \frac{1}{\sqrt{3}}x$	B1B1	2	OE
(c)	Attempt at **BA** or **AB**	M1		
	BA = $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$	m1A1		m1 if zeros in correct positions
	Enlargement SF 4	B1F		ft use of **AB** (answer still 4) or after **BA** = $\begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix}$
	... and reflection in line $y = x$	B1F	5	ft only from **BA** = $\begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix}$
	Total		**11**	
8(a)	Asymptotes $x = 1$, $x = 5$, $y = 1$	B1 × 3	3	
(b)	$y = -1 \Rightarrow (x-1)(x-5) = -x^2$	M1		OE
	... $\Rightarrow 2x^2 - 6x + 5 = 0$	m1		OE
	Disc't = 36 − 40 < 0, so no pt of int'n	A1	3	convincingly shown (AG)
(c)(i)	$y = k \Rightarrow x^2 = k(x^2 - 6x + 5)$	M1		OE
	... $\Rightarrow (k-1)x^2 - 6kx + 5k = 0$	A1	2	convincingly shown (AG)
(ii)	Discriminant = $36k^2 - 20k(k-1)$	M1		OE
	... = 0 when $k(4k + 5) = 0$	A1	2	convincingly shown (AG)
(d)	$k = 0$ gives $x = 0$, $y = 0$	B1		
	$k = -\frac{5}{4}$ gives $-\frac{9}{4}x^2 + \frac{30}{4}x - \frac{25}{4} = 0$	M1A1		OE
	$(3x-5)^2 = 0$, so $x = \frac{5}{3}$	A1		
	$y = -\frac{5}{4}$	B1	5	
	Total		**15**	
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1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{1}{3}$	B1B1	2	
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		or other appropriate formula
	$\ldots=8-3(\frac{1}{3})(2)=6$	m1A1	3	m1 for substn of numerical values; A1 for result shown (AG)
(c)	Sum of roots = $\dfrac{\alpha^3+\beta^3}{\alpha\beta}$	M1		
	$\ldots=\dfrac{6}{1/3}=18$	A1F		ft wrong value for $\alpha\beta$
	Product = $\alpha\beta=\frac{1}{3}$	B1F		ditto
	Equation is $3x^2-54x+1=0$	A1F	4	Integer coeffs and "= 0" needed; ft wrong sum and/or product
	Total		**9**	
2(a)	$z^2=1+2i+i^2=2i$	M1A1	2	M1 for use of $i^2=-1$
(b)	$z^8=(2i)^4$	M1		or equivalent complete method
	$\ldots=16i^4=16$	A1	2	convincingly shown (AG)
(c)	$(z^*)^2=(1-i)^2$	M1		for use of $z^*=1-i$
	$\ldots=-2i=-z^2$	A1	2	convincingly shown (AG)
	Total		**6**	
3	$\sin\dfrac{\pi}{2}=1$ stated or used	B1		Deg/dec penalised in 4th mark
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x+\dfrac{\pi}{4}$ to x	m1		incl division of all terms by 4
	$x=\dfrac{\pi}{16}+\dfrac{1}{2}n\pi$	A1	4	or equivalent unsimplified form
	Total		**4**	
4(a)	$\mathbf{I}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	B1		stated or used at any stage
	Attempt at $(\mathbf{A}-\mathbf{I})^2$	M1		with at most one numerical error
	$(\mathbf{A}-\mathbf{I})^2=\begin{bmatrix}0&4\\3&0\end{bmatrix}\begin{bmatrix}0&4\\3&0\end{bmatrix}=12\mathbf{I}$	A1	3	
(b)	$\mathbf{A}-\mathbf{B}=\begin{bmatrix}0&1\\3-p&0\end{bmatrix}$	B1		
	$(\mathbf{A}-\mathbf{B})^2=\begin{bmatrix}3-p&0\\0&3-p\end{bmatrix}$	M1A1		M1 A0 if 3 entries correct
	$\ldots=(\mathbf{A}-\mathbf{I})^2$ for $p=-9$	A1F	4	ft wrong value of k
	Total		**7**	

MFP1

Q	Solution	Mark	Total	Comments
5(a)	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$	E1	1	Condone "$x^{-\frac{1}{2}}$ has no value at $x = 0$"
(b)(i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\ (+c)$	M1A1		M1 for correct power of x
	$\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,dx = \frac{1}{2}$	A1F	3	ft wrong coefficient of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{5}{4}}\,dx = -4x^{-\frac{1}{4}}\ (+c)$	M1A1		M1 for correct power of x
	$x^{-\frac{1}{4}} \to \infty$ as $x \to 0$, so no value	E1F	3	ft wrong coefficient of $x^{-\frac{1}{4}}$
	Total		**7**	
6(a)(i)	Coords (3, 2), (9, 2), (9, 4), (3, 4)	M1A1	2	M1 for multn of x by 3 or y by 2 (PI)
(ii)	R_2 shown correctly on insert	B1	1	
(b)(i)	R_3 shown correctly on insert	B2,1F	2	B1 for rectangle with 2 vertices correct; ft if c's R_2 is a rectangle in 1st quad
(ii)	Matrix of rotation is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(c)	Multiplication of matrices	M1		(either way) or other complete method
	Required matrix is $\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$	A1	2	
	Total		**8**	
7(a)(i)	Asymptotes $x = 2$, $y = 0$	B1B1	2	
(ii)	One correct branch	B1		
	Both branches correct	B1	2	no extra branches; $x = 2$ shown
(b)(i)	$f(3) = -1$, $f(4) = 3$	B1		where $f(x) = (x-3)(x-2)^2 - 1$; OE
	Sign change, so α between 3 and 4	E1	2	
(ii)	$f(3.5)$ considered first	M1		OE but must consider $x = 3.5$
	$f(3.5) > 0$ so $3 < \alpha < 3.5$	A1		Some numerical value(s) needed
	$f(3.25) < 0$ so $3.25 < \alpha < 3.5$	A1	3	Condone absence of values here
	Total		**9**	

MFP1

Q	Solution	Mark	Total	Comments
8(a)	$\Sigma r^3 + \Sigma r = \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$	M1		at least one term correct
	Factor n clearly shown	m1		or $n + 1$ clearly shown to be a factor
	... = $\frac{1}{4}n(n+1)(n^2+n+2)$	A1A1	4	OE; A1 for $\frac{1}{4}$, A1 for quadratic
(b)	Valid equation formed	M1		
	Factors n, $n + 1$ removed	m1		
	$3n^2 - 29n - 10 = 0$	A1		OE
	Valid factorisation or solution	m1		of the correct quadratic
	$n = 10$ is the only pos int solution	A1	5	SC 1/2 for $n = 10$ after correct quad
	Total		**9**	
9(a)	$x = 2, y = 0 \Rightarrow \frac{4}{a^2} - 0 = 1$ so $a = 2$	E2,1		E1 for verif'n or incomplete proof
	Asymps $\Rightarrow \pm\frac{b}{a} = \pm 2$ so $b = 2a = 4$	E2,1	4	ditto
(b)	Line is $y - 0 = m(x - 1)$	B1		OE
	Elimination of y	M1		
	$4x^2 - m^2(x^2 - 2x + 1) = 16$	A1		OE (no fractions)
	So $(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$	A1	4	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$4m^4 - 4m^4 - 64m^2 + 16m^2 + 256 = 0$	A1		OE
	$-3m^2 + 16 = 0$, hence result	A1	3	convincingly shown (AG)
(d)	$m^2 = \frac{16}{3} \Rightarrow \frac{4}{3}x^2 - \frac{32}{3}x + \frac{64}{3} = 0$	M1		
	$x^2 - 8x + 16 = 0$, so $x = 4$	m1A1		
	Method for y-coordinates	m1		using $m = \pm\frac{4}{\sqrt{3}}$ or from equation of hyperbola; dep't on previous m1
	$y = \pm 4\sqrt{3}$	A1	5	
	Total		**16**	
	TOTAL		**75**	

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General Certificate of Education
June 2010

Mathematics MFP1

Further Pure 1

Mark Scheme

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP1

Q	Solution	Marks	Total	Comments
1	First increment is 0.1×2 $(= 0.2)$	M1		variations possible here
	So next value of y is 3.2	A1		PI
	Second inc't is $0.1(1 + 1.1^3) = 0.2331$	m1A1		PI
	Third inc't is $0.1(1 + 1.2^3) = 0.2728$	A1		PI
	So $y \approx 3.7059 \approx 3.706$	A1F	6	ft one numerical error
	Total		**6**	
2(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	Use of $\mathrm{i}^2 = -1$	M1		
	$(1 - 2\mathrm{i})z - z^* = 2y + \mathrm{i}(2y - 2x)$	A2,1	4	A1 if one numerical error made
(b)	$2y = 20$, $2y - 2x = 10$	M1		equate and attempt to solve
	so $z = 5 + 10\mathrm{i}$	A1	2	allow $x = 5$, $y = 10$
	Total		**6**	
3	Introduction of $360n°$	M1		(or $180n°$) at any stage; condone $2n\pi$ (or $n\pi$)
	$5x - 20° = \pm 40°$ $(+360n°)$	B1		OE, eg RHS '40° or 320°'
	Going from $5x - 20°$ to x	m1		including division of all terms by 5
	GS is $x = 4° \pm 8° + 72n°$	A2,1	5	OE; A1 if radians present in answer
	Total		**5**	
4(a)	4, 16, 36, 64 entered in table	B1	1	
(b)	Four points plotted accurately	B1F		ft wrong values in (a)
	Linear graph drawn	B1	2	
(c)(i)	Finding X for $y = 15$ and taking sq root	M1		
	$x \approx 5.3$	A1	2	AWRT 5.2 or 5.3; NMS 1/2
(ii)	Calculation of gradient	M1		
	a = gradient ≈ 0.37	A1		AWRT 0.36 to 0.38; NMS 1/2
	b = y-intercept ≈ 4.5	B1F	3	can be found by calculation; ft c's y-intercept
	Total		**8**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	At B, $y = (2 + h)^3 - 12(2 + h)$	M1		with attempt to expand and simplify
	$= (8 + 12h + 6h^2 + h^3) - (24 + 12h)$ $(= -16 + 6h^2 + h^3)$	B1		correct expansion of $(2 + h)^3$
	Grad $AB = \dfrac{(-16+6h^2+h^3)-(-16)}{(2+h)-2}$	m1		
	$= \dfrac{6h^2+h^3}{h} = 6h + h^2$	A1	4	convincingly shown (AG)
(b)	As $h \to 0$ this gradient $\to 0$ so gradient of curve at A is 0	E2,1	2	E1 for '$h = 0$'
	Total		**6**	
6(a)	Rotation 45° (anticlockwise)(about O)	M1A1	2	M1 for 'rotation'
(b)	Reflection in $y = x \tan 22.5°$	M1A1	2	M1 for 'reflection'
(c)	Rotation 90° (anticlockwise)(about O)	M1A1F	2	M1 for 'rotation' or correct matrix; ft wrong angle in (a)
(d)	Identity transformation	B2,1F	2	ft wrong mirror line in (b); B1 for $\mathbf{B}^2 = \mathbf{I}$
(e)	$\mathbf{AB} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	M1A1		allow M1 if two entries correct
	Reflection in $y = x$	A1	3	
	Total		**11**	
7(a)(i)	Asymptotes $x = 3$ and $y = 0$	B1,B1	2	may appear on graph
(ii)	Complete graph with correct shape	B1		
	Coordinates $\left(0, -\dfrac{1}{3}\right)$ shown	B1	2	
(iii)	Correct line, $(0, -5)$ and $(2.5, 0)$ shown	B1	1	
(b)(i)	$2x^2 - 11x + 14 = 0$	B1		
	$x = 2$ or $x = 3.5$	M1A1	3	M1 for valid method for quadratic
(ii)	$2 < x < 3$, $x > 3.5$	B2,1F	2	B1 for partially correct solution; ft incorrect roots of quadratic (one above 3, one below 3)
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\alpha + \beta = 4$, $\alpha\beta = 10$	B1,B1	2	
(b)	$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta}$	M1		
	$= \dfrac{4}{10} = \dfrac{2}{5}$	A1	2	convincingly shown (AG)
(c)	Sum of roots = $(\alpha + \beta) + 2$(ans to (b))	M1		
	$= 4\dfrac{4}{5}$	A1F		ft wrong value for $\alpha + \beta$
	Product = $\alpha\beta + 4 + \dfrac{4}{\alpha\beta}$	M1A1		M1 for attempt to expand product (at least two terms correct)
	$= 14\dfrac{2}{5}$	A1F		ft wrong value for $\alpha\beta$
	Equation is $5x^2 - 24x + 72 = 0$	A1F	6	integer coeffs and '= 0' needed here; ft one numerical error
	Total		**10**	
9(a)(i)	Parabola drawn passing through (2, 0)	M1 A1	2	with x-axis as line of symmetry
(ii)	Two tangents passing through $(-2, 0)$	B1B1	2	to c's parabola
(b)(i)	Elimination of y	M1		
	Correct expansion of $(x + 2)^2$	B1		
	Result	A1	3	convincingly shown (AG)
(ii)	Correct discriminant	B1		
	$16m^4 - 8m^2 + 1 = 16m^4 + 8m^2$	M1		OE
	Result	A1	3	convincingly shown (AG)
(iii)	$\dfrac{1}{16}x^2 - \dfrac{3}{4}x + \dfrac{9}{4} = 0$	M1		OE
	$x = 6$, $y = \pm 2$	A1,A1	3	
	Total		**13**	
	TOTAL		**75**	

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Mathematics MFP1

(Specification 6360)

Further Pure 1

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√or ft or F	follow through from previous incorrect result
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AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
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$-x$ EE	deduct x marks for each error
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Otherwise we require evidence of a correct method for any marks to be awarded.

MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha+\beta=6$, $\alpha\beta=18$	B1B1	2	
(b)	Sum of new roots = $6^2-2(18)=0$	M1A1F		ft wrong value(s) in (a)
	Product = $18^2=324$	B1F		ditto
	Equation $x^2+324=0$	A1F	4	'= 0' needed here; ft wrong value(s) for sum/product
(c)	α^2 and β^2 are $\pm 18\text{i}$	B1	1	
	Total		**7**	
2(a)	$\int 2x^{-3}\,\mathrm{d}x=-x^{-2}\ (+c)$	M1A1		M1 for correct index
	$\int_p^q 2x^{-3}\,\mathrm{d}x=p^{-2}-q^{-2}$	A1F	3	OE; ft wrong coefficient of x^{-2}
(b)(i)	As $p\to 0$, $p^{-2}\to\infty$, so no value	B1		
(ii)	As $q\to\infty$, $q^{-2}\to 0$, so value is ¼	M1A1F	3	ft wrong coefficient of x^{-2} or reversal of limits
	Total		**6**	
3(a)(i)	$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	B1	1	
(b)(i)	$\mathbf{AB}=\begin{bmatrix}-20 & 14\\ 14 & -10\end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(ii)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 5\\ -5 & 0\end{bmatrix}$	B1		
	$(\mathbf{A}+\mathbf{B})^2=\begin{bmatrix}-25 & 0\\ 0 & -25\end{bmatrix}$	B1		
	$\ldots=-25\mathbf{I}$	B1F	3	ft if c's $(\mathbf{A}+\mathbf{B})^2$ is of the form $k\mathbf{I}$
(c)(i)	Rot'n 90° clockwise, enlargem't SF 5	B2, 1	2	OE
(ii)	Rotation 180°, enlargement SF 25	B2, 1F	2	Accept 'enlargement SF −25'; ft wrong value of k
(iii)	Enlargement SF 625	B2, 1F	2	B1 for pure enlargement; ft ditto
	Total		**13**	
4	$\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$	B1		OE; dec/deg penalised at 6th mark
	$\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2}$	B1F		OE; ft wrong first value
	Use of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x-\frac{2\pi}{3}$ to x	m1		including division of all terms by 4
	GS $x=\frac{\pi}{8}+\frac{1}{2}n\pi$ or $x=-\frac{\pi}{24}+\frac{1}{2}n\pi$	A1A1	6	OE
	Total		**6**	

MFP1(cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	${z_1}^2 = \frac{1}{4} - \mathrm{i} + \mathrm{i}^2 = -\frac{3}{4} - \mathrm{i}$	M1A1	2	M1 for use of $\mathrm{i}^2 = -1$
(ii)	LHS $= -\frac{3}{4} - \mathrm{i} + \frac{1}{2} + \mathrm{i} + \frac{1}{4} = 0$	M1A1	2	AG; M1 for z^* correct
(b)	LHS $= -\frac{3}{4} + \mathrm{i} + \frac{1}{2} - \mathrm{i} + \frac{1}{4} = 0$	M1A1	2	AG; M1 for ${z_2}^2$ correct
(c)	z real $\Rightarrow z^* = z$ Discr't zero or correct factorisation	M1 A1	 2	Clearly stated AG
	Total		**8**	
6(a)	Sketch of ellipse	M1		centred at origin
	Correct relationship to circle	A1		
	Coords $\left(\pm 2\sqrt{2}, 0\right), \left(0, \pm\sqrt{2}\right)$	B2,1	4	Accept $\sqrt{8}$ for $2\sqrt{2}$; B1 for any 2 of $x = \pm 2\sqrt{2}$, $y = \pm\sqrt{2}$ allow B1 if all correct except for use of decimals (at least one DP)
(b)(i)	Replacing x by $\dfrac{x}{2}$	M1		or by $2x$
	E is $\left(\frac{x}{2}\right)^2 + y^2 = 2$	A1	2	OE
(ii)	Tangent is $\frac{x}{2} + y = 2$	M1A1	2	M1 for complete valid method
	Total		**8**	
7(a)	Denom never zero, so no vert asymp	E1		
	Horizontal asymptote is $y = 0$	B1	2	
(b)	$x - 4 = k(x^2 + 9)$ Hence result clearly shown	M1 A1	 2	 AG
(c)	Real roots if $b^2 - 4ac \geq 0$ Discriminant $= 1 - 4k(9k + 4)$ **...** $= -(36k^2 + 16k - 1)$ ... $= -(18k - 1)(2k + 1)$ Result (AG) clearly justified	E1 M1 m1 m1 A1	 5	PI (at any stage) m1 for expansion m1 for correct factorisation eg by sketch or sign diagram
(d)	$k = -\frac{1}{2} \Rightarrow -\frac{1}{2}x^2 - x - \frac{1}{2} = 0$	M1A1		or equivalent using $k = \frac{1}{18}$
	$\ldots \Rightarrow (x+1)^2 = 0 \Rightarrow x = -1$	A1		
	$k = \frac{1}{18} \Rightarrow \frac{1}{18}x^2 - x + \frac{9}{2} = 0$	A1		
	$\ldots \Rightarrow (x-9)^2 = 0 \Rightarrow x = 9$	A1		
	SPs are $\left(-1, -\frac{1}{2}\right), \left(9, \frac{1}{18}\right)$	A1	6	correctly paired
	Total		**15**	

MFP1(cont)

Q	Solution	Marks	Total	Comments
8(a)	$x_2 = 50 - \dfrac{50^3 + 2(50^2) + 50 - 100\ 000}{3(50^2) + 4(50) + 1}$	B1 B1		For numerator (PI by value 30050) For denominator (PI by value 7701)
	$x_2 \approx 46.1$	B1	3	Allow AWRT 46.1
8(b)(i)	$\Sigma r(3r + 1) = 3\Sigma r^2 + \Sigma r$	M1		
	$\ldots = 3\left(\frac{1}{6}n\right)(n+1)(2n+1) + \frac{1}{2}n(n+1)$	m1		correct formulae substituted
	$\ldots = \frac{1}{2}n(n+1)(2n+1+1)$	m1m1		m1 for each factor (n and $n + 1$)
	$\ldots = n(n+1)^2$ convincingly shown	A1	5	AG
(ii)	Correct expansion of $n(n + 1)^2$	B1	1	and conclusion drawn (AG)
(c)	Attempt at value of S_{46} Attempt at value of S_{45} $S_{45} < 100000 < S_{46}$, so $N = 46$	M1 m1 A1	3	
	Alternative method Root of equation in (a) is 45.8 So lowest integer value is 46	(B3)		Allow AWRT 45.7 or 45.8
	Total		**12**	
	TOTAL		**75**	

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General Certificate of Education (A-level)
June 2011

Mathematics **MFP1**

(Specification 6360)

Further Pure 1

Final

Mark Scheme

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Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

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Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	Attempt at $0.5 \times y'(2)$ $(= 0.25)$	M1		Other variations are allowed
	$y(2.5) \approx 3.25$	A1		
	$y(3) \approx 3.25 + 0.5\,y'(2.5)$	m1		
	$\approx 3.25 + 0.2357(0)$	A1F		PI; OE; ft c's value for $y(2.5)$
	≈ 3.4857	A1	5	4 dp needed
	Total		**5**	
2(a)	$\alpha + \beta = -\frac{3}{2}$, $\alpha\beta = \frac{3}{4}$	B1B1	2	
(b)	$\alpha^2 + \beta^2 = \left(-\frac{3}{2}\right)^2 - 2\left(\frac{3}{4}\right) = \frac{3}{4}$	M1A1	2	AG; A0 if $\alpha + \beta$ has wrong sign
(c)	Sum $= 2(\alpha + \beta) = -3$	B1F		ft wrong value for $\alpha + \beta$
	Product $= 10\alpha\beta - 3\left(\alpha^2 + \beta^2\right) = \frac{21}{4}$	M1A1F		ft wrong values
	$x^2 - Sx + P$ $(= 0)$	M1		Signs must be correct for the M1
	Eqn is $4x^2 + 12x + 21 = 0$	A1	5	Integer coeffs and '= 0' needed
	Total		**9**	
3(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	$(z - \mathrm{i})(z^* - \mathrm{i}) = (x^2 + y^2 - 1) - 2\mathrm{i}x$	m1A1	3	A1 may be earned in (b)
(b)	Equating R and I parts	M1		
	$-2x = -8$ so $x = 4$	A1		
	$16 + y^2 - 1 = 24$ so $y = \pm 3$ $(z = 4 \pm 3\mathrm{i})$	m1A1	4	A0 if $x = -4$ used
	Total		**7**	
4(a)	Use of one law of logs or exponentials	M1		
	$\lg a = c$ and $\lg b = m$	A1		OE; both needed
	So $a = 10^c$ and $b = 10^m$	A1	3	
(b)	Points (1, 1.08), (5, 1.43) plotted	M1A1		M1 A0 if one point correct
	Straight line drawn through points	A1F	3	ft small inaccuracy
(c)(i)	Attempt at antilog of $Y(3)$	M1		OE
	When $x = 3$, $Y \approx 1.25$ so $y \approx 18$	A1	2	Allow AWRT 18
(ii)	Attempt at a as antilog of Y-intercept	M1		OE
	$a \approx 9.3$ to 10	A1	2	AWRT
	Total		**10**	
5(a)	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	B1		OE stated or used; deg/dec penalised at 5th mark
	$\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$	B1F		OE; ft wrong first value
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $3x - \frac{\pi}{6}$ to x	m1		incl division of all terms by 3
	GS: $x = \frac{\pi}{18} \pm \frac{\pi}{18} + \frac{2}{3}n\pi$	A1F	5	ft wrong first value
(b)	$n = 8$ will give the required solution	M1		GS must include $\frac{2}{3}n\pi$ for this
	... which is $\frac{16}{3}\pi$ (≈ 16.755)	A1	2	from correct GS; allow $\frac{48}{9}\pi$ or dec approx
	Total		**7**	

Q	Solution	Marks	Total	Comments
6(a)	$(5+h)^3 = 125 + 75h + 15h^2 + h^3$	B1	1	Accept unsimplified coefficients
(b)(i)	$y(5+h) = 100 + 65h + 14h^2 + h^3$	B1F		PI; ft numerical error in (a)
	Use of correct formula for gradient	M1		
	Gradient is $65 + 14h + h^2$	A2,1F	4	A1 if one numerical error made; ft numerical error already penalised
(ii)	As $h \to 0$ this $\to 65$	E2,1F	2	E1 for '$h = 0$'; ft wrong values for p, q, r
	Total		**7**	
7(a)(i)	$\mathbf{A}^2 = \begin{bmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{bmatrix}$	M1A1	2	M1 if at least two entries correct
(ii)	$\mathbf{A}^3 = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$	M1		if at least two entries correct
	……..$= 8\mathbf{I}$	A1	2	
(b)(i)	$\mathbf{A}^3$ gives enlargement with SF 8 (centre the origin)	M1A1F	2	M1 for enlargement (only); ft wrong value for k
(ii)	Enlargement and rotation Enlargement scale factor 2 Rotation through 120° (antic'wise)	M1 A1 A1	3	Some detail needed
	Total		**9**	
8(a)(i)	Asymptotes $x = -2$, $x = 2$, $y = 0$	B1 × 3	3	
(ii)	Middle branch generally correct Other branches generally correct All branches approaching asymps Intersection at $\left(0, -\frac{1}{4}\right)$ indicated	B1 B1 B1 B1	4	Allow if max pt not in right place Asymps must be shown correctly on diagram or elsewhere; B0 if any other intersections are shown
(b)	$y = -2$ when $x = \pm\sqrt{3.5}$	B1		Allow NMS
	Sol'n $-2 < x < -\sqrt{3.5}, \sqrt{3.5} < x < 2$	B2,1	3	Condone dec approx'n for $\sqrt{3.5}$; B1 if $\le$ used instead of $<$
	Total		**10**	
9(a)(i)	Elimination to give $x = \frac{1}{8}x^2$	M1		OE
	A is (8, 8)	A1	2	NMS 2/2
(ii)	Equation of Q is $x = \frac{1}{8}y^2$	B1	1	OE; condone $y = \sqrt{8x}$
(iii)	Points of contact are images in $y = x$	E1	1	
(b)(i)	Eliminating y to give $-x + c = \frac{1}{8}x^2$ (ie $x^2 + 8x - 8c = 0$)	M1		
	Distinct roots if $\Delta > 0$	E1		stated or implied
	$\Delta = 64 + 32c$, so $c > -2$	A1	3	convincingly shown (AG)
(ii)	For tangent $c = -2$, so $x^2 + 8x + 16 = 0$	M1		OE
	... and $x = -4$, $y = 2$	A1		
	Reflection in $y = x$	M1		or other complete method
	$x = 2$, $y = -4$	A1F	4	ft wrong answer for first point; allow NMS 2/2
	Total		**11**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
January 2012

Mathematics MFP1

(Specification 6360)

Further Pure 1

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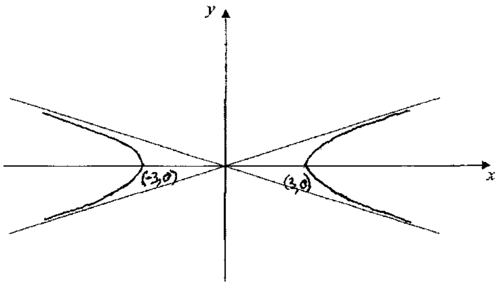
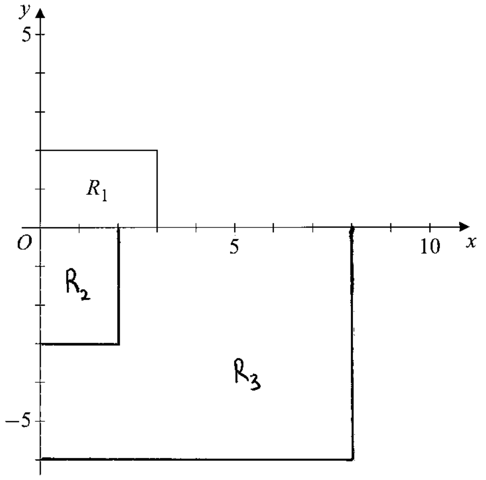
1(a)	$\alpha+\beta=-\frac{7}{2}$	B1		
	$\alpha\beta=4$	B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{7}{2}\right)^2-2(4)$	M1		Using correct identity with ft or correct substitution
	$=\frac{49}{4}-8=\frac{17}{4}$	A1	2	CSO AG. A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}=\frac{17/4}{16}\left(=\frac{17}{64}\right)$	M1		Writing $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ in a correct suitable form with ft or correct substitution
	$=\frac{17}{64}$	A1F		ft wrong value for $\alpha\beta$
	(Product =) $\frac{1}{(\alpha\beta)^2}=\frac{1}{16}\left(=\frac{4}{64}\right)$	B1F		ft wrong value for $\alpha\beta$
	x^2-Sx+P $(=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values. PI
	Eqn is $64x^2-17x+4=0$	A1	5	CSO Integer coefficients and '= 0' needed
	Total		**9**	
2(a)	$\int x^{-2/3}\,dx=3x^{1/3}(+c)$	B1		$kx^{\frac{1}{3}}$, $k\neq 0$ ie condone incorrect non-zero coefficient here
	$(3)x^{1/3}\to\infty$ as $x\to\infty$, so no finite value	E1		
(b)	$\int x^{-4/3}\,dx=-3x^{-1/3}(+c)$	M1		$\lambda x^{-1/3}$, $\lambda\neq 0$
		A1		$-3x^{-1/3}$ OE
	$\int_8^\infty x^{-4/3}\,dx=-3(0-\frac{1}{2})=\frac{3}{2}$	A1	5	CSO
	Total		**5**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$x = \pm 3\text{i}$	B1	1	$\pm 3\text{i}$ $(a = 0, b = \pm 3)$
(ii)	$x = -2 \pm 3\text{i}$	B1F	1	If not correct, ft wrong answer(s) to (i) provided (i) has a non-zero b value
(b)(i)	$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$	B1	1	Terms simplified in any order.
(ii)	$(1 + 2\text{i})^3 = 1 + 3(2\text{i}) + 3(2\text{i})^2 + (2\text{i})^3$ $= 1+3(2\text{i}) + 3(4\text{i}^2) + (8\text{i}^3)$	B1F		Replacing x in (b)(i) by 2i, squaring and cubing correctly, only ft on c's wrong non-zero coefficients from (b)(i).
	$= 1 + 3(2\text{i}) + 3(4)(-1) + (8)(-\text{i})$ $= -11 - 2\text{i}$	M1 A1	 3	Use of $\text{i}^2 = -1$ at least once. $-11 - 2\text{i}$ $(a = -11,\ b = -2)$
(iii)	$z^* - z^3 = 1 - 2\text{i} - (-11 - 2\text{i})$ $= 12$	M1 A1F	 2	Use of $z^* = 1 - 2\text{i}$ If not correct, only ft on $1 - 2\text{i}$ − c's (b)(ii) if c's (b)(ii) answer is of the form $a + b\text{i}$ with $a \neq 0$ and $b \neq 0$
	Total		**8**	
4(a)	$\Sigma r^2(4r-3) = 4\Sigma r^3 - 3\Sigma r^2$ ...	M1		Splitting up the sum into two separate sums. PI by next line.
	$= 4\left(\frac{1}{4}\right)n^2(n+1)^2 - 3\left(\frac{1}{6}\right)n(n+1)(2n+1)$	m1		Substitution of the two summations from FB
	$= n(n+1)\left[n(n+1) - \frac{1}{2}(2n+1)\right]$	m1		Taking out common factors n and $n + 1$.
		A1		Remaining expression eg our […] in ACF not just simplified to AG
	Sum $= \frac{1}{2}n(n+1)(2n^2-1)$	A1	5	Be convinced as form of answer is given, penalise any jumps or backward steps
(b)	$\sum_{r=20}^{40} r^2(4r-3)$ $= \sum_{r=1}^{40} r^2(4r-3) - \sum_{r=1}^{19} r^2(4r-3)$	M1		Attempt to take S(19) from S(40) using part (a)
	$= 20(41)(3199) - 9.5(20)(721)$ $= 2623180 - 136990$ $\sum_{r=20}^{40} r^2(4r-3) = 2486190$	A1	2	2486190 ; Since 'Hence' NMS 0/2. **SC** $\sum_{r=1}^{40}....... - \sum_{r=1}^{20}......$ clearly attempted and evaluated to 2455390 scores B1
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	Line joining points A and B	B1	1	Must not be linked to Q
(ii)	$x_P = 2 + w$, $\frac{w}{10} = \frac{5-2}{22-(-10)}$	M1		OE eg correct equation for AB with y replaced by 0
	$x_P = 2 + 10\times\frac{3}{32}$	A1		$2+10\times\frac{3}{32}$ OE
	$x_P = 2.9375 = 2.9$ (to 1dp)	A1	3	CAO Must be 2.9
(b)(i)	Tangent at A drawn	B1	1	At least as far as meeting the x-axis. Accept reasonable attempt. Must not be linked to P.
(ii)	$x_Q = 2 - \frac{-10}{8}$	M1		PI by 3.25 or 26/8 OE
	... $= 3.25$	A1	2	CAO Must be 3.25
	Total		**7**	
6(a)	$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$	B1		OE (PI) Stated or used. A correct angle in 1st or 3rd quadrant for $\tan^{-1}(1/\sqrt{3})$. Condone degrees / decimal equivs.
	$\left(\frac{x}{2} - \frac{\pi}{4}\right) = n\pi + \frac{\pi}{6}$;	M1		Correct use of either $n\pi$ or $2n\pi$. Eg either $n\pi + \alpha$ or **both** $2n\pi + \alpha$ and $2n\pi + \pi + \alpha$ OE where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right)$ $\left(= 2n\pi + \frac{5\pi}{6}\right)$	m1		Either correct rearrangement of $\frac{x}{2} - \frac{\pi}{4} = n\pi + \alpha$ to $x = \ldots$, or correct rearrangements of both the equivalents above in the M1 line involving $2n\pi$, where α is c's $\tan^{-1}(1/\sqrt{3})$. Condone degrees/decimals/mixture
		A1	4	ACF, but must now be exact and in terms of π.
(b)	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \pm\sqrt{\frac{1}{3}}$	M1		PI. Taking square roots, must include the $\pm$ or evidence of its use
	$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = -\sqrt{\frac{1}{3}}$ $\Rightarrow \frac{x}{2} - \frac{\pi}{4} = n\pi - \frac{\pi}{6}$;	m1		OE If not correct, ft on c's working in (a) with c's α replaced by $-\alpha$. Condones as in m1 above.
	$x = 2\left(n\pi + \frac{\pi}{6} + \frac{\pi}{4}\right)$, $x = 2\left(n\pi - \frac{\pi}{6} + \frac{\pi}{4}\right)$ $\{x = 2n\pi + \frac{5\pi}{6}$, $x = 2n\pi + \frac{\pi}{6}\}$	A1F	3	Any valid form, but only ft on c's exact value for $\tan^{-1}(1/\sqrt{3})$ in terms of π.
	Total		**7**	

Q	Solution	Marks	Total	Comments
7(a)	$y = \pm \frac{1}{3}x$	B1	1	ACF Need both
(b)		B1 B1 B1	 3	2-branch curve with branches in correct regions above and below x-axis Curve approaching asymptotes Coords (±3, 0), as **only** points of intersection with coordinate axes, indicated. Condone −3 and +3 marked on x-axis at points of intersection as (±3, 0) indicated.
(c)(i)	$\frac{(x+3)^2}{9} - y^2 = 1$	M1 A1	 2	Replacing x by either $x + 3$ or $x - 3$ ACF
(ii)	$\frac{(x+3)^2}{9} - x^2 = 1$	M1		Substitution into c's (c)(i) eqn of $y = x$ to eliminate y or of $x = y$ to eliminate x
	$x^2 + 6x + 9 = 9(x^2 + 1)$	A1F		Correct expansion of $(x \pm 3)^2$ equated to $9(x^2 + 1)$ OE ft; [OE in y]
	$8x^2 - 6x = 0 \quad (8x^2 = 6x)$	A1F		Ft on error $(x - 3)$ for $(x + 3)$ in (c)(i) which gives $8x^2 + 6x = 0 \quad (8x^2 = -6x)$ [OE in y]
	Points are (0, 0), $\left(\frac{3}{4}, \frac{3}{4}\right)$	A1	4	Both. ACF
(d)	 Points are (3, 0), $\left(3\frac{3}{4}, \frac{3}{4}\right)$	M1 A1F	 2	Adding 3 to c's (c)(ii) **two** x-coords keeping y-coordinates unchanged. Ft on c's (c)(ii) coordinates for the **two** points If not deduced then M0A0
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)(i)		B1	1	Rectangle with vertices (0, 0), (0, −3), (2, −3), (2, 0)
(ii)		M1		Rectangle with vertices **either** whose x-coords are c's (a)(i) x-coord vertices multiplied by 4 **or** whose y-coords are c's (a)(i) y-coord vertices multiplied by 2
		A2,1	3	A2 if rectangle with vertices (0, 0), (0, −6), (8, −6), (8, 0) (A1 if either the four x-coords are correct or the four y-coords are correct)
(b)(i)	Matrix is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} =$	M1		Attempt to multiply $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ with c's (b)(i) matrix in **either** order.
		m1		Multiplication in **correct** order with at least two of the four ft multiplications carried out correctly.
	$\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$	A1	3	For $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ NMS $\begin{bmatrix} 0 & 4 \\ -2 & 0 \end{bmatrix}$ scores B3 $\begin{bmatrix} 0 & 2 \\ -4 & 0 \end{bmatrix}$ scores B1
	Total		**8**	

Q	Solution	Marks	Total	Comments
9(a)	Asymptotes $x = 1$ $y = 1$	B1 B1	 2	$x = 1$ OE $y = 1$ OE
(b)	$-4x + c = \dfrac{x}{x-1}$	M1		Elimination of y PI by next line
	$(-4x + c)(x - 1) = x$ $-4x^2 + cx + 4x - c = x$ $-4x^2 + cx + 3x - c = 0$	A1		OE (denominators cleared)
	$4x^2 - (c+3)x + c = 0$	A1	3	CSO AG No incorrect algebraic expressions etc
(c)(i)	Discriminant is $(c + 3)^2 - 4(4c)$	B1		OE
	For tangency $c^2 - 10c + 9 = 0$	M1		Forming a quadratic eqn in c after equating discriminant to zero
	$(c - 9)(c - 1) = 0 \Rightarrow c = 1,\ c = 9$	A1	3	Correct values 1, 9 for c.
(ii)	$\underline{c = 1}$: $4x^2 - 4x + 1 = 0$ $\underline{c = 9}$: $4x^2 - 12x + 9 = 0$	M1		Substitutes at least one of c's values for c from (c)(i) either into the given quadratic in (b) OE or into $\dfrac{c+3}{8}$
	$4x^2 - 4x + 1 = 0 \quad \Rightarrow \quad x = 1/2 \quad (= 0.5)$	A1		No other root from quadratic
	$4x^2 - 12x + 9 = 0 \quad \Rightarrow \quad x = 3/2 \quad (= 1.5)$	A1		No other root from quadratic
	When $x = 1/2$, $y = -1$; when $x = 3/2$, $y = 3$ $\left(\dfrac{1}{2}, -1\right) \quad \left(\dfrac{3}{2}, 3\right)$	A1	4	Accept in either format
	Total		**12**	
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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

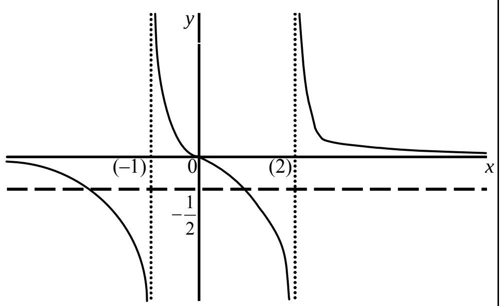
Otherwise we require evidence of a correct method for any marks to be awarded.

General Certificate of Education
MFP1 June 2012

Q	Solution	Marks	Total	Comments
				Accept correct equivalent decimals in place of some/all fractions in the scheme
1(a)	$\alpha+\beta=\frac{7}{5}\ (=1.4)$	B1		
	$\alpha\beta=\frac{1}{5}\ (=0.2)$	B1	2	
(b)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	M1		OE eg $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\frac{1}{5}[7(\alpha+\beta)-1-1]}{\alpha\beta}$ scores M1 m1
	$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\quad=\frac{\left(\frac{7}{5}\right)^2-2\left(\frac{1}{5}\right)}{\frac{1}{5}}$	m1		Correct expression for $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ in terms of either $(\alpha+\beta)$ and $\alpha\beta$ or with numerical substitution of correct/c's values from (a)
	$=\frac{\frac{49}{25}-2\left(\frac{1}{5}\right)}{\frac{1}{5}}\ =\frac{\frac{49}{25}-\frac{2}{5}}{\frac{1}{5}}=\frac{\frac{39}{25}}{\frac{1}{5}}=\frac{39}{5}$	A1	3	CSO AG must see some intermediate evaluation, must see one of the first three expressions A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}\ =\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$	M1		Writing $\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}$ in a correct suitable form or with numerical values
	$\left(=\frac{7}{5}+\frac{\frac{7}{5}}{\frac{1}{5}}\right)$			
	(Product =) $\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}$ $=\frac{1}{5}+\frac{39}{5}+5$	M1		Correct expression for product into which substitution of numbers attempted for all terms, at least one either correct/correct ft
	Sum = $\frac{42}{5}$, Product = 13	A1		OE <u>Both</u> **SC** If B0 for $\alpha+\beta=-\frac{7}{5}$ in (a), and (c) S= $-\frac{42}{5}$ oe, P = 13 award this A1
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of equation **with** ft substitution of c's S and P values. PI. M0 if either $S=\alpha+\beta$ or $P=\alpha\beta$ values
	Equation is $5x^2-42x+65=0$	A1	5	CSO Integer coefficients and '= 0' needed. Dependent on B1B1 in (a) and previous 4 marks in (c) scored
	Total		**10**	

Q	Solution	Marks	Total	Comments
2(a)	$y = x^4 + x$			
	$\{y(-2+h) =\}$ $(-2+h)^4 + (-2+h)$	M1		$(-2+h)^4 + (-2+h)$ PI
	$= h^4 - 8h^3 + 24h^2 - 32h + 16 - 2 + h$	B1		Correct expansion of $(-2+h)^4$ as $h^4 - 8h^3 + 24h^2 - 32h + 16$ PI
	$= h^4 - 8h^3 + 24h^2 - 31h + 14$	A1F		Seen separately or as part of the gradient expression. Ft one incorrect term in expansion of $(-2+h)^4$
	Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$			
	$= \dfrac{h^4 - 8h^3 + 24h^2 - 31h + 14 - (14)}{-2 + h - (-2)}$	M1		Use of correct formula for gradient PI
	$= \dfrac{h^4 - 8h^3 + 24h^2 - 31h}{h} =$ $h^3 - 8h^2 + 24h - 31$	A1	5	The four correct terms in any order A0 if incorrect (constant/h) term ignored due printed form of answer
(b)	As $h \to 0$, gradient of line in **(a)** $\to$ gradient of curve at point $(-2, 14)\}$	E1		Lim [c's$(p+qh+rh^2+h^3)$] OE $h \to 0$ NB '$h=0$' instead of '$h \to 0$' gets E0
	{Gradient of curve at point $(-2, 14)$ is} -31	E1	2	Dependent on previous E1 and printed **form** of answer in (a) obtained convincingly but then ft on c's p value
	Total		**7**	
3(a)	$\mathrm{i}(z+7) + 3(z^* - \mathrm{i}) =$ $\mathrm{i}(x + \mathrm{i}y + 7) + 3(x - \mathrm{i}y - \mathrm{i})$	M1		M1 for use of $z^* = x - \mathrm{i}y$
	$= \mathrm{i}x - y + 7\mathrm{i} + 3x - 3\mathrm{i}y - 3\mathrm{i}$	M1		M1 for $\mathrm{i}^2 y = -y$
	$= 3x - y + \mathrm{i}(x - 3y + 4)$	A1	3	If the five terms correct but not grouped into Real and Imaginary parts, allow A1 retrospectively provided the correct two expressions used in the M1 line in (b)
(b)	$3x - y = 0, \quad x - 3y + 4 = 0$	M1		Attempting to equate all Real parts to zero and all Imaginary parts to zero
	$x - 9x + 4 = 0$ (or eg $y - 9y + 12 = 0$)	A1		A correct equation in either x or y PI by correct final answer
	Solving to give $z = \frac{1}{2} + \frac{3}{2}\mathrm{i}$	A1	3	Allow $x = \frac{1}{2}$, $y = \frac{3}{2}$
	Total		**6**	

Q	Solution	Marks	Total	Comments
				Watch out for the many correct different forms of the general solutions
4	$\sin\left(70^\circ - \frac{2}{3}x\right) = \cos 20^\circ = \sin 70°$	B1		OE cos20 = sin70; or cos20 = sin110 etc PI
	$\sin\left(70^\circ - \frac{2}{3}x\right) = \sin 110^\circ$	B1		OE; Use of a correct angle, in degrees, in other relevant quadrant PI
	$70^\circ - \frac{2}{3}x = 360n^\circ + "70^\circ"$ $70^\circ - \frac{2}{3}x = 360n^\circ + "110^\circ"$	M1		OE; Either one, showing a correct use of 360n in forming a general solution. Condone $2n\pi$ in place of 360n
	$x = \frac{3}{2}\left(70^\circ - 70^\circ - 360n^\circ\right)$ $x = \frac{3}{2}\left(70^\circ - 110^\circ - 360n^\circ\right)$	m1		Rearrangement of $70 - \frac{2}{3}x = 360n + \alpha$ OE to $x = -\frac{3}{2}\left(\pm 360n + \alpha - 70\right)$ OE, where α is from c's $\sin\alpha = \cos 20$ Condone $2n\pi$ in place of 360n
	$x = -540n^\circ$; $x = -540n^\circ - 60^\circ$	A2,1,0	6	OE eg 540nº, 540nº−60º. Condone 0 ± 540n for ± 540n. If not A2, award (i) A1 for either correct unsimplified full general solution or (ii) A1F for correct ft full general solution, ft c's wrong angle(s) after award of B0, may be left in unsimplified form(s) or (iii) A1 for 'correct' simplified full general solution but with radians present A0 for only a partial correct solution
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)	Asymptotes			
	$x = -1$	B1		$x = -1$ OE
	$x = 2$	B1		$x = 2$ OE
	$y = 0$	B1	3	$y = 0$
(b)	$-\frac{1}{2} = \frac{x}{x^2 - x - 2} \Rightarrow x^2 - x - 2 = -2x$	M1		Correctly removing brackets and fractions to reach $x^2 - x - 2 = -2x$ OE
	$x^2 + x - 2 = 0 \Rightarrow x = 1,\ x = -2$	A1	2	Correct two values for x-coordinates. NMS 2 or 0 marks
(c)		M1		Three branches shown on sketch of C with either middle branch or outer two branches correct in shape
		A1		All three branches, correct shape and positions and approaching correct asymptotes in a correct manner. If middle branch does **clearly** not go through the origin, then A0
		B1	3	Correct sketch of line (L), $y = -0.5$ identified
(d)	$-2 \le x < -1$	B1		Condone < for ≤ or vice versa
	$1 \le x < 2$	B1		Condone < for ≤ or vice versa
	$-2 \le x < -1,\ 1 \le x < 2$	B1	3	All complete and correct
	Total		**11**	

Q	Solution	Marks	Total	Comments
6(a)	$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	M1 A1	2	If A1 not scored, award M1A0 for all correct entries expressed in trig form eg $\begin{bmatrix} \cos 135 & -\sin 135 \\ \sin 135 & \cos 135 \end{bmatrix}$
(b)(i)	**M**= $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \sqrt{2} \times \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $= \left(= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \right)$	M1		Or better PI by cand. having both a correct scale factor of enlargement and a correct corresponding angle of rotation
	Scale factor of enlargement is $\sqrt{2}$	A1		SF = $\sqrt{2}$ OE surd form
	Angle of rotation is 135 (degrees anticlockwise)	A1	3	Angle = 135 OE eg −225 If M0 give B1 for SF=$\sqrt{2}$ OE surd and B1 for angle = 135 OE
(b)(ii)	For $\mathbf{M}^2$, SF of enlargement = 2	B1F		OE If incorrect, ft on [c's SF in (b)(i)]^2
	Angle of rotation is 270 (degrees anticlockwise)	B1F	2	OE, eg − 90(degrees), eg 90 (degrees) clockwise If incorrect, ft on 2×c's angle in (b)(i) (neither B1F B1 nor B1 B1F is possible)
(iii)	$\mathbf{M}^2 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $\mathbf{M}^4 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$	M1		For complete method (matrix calculation or geometrical reasoning) Matrix for $\mathbf{M}^2$ could be seen earlier (M0 if >1 independent error in matrix multiplication) Geometrically SF = 4, rotation angle= 540 OE scores M1 and completion scores A1
	$\mathbf{M}^4 = -4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -4\mathbf{I}$	A1	2	Either of these two forms convincingly shown
(iv)	$\mathbf{M}^{2012} = (\mathbf{M}^4)^{503} = (-4\mathbf{I})^{503} =$ $-(2^2)^{503}\mathbf{I} = -2^{1006}\mathbf{I}$	E1		OE Fully explained, algebraically from $(-4\mathbf{I})^{503}$, or geometrically
	$\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$ (Geometrically: $\mathbf{M}^{2012}$ represents an enlargement with SF 2^{1006} followed by a rotation of angle 2012×135º ie 754.5 revolutions, being equivalent to rotation of 180º ie matrix is $-\mathbf{I}$ so $\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$)	B1	2	$\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$ (n = 1006) (B0 if FIW)
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	Let $\mathrm{f}(x) = 24x^3 + 36x^2 + 18x - 5$			
	$\mathrm{f}(0.1) = -2.816$, $\mathrm{f}(0.2) = 0.232$	M1		Both attempted and at least one evaluated correctly to at least 1sf rounded or truncated OE fraction
	Change of sign so α lies between 0.1 and 0.2	A1	2	Need both evaluations correct to above degree of accuracy and 'change of sign OE' <u>and</u> relevant reference to 0.1 and 0.2
(b)	$\mathrm{f}(0.15) = -1.409$ (< 0 so root > 0.15)	M1		f(0.15) considered first
	$\mathrm{f}(0.175) \approx -0.619$ (< 0 so root > 0.175)	A1		f(0.15) then f(0.175) both evaluated correctly to at least 1sf OE fractions
	α lies between 0.175 and 0.2	A1	3	Dependent on both previous marks gained and no other additional evaluations other than at 0.15 and 0.175
(c)	$\mathrm{f}'(x) = 72x^2 + 72x + 18$	B1		PI
	$(x_2 =)$ $0.2 - \dfrac{24(0.2)^3 + 36(0.2)^2 + 18(0.2) - 5}{72(0.2)^2 + 72(0.2) + 18}$	B1 B1		B1 for numerator in correct formula B1 for denominator in correct formula
	$= 0.1934$ (to 4dp)	B1	4	CAO Must be 0.1934 Do not apply ISW NMS scores 0/4
	Total		**9**	

Q	Solution	Marks	Total	Comments
8(a)	$(\pm\sqrt{5}\ ,\ 0)$, $(0\ ,\ \pm 2)$	B2,1	2	If not B2, award B1 if either at least two of these 4 correct pts or if ' $x = \pm\sqrt{5}$ and $y = \pm 2$'
8(b)	$\frac{(x-p)^2}{5} + \frac{y^2}{4} = 1$	M1 A1	2	Replacing x by either $x + p$ or $x - p$ and keeping y unchanged or as $y \pm 0$ ACF
8(c)	$\frac{(x-p)^2}{5} + \frac{(x+4)^2}{4} = 1$	M1		Substitution into c's (**b**) eqn of $y = x+4$ to eliminate y
	$4(x-p)^2 + 5(x+4)^2 = 4 \times 5$ $4(x^2 - 2px + p^2) + 5(x^2 + 8x + 16) = 20$	m1		Denominators 5 and 4 cleared in a correct manner and at least either a correct expansion of $(x \pm p)^2$ or a correct expansion of $(x + 4)^2$
	$4x^2 - 8px + 4p^2 + 5x^2 + 40x + 80 = 20$ $9x^2 - (8p - 40)x + 4p^2 + 60 = 0$	A1	3	CSO No errors in any line of working. AG. Must see brackets correctly removed and all terms involving x, p correctly rearranged to same side before the printed answer is stated. Must have '= 0' although brackets around $4p^2 + 60$ may be omitted
(d)	Discriminant is $(8p - 40)^2 - 4\,(9)\,(4p^2 + 60)$	B1		OE Must be isolated, not just within the quadratic formula
	For tangency $(8p - 40)^2 - 4\,(9)\,(4p^2 + 60) = 0$	M1		OE Equating c's discriminant to zero before obtaining any values for p
	$p^2 + 8p + 7 = 0$	A1		ACF with like terms collected
	$\{(p+1)(p+7) = 0 \Rightarrow\}\ p = -1, p = -7$ (*)	B1		Correct values $-1, -7$ for p
	$\underline{p=-1}$: $9x^2 + 48x + 64\ (= 0)$ $\underline{p=-7}$: $9x^2 + 96x + 256\ (= 0)$	M1		Substitutes at least one of c's **two** values for p either into the given quadratic in (c) OE or into $\frac{8p-40}{18}$
	$\underline{p=-1}$: $9x^2 + 48x + 64\ (= 0) \Rightarrow x = -\frac{8}{3}$	A1		$x = -\frac{8}{3}$ OE as only root from the quadratic or from $\frac{8p-40}{18}$. Apply FIW if (*) is B0
	$\underline{p=-7}$: $9x^2 + 96x + 256\ (=0) \Rightarrow x = -\frac{16}{3}$	A1		$x = -\frac{16}{3}$ OE as only root from the quadratic or from $\frac{8p-40}{18}$. Apply FIW if (*) is B0
	$x = -\frac{8}{3}$, $y = \frac{4}{3}$; $x = -\frac{16}{3}$, $y = -\frac{4}{3}$ $\left(-\frac{8}{3}, \frac{4}{3}\right)$ $\left(-\frac{16}{3}, -\frac{4}{3}\right)$	A1	8	CSO Previous 7 marks must have been awarded and coordinates of both points need to be correct and exact but accept in either format
	Total		**15**	
	TOTAL		**75**	

Version

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Mathematics MFP1

(Specification 6360)

Further Pure 1

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

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1	$y_{n+1} \approx y_n + h\,\mathrm{f}(x_n)$			OE
	$h\,y'(1) = 0.1 \times y'(1)$ (=0.05)	M1		Attempt to find $h\,y'(1)$. PI by eg 3.05 for $y(1.1)$
	$y(1.1) \approx 3 + 0.05 = 3.05$	A1		
	$y(1.2) \approx y(1.1) + 0.1 \times y'(1.1) = 3.05 + 0.1 \times y'(1.1)$ $\approx 3.05 + 0.1 \times \frac{1.1}{1+1.1^3}$ $\left(= 3.05 + 0.1 \times \frac{1100}{2331}\right)$	m1		Attempt to find $y(1+0.1)+0.1\times y'(1+0.1)$ must see evidence of calculation if correct ft [0.047..+c's $y(1.1)$] value not obtained
	$\approx 3.05 + 0.047(19\ldots..)$	A1F		OE; ft on [0.047..+c's $y(1.1)$] value; PI
	≈ 3.0972 (to 4 d.p.)	A1	5	Must be 4 dp.
	Total		**5**	
2(a)	$(w=)\ \frac{-6 \pm \sqrt{36-4(34)}}{2}$ $\left\{= \frac{-6 \pm \sqrt{-100}}{2}\right\}$	M1		Correct substitution into quadratic formula OE
	$= \frac{-6 \pm 10\mathrm{i}}{2}$	B1		$\sqrt{-100} = 10\mathrm{i}$ or $\sqrt{-100}/2 = 5\mathrm{i}$
	$= -3 \pm 5\mathrm{i}$	A1	3	$-3 \pm 5\mathrm{i}$ ($p=-3$, $q=\pm 5$) NMS mark as 3/3 or 0/3
(b)(i)	$z = \mathrm{i}(1+\mathrm{i})(2+\mathrm{i}) = \mathrm{i}(2+3\mathrm{i}+\mathrm{i}^2) = 2\mathrm{i} + 3\mathrm{i}^2 + \mathrm{i}^3$	M1		Attempt to expand all brackets.
	$= 2\mathrm{i} + 3(-1) + \mathrm{i}(-1)$	B1		$\mathrm{i}^2 = -1$ used at least once
	$= -3 + \mathrm{i}$	A1	3	$-3 + \mathrm{i}$ ($a = -3$, $b = 1$)
(ii)	$z^* = -3 - \mathrm{i}$	B1F		OE Ft c's $a - b\mathrm{i}$
	$-3 + \mathrm{i} + m(-3 - \mathrm{i}) = n\mathrm{i}$ $\Rightarrow -3-3m=0;\ 1-m=n$	M1		Equating **both** real parts and the imag. parts, PI by next line
	$\Rightarrow m = -1,\ n = 2$	A1	3	Both correct
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$	B1		OE (PI) Stated or used. A correct angle in 1st or 2nd quadrant for $\sin^{-1}(\surd 3/2)$.
	$\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$	B1F		OE (PI) Stated or used. A correct ft angle in remaining quadrant for $\sin^{-1}(\surd 3/2)$. B0F if angle used is in an incorrect quadrant
	$2x+\frac{\pi}{4}=2n\pi+\frac{\pi}{3}$; $2x+\frac{\pi}{4}=2n\pi+\frac{2\pi}{3}$	M1		OE Either. Ft on c's $\sin^{-1}(\surd 3/2)$.
	$x=\frac{1}{2}\left(2n\pi+\frac{\pi}{3}-\frac{\pi}{4}\right)$; $x=\frac{1}{2}\left(2n\pi+\frac{2\pi}{3}-\frac{\pi}{4}\right)$	m1		Either. Correct rearrangement of $2x+\frac{\pi}{4}=2n\pi+\alpha$ to $x=\dots$, where α is c's $\sin^{-1}(\surd 3/2)$.
	GS: $x=n\pi+\frac{\pi}{24}$; $x=n\pi+\frac{5\pi}{24}$	A2,1,0	6	Both in ACF, but must now be exact and in terms of π for A2. A1 if decimal approx used.
(b)	$n=5$ (gives greatest soln $<6\pi$) $=5\pi+\frac{5\pi}{24}$	M1		Applying a correct value for n which gives greatest soln. $<6\pi$ for c's GS dep on GS, using above method, having two expressions of the form $n\pi+\lambda$, for different λ and m1 scored in (a).
	$=\frac{125\pi}{24}$	A1	2	Dep on correct full GS.
	Total		**8**	
4	$\int\frac{1}{x\sqrt{x}}\,dx=\int x^{-\frac{3}{2}}\,(dx)$	M1		$\int x^{-\frac{3}{2}}$ PI
	$=-2x^{-\frac{1}{2}}\ (+c)$	A1		ACF, can be unsimplified. Condone absence of $+c$
	$-2x^{-\frac{1}{2}}\to 0$ as $x\to\infty$	E1		OE Ft on $k\,x^{-n}$, $n>0$
	$\int_{25}^{\infty}\frac{1}{x\sqrt{x}}\,dx=\frac{2}{5}$	A1	4	
	Total		**4**	

Q	Solution	Marks	Total	Comments
5(a)	$\alpha + \beta = -2$	B1		
	$\alpha\beta = -5$	B1	2	
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - 2(-5)$	M1		OE Using correct identity for $\alpha^2+\beta^2$ with ft or correct substitution
	$= 14$	A1	2	CSO A0 if $\alpha+\beta$ has wrong sign
(c)	$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2+\beta^2)$	M1		PI Seen at least once in part (c). OE eg $\alpha^3\beta + \alpha\beta^3 = \alpha\beta[(\alpha+\beta)^2 - 2\alpha\beta]$
	S(um)$= \alpha^3\beta + \alpha\beta^3 + 2 = (-5)(14)+2 = -68$	A1F		Correct or ft c's $\alpha\beta \times$ c's [answer (b)] +2
	P(roduct) $= (\alpha\beta)^4 + \alpha^3\beta + \alpha\beta^3 + 1$ $= (-5)^4+(-5)(14)+1 = 556$	A1F		Correct or ft [c's $\alpha\beta$]^4 + c's $\alpha\beta \times$ c's [answer (b)] +1
	$x^2 - Sx + P$ (=0)	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.
	Eqn.: $x^2 + 68x + 556 = 0$	A1	5	CSO ACF
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathbf{X}^2 = \begin{bmatrix} 7 & 2 \\ 3 & 6 \end{bmatrix}$; $(m =)\ 7$	B1	1	$(m =)7$ or 7 as top left element of $\mathbf{X}^2$
(ii)	$\mathbf{X}^3 = \begin{bmatrix} 13 & 14 \\ 21 & 6 \end{bmatrix}$;	M1		At least 2 elements correct
	$7\mathbf{X} = \begin{bmatrix} 7 & 14 \\ 21 & 0 \end{bmatrix}$	B1		PI
	$\mathbf{X}^3 - 7\mathbf{X} = \begin{bmatrix} 13-7 & 14-14 \\ 21-21 & 6-0 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$	A1F		Ft on c's m value
	$= 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 6\,\mathbf{I}$	A1	4	CSO Accept either form but at least one must be shown explicitly
(b)(i)	Reflection in the x-axis	B1	1	OE
(ii)	$\mathbf{B} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$	M1		Either OE. For M mark, accept dec. equiv. (at least 3sf) for $\frac{1}{\sqrt{2}}$
	$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$	A1	2	NMS SC1 for $k = \frac{1}{\sqrt{2}}$ or better.
(iii)	$\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$	M1		Attempt to find $\mathbf{AB}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
	$= k\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\left\{\text{or } k\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right\}$	A1		Either $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$
	$= k\begin{bmatrix} -3 \\ -1 \end{bmatrix}$	m1		Completing the matrix mult. to reach a 2×1 matrix
	(Image of P is the point) $\left(-\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	A1	4	**CSO** **SC** Wrong order, works with $\mathbf{BA}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, mark out of a max of M1A0 m1A0
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)	$y = ax^n \Rightarrow \log_{10} y = \log_{10} ax^n$ $\log_{10} y = \log_{10} a + \log_{10} x^n$	M1		Take logs and apply one log law in soln. correctly PI.
	$\log_{10} y = \log_{10} a + n\log_{10} x$	m1		Apply a further log law correctly.
	$Y = \log_{10} a + nX$ (which is a linear relationship between Y and X.)	A1	3	Correct eqn. with base 10 (or lg or later evidence of use of base 10 if log without base here)
(b)	n = gradient of line	M1		Stated or used. Accept $n = \pm\frac{2}{3}$ OE as evidence
	$n = -\frac{2}{3}$	A1		$n = -\frac{2}{3}$ (OE 3sf)
	$\log_{10} a = 4$	M1		Equating c's constant term [must involve a log] in c's **(a)** eqn. to the Y-intercept value PI by correct value of a
	$a = 10^4$ (= 10 000)	A1	4	
	Total		**7**	

Q	Solution	Marks	Total	Comments
8(a)	$\sum_{r=1}^{n} 2r(2r^2-3r-1) = \sum_{r=1}^{n} 4r^3 - \sum_{r=1}^{n} 6r^2 - \sum_{r=1}^{n} 2r$ $= 4\sum_{r=1}^{n} r^3 - 6\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r$	M1		Splitting up the sum into three separate sums. PI by m1 line below.
	$= 4\times\frac{1}{4}n^2(n+1)^2 - 6\times\frac{1}{6}n(n+1)(2n+1) - 2\times\frac{1}{2}n(n+1)$	m1		Substitution of the three summations from FB into $a\sum_{r=1}^{n} r^3 + b\sum_{r=1}^{n} r^2 + c\sum_{r=1}^{n} r$
	$= n^2(n+1)^2 - n(n+1)(2n+1) - n(n+1)$	A1		PI by later expressions
	$= n(n+1)[n(n+1)-(2n+1)-1]$	m1		Taking out factor $n(n+1)$ from correct expressions
	$= n(n+1)[n^2-n-2]$	A1		
	$= n(n+1)(n+1)(n-2)$ $(= n(n-2)(n+1)^2$ $(p=-2, q=1))$	A1	6	
(b)	$\sum_{r=11}^{20} 2r(2r^2-3r-1)$ $= \sum_{r=1}^{20} 2r(2r^2-3r-1) - \sum_{r=1}^{10} 2r(2r^2-3r-1)$	M1		$\sum_{r=1}^{20}\ldots - \sum_{r=1}^{10}\ldots$ PI by next line (ft c's p&q)
	$= 20(20+p)(20+q)^2 - 10(10+p)(10+q)^2$			
	$= 20\times18\times21^2 - 10\times8\times11^2 = 158760 - 9680 = 149080$	A1	2	NMS 0/2 A0 if not showing use of fully factorised form.
	Total		**8**	

Q	Solution	Marks	Total	Comments
9(a)	$y=0,\ \dfrac{(x-4)^2}{4}=1;\quad (x-4)^2=4$	M1		OE Sub y=0 in eqn of ellipse and either eliminate fraction **or** take sq root, condoning missing ±, ie $\dfrac{(x-4)}{2}=(\pm)1$
	$\Rightarrow x=2,6\ \ (x_A=2,\ x_B=6)$	A1	2	Both 2 and 6 NMS Mark as B2 or B0
(b)(i)	$\dfrac{(x-4)^2}{4}+(mx)^2=1\quad\Rightarrow$	M1		Substitute y=mx to eliminate y
	$(x-4)^2+4(mx)^2=4 \Rightarrow x^2-8x+16+4(mx)^2=4$	A1		Eliminate fractions correctly and expand $(x-4)^2$ correctly
	$\Rightarrow x^2-8x+16+4m^2x^2-4=0$ $\Rightarrow (1+4m^2)x^2-8x+12=0$	A1	3	CSO AG
(ii)	Discriminant $b^2-4ac\ \ \{(-8)^2-4(1+4m^2)(12)\}$	M1		b^2-4ac in terms of m condone one sign or copying error OE
	For tangency, $(-8)^2-4(1+4m^2)(12)=0$	A1		A correct equation with m^2 being the only unknown at any stage.
	$192m^2-16\ \ (=0)$	A1		OE eg $12m^2-1(=0)$ OE PI by a correct value for m condoning wrong sign
	(m>0 so) $m=\dfrac{1}{\sqrt{12}}$	A1	4	ACF of an **exact** value for m eg $\dfrac{1}{2\sqrt{3}}$, $\dfrac{\sqrt{3}}{6}$. Dep on prev 3 mrks
(iii)	$\left(1+4\times\left\{\dfrac{1}{\sqrt{12}}\right\}^2\right)x^2-8x+12\ \ (=0)$	M1		Subst value for m in LHS of eqn (b)(i); ft on c's value of m.
	$\dfrac{4}{3}x^2-8x+12=0;\qquad 4x^2-24x+36=0$ $x^2-6x+9=0$ $x=\dfrac{-(-8)\pm\sqrt{0}}{\frac{8}{3}};\qquad (x-3)^2\ \ (=0)$	m1		Valid method to solve a **correct** quadratic <u>equation</u>; as far as either correct subst into quadratic formula with b^2-4ac evaluated to 0 or correct factorisation or correct value of x after $\dfrac{4}{3}x^2-8x+12=0$ or better seen.; OE, correct use of $-b/2a$
	$x=3$	A1		Must see earlier justification
	Coordinates of P are $\left(3,\ \dfrac{3}{\sqrt{12}}\right)$	A1	4	Correct coordinates with the y-coord in any correct exact form eg $\dfrac{\sqrt{3}}{2}$. NMS SC 1 for $\left(3,\ \dfrac{3}{\sqrt{12}}\right)$
	Total		**13**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
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Mathematics MFP1

(Specification 6360)

Further Pure 1

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$(x_2 =)\ 10 - \dfrac{\left(10^3 - 10^2 + 4\times 10 - 900\right)}{\left(3\times 10^2 - 2\times 10 + 4\right)}$	B1		$10 - \dfrac{\mathrm{f}(10)}{\mathrm{f}'(10)}$ with a correct numerical expression or value PI for f(10).
	$\left(\begin{array}{l} = 10 - \dfrac{1000 - 100 + 40 - 900}{300 - 20 + 4} \\ = 10 - \dfrac{40}{284} = 10 - 0.1408... \end{array} \right)$	B1		$10 - \dfrac{\mathrm{f}(10)}{\mathrm{f}'(10)}$ with a correct numerical expression or value PI for f′(10).
	$(= 9.85915....)\quad = 9.859$ (to 4 sf)	B1	3	Must be 9.859
	Total		**3**	
2(a)(i)	$\mathbf{A} - \mathbf{B} = \begin{bmatrix} p-3 & 1 \\ 2 & p-3 \end{bmatrix}$	B1	1	
(ii)	$\mathbf{AB} = \begin{bmatrix} p & 2 \\ 4 & p \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3p+4 & p+6 \\ 12+2p & 4+3p \end{bmatrix}$	M1		Finding **AB** and at least 2 elements correct
		A1	2	CSO
(b)	$\mathbf{A}-\mathbf{B}+\mathbf{AB} = \begin{bmatrix} 4p+1 & p+7 \\ 14+2p & 1+4p \end{bmatrix}$	B1F		Only ft if all matrices are 2 by 2 PI by later correct work
	$\mathbf{A}-\mathbf{B}+\mathbf{AB} = k\,\mathbf{I} = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1		**I** used as or equated to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ at some stage
	$(p + 7 = 0\ ,\ 14 + 2p = 0 \quad \Rightarrow)\ p = -7$	B1		$p = -7$ provided it gives the relevant two zero elements
	$p = -7 \Rightarrow \mathbf{A}-\mathbf{B}+\mathbf{AB} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} = -27\,\mathbf{I}$ $\Rightarrow\ k = -27$	B1	4	CSO Either -27**I** (no earlier errors) for B1 OR $k = -27$ with either $\begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$ or $27\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ or $-27\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ seen before (no earlier errors) for B1
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)	$\cos\left(5x+40^\circ\right)=\cos 65^\circ$ $5x + 40^\circ = \pm 65^\circ$	B1		Both $\pm 65^\circ$ OE eg $5x + 40 = 65, 295$
	$5x+40^\circ = 360n^\circ + 65^\circ$, $5x+40^\circ = 360n^\circ - 65^\circ$	M1		$5x+40 = 360n \pm \alpha$ Either one, OE Condone $2n\pi$ for $360n$
	$x=\dfrac{360n^\circ + 65^\circ - 40^\circ}{5}$, $x=\dfrac{360n^\circ - 65^\circ - 40^\circ}{5}$	m1		Either one, OE Correct rearrangement of $5x+40 = 360n \pm \alpha$ OE to $x =$. Condone $2n\pi$ for $360n$
	$x = 72n^\circ + 5^\circ$, $x = 72n^\circ - 21^\circ$	A2,1,0	5	OE Full set of correct solns. in degrees written in a simplified form. (A1 if not in a simplified form) (A0 if radians present in answer)
(b)	$\dfrac{\sqrt{3}-1}{2\sqrt{2}} = (\cos\dfrac{\pi}{4})\left[\cos(a\pi)+\cos(b\pi)\right]$			Recognising $\cos\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$ (or $=\dfrac{1}{\sqrt{2}}$)
	$\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\right) = \dfrac{1}{\sqrt{2}}\left[\cos(a\pi)+\cos(b\pi)\right]$	B1		PI eg by seeing $\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\right)$
	$\cos a\pi = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6}$, $(a=\dfrac{1}{6})$	B1		OE ie any correct rational value for a which satisfies $\cos a\pi = \dfrac{\sqrt{3}}{2}$
	$\cos b\pi = -\dfrac{1}{2} = \cos\dfrac{2\pi}{3}$, $(b=\dfrac{2}{3})$	B1	3	OE ie any correct rational value for b which satisfies $\cos b\pi = -\dfrac{1}{2}$
	$\sin\dfrac{\pi}{12} = \cos\left(\dfrac{\pi}{4}\right)\left[\cos\left(\dfrac{1}{6}\pi\right)+\cos\left(\dfrac{2}{3}\pi\right)\right]$			Note: labels a and b could be interchanged.
	Total		**8**	

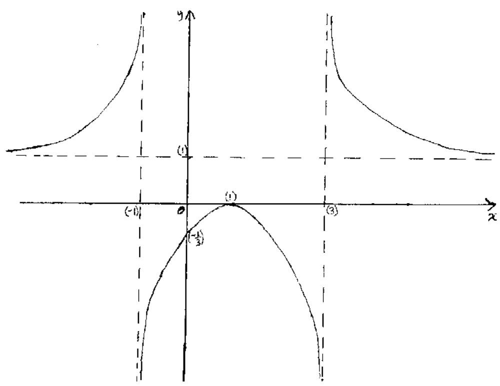
Q	Solution	Marks	Total	Comments
4(a)(i)	$(z-2\mathrm{i})^* = (x+y\,\mathrm{i}-2\mathrm{i})^* = x+(2-y)\mathrm{i}$	B1	1	$x+2\mathrm{i}-y\mathrm{i}$ OE rearrangement
(ii)	$(z-2\mathrm{i})^* = 4\mathrm{i}\,z+3 = 4\mathrm{i}x+4\mathrm{i}^2y+3 = 4\mathrm{i}x-4y+3$	B1		$\mathrm{i}^2=-1$ used
	$x+(2-y)\,\mathrm{i} = 4\mathrm{i}x-4y+3$ (#) Real parts: $x=-4y+3$ Imaginary parts: $2-y=4x$	M1		Attempting to equate, without mixing real and imaginary terms, **both** the real parts and the imag. parts for the c's eqn (#).
		A1F		If not corrected, ft on [c's(a)(i)] $= 4\mathrm{i}x-4y+3$ provided both the resulting linear equations have non zero x, y and const terms
	$y=\frac{2}{3}$, $x=\frac{1}{3}$	A1		Solving correct equations, to obtain either $x=\frac{1}{3}$ OE or $y=\frac{2}{3}$ OE
	$(z=)\ \frac{1}{3}+\frac{2}{3}\mathrm{i}$	A1	5	$\frac{1}{3}+\frac{2}{3}\mathrm{i}$
(b)	(One of the) coefficients (of the quadratic equation is) not real.	E1	1	OE eg Sum of roots is $-10\,\mathrm{i}$ so p cannot be real if roots are $p\pm q\mathrm{i}$
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$y = 2x^2 - 5x$			
	$y_Q = 2(1+h)^2 - 5(1+h) = 2 + 4h + 2h^2 - 5 - 5h$ $(= 2h^2 - h - 3)$	B1		$y_Q = 2(1+h)^2 - 5(1+h)$ with correct expansion of brackets PI.
	Grad. $= \dfrac{y_Q - y_P}{x_Q - x_P} = \dfrac{2(1+h)^2 - 5(1+h) - (-3)}{1+h-1}$	M1		Use of correct formula for gradient
	$= \dfrac{2h^2 - h - 3 - (-3)}{h} = \dfrac{2h^2 - h}{h} = 2h - 1$	A1	3	CSO
(ii)	As $h \to 0$, (grad of $PQ \to$ grad of tangent at P) (ie) gradient (of tangent at P) $= -1$	E1		$h = 0$ scores E0
	Now **gradient** of $x+y=0$ (or $y = -x$) is also **−1** $\Rightarrow$ tangent at P is **parallel** to line $x + y = 0$	E1	2	Dep on $h \to 0$ or $h = 0$ being used earlier
(b)	$\mathrm{I} = \int_1^{\infty} x^{-4}\left(2x^2 - 5x\right)\,\mathrm{d}x = \int_1^{\infty}\left(2x^{-2} - 5x^{-3}\right)\,\mathrm{d}x$			
	$\mathrm{I} = \left[-2x^{-1} - 5\dfrac{x^{-2}}{-2}\right]_1^{\infty}$	M1		At least one term correct
	As $x \to \infty$, $x^{-1} \to 0$ and $x^{-2} \to 0$	E1		OE Ft on $k\,x^{-n}$ provided M1 awarded
	$\mathrm{I} = 0 - (-2 + 5/2) = -\dfrac{1}{2}$	A1	3	$(\mathrm{I} =)\ -\dfrac{1}{2}$ Dep on both terms integrated correctly in the M1 line
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)	$\alpha+\beta=-\frac{3}{2}$	B1		OE
	$\alpha\beta=-3$	B1	2	OE
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		Using correct identity for $\alpha^3+\beta^3$ in terms of $\alpha+\beta$ and $\alpha\beta$.
	$=\left(-\frac{3}{2}\right)^3-3(-3)(-3/2)$	A1F		with ft/or correct substitution
	$=-\frac{27}{8}-\frac{27}{2}=-\frac{135}{8}$	A1	3	CSO AG. Correct evaluation of each of $(-1.5)^3$ and $-3(-3)(-1.5)$ must be seen before the printed answer is stated
(c)	Sum $=\alpha+\frac{\alpha}{\beta^2}+\beta+\frac{\beta}{\alpha^2}$ $=\alpha+\beta+\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}=-\frac{3}{2}+\frac{-135/8}{9}$	M1		Writing $\alpha+\frac{\alpha}{\beta^2}+\beta+\frac{\beta}{\alpha^2}$ in a suitable form with ft/or correct substitution
	Sum $=-\frac{27}{8}$	A1		PI OE exact value eg -3.375 (A0 if $\alpha\beta=3$ used to get $(\alpha\beta)^2=9$)
	Product $=\alpha\beta+\frac{\beta}{\alpha}+\frac{\alpha}{\beta}+\frac{1}{\alpha\beta}$ $=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$ **(*)** Now $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ **(=** $\frac{9}{4}+6$ **)**	M1		**(*)** OE with correct identity for $\alpha^2+\beta^2$ used in (c). Subst of values not required but PI by correct value of Product
	Product $=-3-\frac{1}{3}\left(\frac{9}{4}+6\right)-\frac{1}{3}=-\frac{73}{12}$	A1		PI OE exact value
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's *S* and *P* values.
	Eqn is $24x^2+81x-146=0$	A1	6	OE but integer coefficients and '= 0' needed
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$f(x) = 4x^3 - x - 540\,000$ $f(51) = -9447$ (<0); $f(52) = 22380$ (>0); Since sign change (and f continuous), $51<\alpha<52$	M1 A1	2	f(51) and f(52) both considered All values and working correct plus relevant concluding statement involving '51' and '52'.
(b)(i)	$S_n = \sum_{r=1}^{n}(2r-1)^2 = \sum 4r^2 - \sum 4r + \sum 1$	M1		Splitting up the sum into separate sums. PI by m1 line below or better
	$= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + \sum_{r=1}^{n} 1$	m1		Substitution of correct formulae from FB for the two summations
	$= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + n$	B1 A1		B1 for $\sum_{r=1}^{n} 1 = n$ stated or used
	$= \frac{n}{3}\left[2(2n^2+3n+1) - 6(n+1) + 3\right] = \frac{n}{3}\left[4n^2 - 1\right]$	A1	5	CSO
(ii)	$(6S_n = 2n[4n^2 - 1]) = 2n(2n-1)(2n+1)$ $(2n-1)$, $2n$ and $(2n+1)$ are consecutive integers	B1 E1	2	Terms in any order Terms must be identified and statement 'consecutive integers'
(c)	$S_n = 1^2 + 3^2 + 5^2 + \ldots. + (2n-1)^2$ ie sum of squares of first n odd numbers so need least N such that $S_N > 180\,000$			
	$S_{52} = \frac{52}{3}\left[4 \times 52^2 - 1\right] = 187460$ and $S_{51} = 176851$	M1		Either $\frac{n}{3}\left[4n^2 - 1\right] = 180000$ or $2N(2N-1)(2N+1) = 1080000$ or S_{52} and S_{51} both attempted (or = replaced by > or by ≥)
	Smallest value of N is 52	A1	2	CSO Fully and correctly justified. NMS N=52 scores 0/2
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$	M1		Matrix in form $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$, where $\lambda \neq 0$, $\mu \neq 0$ and $\lambda \neq \mu$
		A1	2	$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$
(b)(i)	$y = \sqrt{3}\,x = \tan 60^{\circ}\,x$ $\quad \begin{bmatrix} \cos 120^{\circ} & \sin 120^{\circ} \\ \sin 120^{\circ} & -\cos 120^{\circ} \end{bmatrix}$	M1		$\begin{bmatrix} \cos 120 & \sin 120 \\ \sin 120 & -\cos 120 \end{bmatrix}$ PI For M mark, condone dec approx 0.86 or 0.87 or better in place of $\sin 120^{\circ}$
	Required matrix is $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	A1	2	OE but must be in exact surd form.
(ii)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \ldots\ldots$	M1		Attempt to multiply c's (b)(i) 2by2 matrix and c's (a) 2by2 matrix in **correct** order.
	$= \begin{bmatrix} -\frac{1}{2} & \frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$	A1	2	OE but must be in exact surd form.
	Total		**6**	

Q	Solution	Marks	Total	Comments
9(a)	(HA) $y = 1$ (VA) $x^2 - 2x - 3 = 0 \quad (x-3)(x+1) = 0$ $x = -1$ and $x = 3$	B1 M1 A1	3	$y = 1$ OE eqn PI OE eg use of quadratic formula Both needed OE eqn(s)
(b)(i)	$k = \dfrac{x^2 - 2x + 1}{x^2 - 2x - 3} \Rightarrow kx^2 - 2kx - 3k = x^2 - 2x + 1$ $kx^2 - 2kx - 3k - x^2 + 2x - 1 = 0$ $(k-1)x^2 - 2(k-1)x - (3k+1) = 0$	B1	1	AG Must see the two stages, correct elimination of fraction and a correct rearrangement to … = 0, along with correct elimination of brackets before printed answer is stated.
(ii)	Discriminant $b^2 - 4ac$ $\{ 4(k-1)^2 + 4(k-1)(1+3k) \}$	M1		$b^2 - 4ac$, OE, in terms of k; condoning one minor error in substitution.
	Line intersects curve $\Rightarrow b^2 - 4ac \geq 0$ $\Rightarrow 4(k-1)^2 + 4(k-1)(1+3k) \geq 0$	A1		A correct inequality where k is the only unknown
	$\Rightarrow 4(k-1)[k-1+1+3k] \geq 0, \quad 16k(k-1) \geq 0$ ie $k^2 - k \geq 0$	A1	3	CSO AG Must be convinced
(iii)	$k^2 - k \geq 0, \quad k(k-1) \geq 0,$ $k \leq 0, \quad k \geq 1$ Critical values $k = 0$, $(k = 1)$	B1		For $k = 0$ either as an equation or inequality.
	$k \neq 1$ since there is no point on the curve where y=1 ($x^2 - 2x - 3 \neq x^2 - 2x + 1$)	E1		OE Valid explanation, with no accuracy errors, to discount k=1
	k=0, $-x^2 + 2x - 1 = 0$ or $y = 0$, $x^2 - 2x + 1 = 0$	M1		OE
	(Only one) **stationary** point (and its coordinates are) **(1, 0)**	A1	4	'stationary' with either (1, 0) or $\{x=1, y=0\}$
(c)		B1		Curve with three distinct branches
		B1		Branch between VAs, correct shape, no part of the branch above the x-axis, only intersection with y-axis at a point below the origin, and its max pt on the positive x-axis
		B1	3	Fully correct curve drawn with each branch correctly approaching its relevant asymptotes
	Total		**14**	
	TOTAL		**75**	

A-LEVEL

MATHEMATICS

Further Pure 1 – MFP1

Mark scheme

6360

June 2014

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

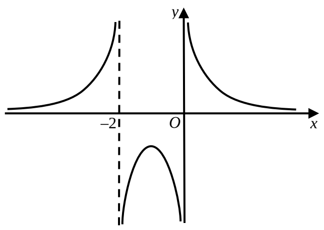
Q	Solution	Mark	Total	Comment
1	$h\,y'(9) = 0.25 \times \frac{1}{2+\sqrt{9}}$ (=0.05)	M1		Attempt to find $h\,y'(9)$.
	$\{y(9.25)\} \approx 6 + 0.05 = 6.05$	A1		6.05 OE
	$\{y(9.5)\} \approx y(9.25) + 0.25 \times y'(9.25)$ $\approx 6.05 + 0.25 \times \frac{1}{2+\sqrt{9.25}}$	m1		Attempt to find $y(9.25) + 0.25 \times y'(9.25)$, must see evidence of numerical expression if correct ft [0.049(5..)+c's $y(9.25)$] value is not obtained.
	$\approx 6.05 + 0.25 \times 0.1983(5\ldots)$			
	$\approx 6.05 + 0.0495(8\ldots..)$	A1F		PI; ft on c's value for $y(9.25)$; 4dp value (rounded or truncated) or better.
	$y(9.5) = 6.0996$ (to 4 d.p.)	A1	**5**	$y(9.5) = 6.0996$
	Total		**5**	
	In this Q1, misreads lose all those A marks that are affected.			

Q	Solution	Mark	Total	Comment
2(a)	$\alpha + \beta = -4;\quad \alpha\beta = \frac{1}{2}$	B1; B1	**2**	Answers −4 & ½ with LHS missing, look for later evidence before awarding B1B1
(b)(i)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	M1		PI
	$= 16 - 1 = 15$	A1	**2**	CSO
(b)(ii)	$\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$	M1		OE identity enabling direct substitution.
	$= 225 - 2\times\frac{1}{4} = 225 - \frac{1}{2} = \frac{449}{2}$	A1	**2**	CSO AG Must see evaluations (eg as indicated by either of these two alternatives) before the printed answer.
(c)	$S = 2(\alpha^4+\beta^4) + \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$	M1		OE identity enabling direct substitution, seen or used.
	$P = 4\alpha^4\beta^4 + 2(\alpha^2+\beta^2) + \frac{1}{\alpha^2\beta^2}$	M1		OE identity enabling direct substitution, seen or used.
	$S = 509,\quad P = \frac{137}{4}$ (= 34.25)	A1F		Both values correct; ft only on $\alpha+\beta=4$
	Quadratic is $x^2 - 509x + 34.25 (= 0)$	M1		$x^2 - Sx + P$ ft c's vals for S and P. M0 if either $S = \alpha+\beta$ or $P = \alpha\beta$ values
	$4x^2 - 2036x + 137 = 0$	A1F	**5**	ACF of the equation, but must have integer coefficients; ft only on $\alpha+\beta=4$
	Total		**11**	
Alt (b)(ii)	$\alpha^4+\beta^4 = (\alpha+\beta)^4 - 4\alpha\beta(\alpha^2+\beta^2) - 6\alpha^2\beta^2$ (M1) $= 256 - 4\times\frac{15}{2} - 6\times\frac{1}{4} = 256 - 30 - \frac{3}{2} = \frac{449}{2}$ (A1) AG Cand whose only error is $\alpha+\beta=4$ in (**a**) can score B0B1; M1A0; M1A0; 5			

Q	Solution	Mark	Total	Comment
3	$\sum_{r=3}^{60} r^2(r-6) = \sum_{r=3}^{60} r^3 - 6\sum_{r=3}^{60} r^2$	M1		$\sum r^2(r-6) = \sum r^3 - 6\sum r^2$ seen or used
	$= \sum_{r=1}^{60} r^3 - 6\sum_{r=1}^{60} r^2 - \left[\sum_{r=1}^{2} r^3 - 6\sum_{r=1}^{2} r^2\right]$			
	$= \sum_{r=1}^{60} r^3 - 6\sum_{r=1}^{60} r^2 - [\,9-30\,]$	B1		B1 for $\left[\sum_{r=1}^{2} r^3 - 6\sum_{r=1}^{2} r^2\right] = 9-30$ OE PI
	$= \frac{1}{4}(60)^2(61)^2 - 6\frac{1}{6}(60)(61)(2\times 60+1) + 21$	M1		Substitution of n=60 into either (i) the correct formula $\sum_{r=1}^{n} r^3$ or (ii) the correct formula for $\sum_{r=1}^{n} r^2$ or (iii) the c's rearrangement of $\frac{1}{4}n^2(n+1)^2 - 6\frac{n}{6}(n+1)(2n+1)$
	$= 3348900 - 442860 + 21 = 2906061$	A1	4	2906061 NMS Answer only of 2906061 scores 0/4
	Total		4	
	Cand who works with Q as $\sum_{r=1}^{60} r^2(r-6)$ can score max of M1B0M1A0 Condone notation $\sum_{1}^{60} r^3$ for $\sum_{r=1}^{60} r^3$ etc **SC** : Let s=r–2; $\sum_{r=3}^{60} r^2(r-6) = \sum_{s=1}^{58}(s+2)^2(s-4) = \sum_{s=1}^{58} s^3 - 12\sum_{s=1}^{58} s - 16\sum_{s=1}^{58} 1$ (**M1** relevant split following expn of $(s+2)^2(s-4)$ into the form $as^3 + (bs^2+)\,cs + d$, ft wrong coeffs provided at least 3 non-zero coefficients.) $= \frac{1}{4}(58)^2(59)^2 - 12\frac{1}{2}(58)(59) - 16(58)$ (**M1** Substitution of n=58 into correct formula for either $\sum_{s=1}^{n} s^3$ or $\sum_{s=1}^{n} s$) (**B1** for $16\sum_{s=1}^{58} 1 = 16(58)$ (=928)) $= 2927521 - 20532 - 928 = 2906061$ (**A1**)			

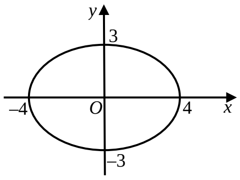
Q	Solution	Mark	Total	Comment
4	5i(a+bi) + 3(a–bi) + 16 = 8i	M1		Use of $z^* = a - b\mathrm{i}$ for $z = a+b\mathrm{i}$ OE
	5ai –5b + 3(a–bi) + 16 = 8i	M1		Use of $\mathrm{i}^2 = -1$
	5ai–5b+3a–3bi+16=8i	A1		5ai–5b+3a–3bi+16=8i OE PI
	3a –5b + 16 = 0, 5a –3b = 8	M1		Equating both the real parts and the imag. parts for the c's eqn.
	16b = 104 (or 16a = 88 etc)	A1		Correct elimination of either a or b from two correct equations involving a and b. OE PI
	$(z=)\frac{11}{2} + \frac{13}{2}\mathrm{i}$	A1	6	ACF isolated, not embedded.
	Total		6	

Q		Solution	Mark	Total	Comment
5	**(a)**	$\{y(-5+h)=\}\ \ (-5+h)(-5+h+3)$	M1		Attempt to find y when $x=-5+h$ PI
		Gradient $=\dfrac{(-5+h)(-2+h)-10}{-5+h-(-5)}$	M1		Use of gradient $=\dfrac{y_2-y_1}{x_2-x_1}$ OE to obtain an expression in terms of h.
		$=\dfrac{-7h+h^2}{h}\ =-7+h$	A1	**3**	CSO $-7+h$ or $h-7$
	(b)	As $h\to 0$, {grad of line in **(a)** $\to$ grad of curve at point $(-5, 10)$}	E1		Lim [c's($a+bh$)] OE $h\to 0$ NB 'h=0' instead of '$h\to 0$' gets E0
		{Gradient of curve at point $(-5, 10)$ =} -7	A1F	**2**	ft on c's a value only if both Ms have been scored in part **(a)** and $a+bh$ has been obtained convincingly. Final answer must be -7 not '$\to -7$ OE'
		Total		**5**	
	(b) **(b)**	Note: E0, A1F is possible. OE wording for '$\to$' eg 'tends to', 'approaches', 'goes towards'. Do NOT accept '='.			

Q		Solution	Mark	Total	Comment
6	**(a)**	$x=0,\quad x=-2,\quad y=0$	B2,1,0	**2**	OE (eg x+2=0) B1 for two correct.
	(b)(i)	$(y=)-1$	B1	**1**	
	(b)(ii)		M1		Three branches shown on sketch of C with either middle branch or outer two branches correct in shape.
			A1	**2**	All three branches, correct shape and positions and approaching correct asymptotes in a correct manner.
	(c)	Critical values: $(x+4)(x-2)=0$	M1		PI Valid method to find critical values. Condone corresponding inequality. Alternatives must reach an equivalent stage where critical values can be stated.
		Critical values are $x=-4,\ x=2$	A1		Both correct with no extras remaining. Seen or used.
		$x\le -4,\quad x\ge 2$	B1		Both inequalities
		$-2<x<0$	B2,1,0	**5**	B1 if either or both '<' replaced by '≤'
		Total		**10**	
	(a)	Must be equations. If more than 3 equations deduct 1 mark for each extra to a minimum of B0			

Q	Solution	Mark	Total	Comment
7(a)(i)	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	B1	**1**	
(a)(ii)	$\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}$	B1	**1**	
(b)	$\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -7 & 0 \end{bmatrix}$	M1 A1	 **2**	Multiplication of c's matrices from **(a)(i) and (a)(ii)** in correct order. CAO
(c)(i)	$\mathbf{A}^2 = \begin{bmatrix} 9+3 & 3\sqrt{3}-3\sqrt{3} \\ 3\sqrt{3}-3\sqrt{3} & 3+9 \end{bmatrix} = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$ $= 12\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 12\mathbf{I}$	B1	**1**	Accept either of these two final forms.
(c)(ii)	$\mathbf{A} = \sqrt{12}\begin{bmatrix} -\frac{3}{\sqrt{12}} & -\frac{\sqrt{3}}{\sqrt{12}} \\ -\frac{\sqrt{3}}{\sqrt{12}} & \frac{3}{\sqrt{12}} \end{bmatrix}$	M1		OE eg $-2\sqrt{3}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$
	$= \begin{bmatrix} \sqrt{12} & 0 \\ 0 & \sqrt{12} \end{bmatrix}\begin{bmatrix} \cos 210^\circ & \sin 210^\circ \\ \sin 210^\circ & -\cos 210^\circ \end{bmatrix}$	A1		Either order. OE
	Scale factor of enlargement $= \sqrt{12}$ $(= 2\sqrt{3})$ (line of reflection) $y = \tan 105^\circ\ x$	B1 B1		OE. If not $\sqrt{12}$ OE, ft on $\sqrt{k}$ from (c)(i). OE in form $y = (\tan\theta)x$ ACF
	Combination of enlargement sf $\sqrt{12}$ and reflection in line $y = \tan 105^\circ\ x$	A1	**5**	OE CSO Need correct combination of sf and eqn and also convincingly shown that the matrix corresponds to a combination of an enlargement and reflection
	<u>**Altn for M1A1 in (c)(ii)**</u> $\begin{bmatrix} -3 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} =$	(M1)		Attempting to find the image of vertices of a square under **A** with at least two non-origin images obtained and correct.
	$= \begin{bmatrix} 0 & -3 & -\sqrt{3} & -3-\sqrt{3} \\ 0 & -\sqrt{3} & 3 & -\sqrt{3}+3 \end{bmatrix}$	(A1)		Correct image of square under **A** (seen or used) with evidence of either correct length of side of the square or correct angle between a side and an axis.
	Total		**10**	
(c)(ii)	Other correct alternatives' include eg Enlargement sf $-\sqrt{12}$, reflection in $y = \tan 15^\circ\ x$			
(c)(ii)	Other acceptable answers for final B mark above include $y = (\tan\frac{7\pi}{12})\,x$; Condone eg $y = -\tan 75^\circ\ x$, $y = -(\tan\frac{5\pi}{12})\,x$; Apply ISW after a correct form is given			

Q	Solution	Mark	Total	Comment
8(a)	$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	B1		$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$ OE stated or used. B0 if any incorrect angle also used. Condone degrees or decs (3sf or better)
	$\frac{5}{4}x-\frac{\pi}{3}=2n\pi+"\frac{\pi}{4}"$; $\frac{5}{4}x-\frac{\pi}{3}=2n\pi-"\frac{\pi}{4}"$	M1		OE; Either one, showing a correct use of $2n\pi$ in forming a general soln. ft c's $\cos^{-1}(\sqrt{2}/2)$. Condone $360n$ in place of $2n\pi$
	$x=\frac{4}{5}\left(2n\pi+\frac{\pi}{4}+\frac{\pi}{3}\right)$, $x=\frac{4}{5}\left(2n\pi-\frac{\pi}{4}+\frac{\pi}{3}\right)$	m1		Correct rearrangement of $\frac{5}{4}x-\frac{\pi}{3}=2n\pi+\alpha$ OE to $x=$ .... , where an α is from c's $\cos\alpha=\sqrt{2}/2$. Condone $360n$ in place of $2n\pi$
	$x=\frac{24n\pi+7\pi}{15}$, $x=\frac{24n\pi+\pi}{15}$	A2,1,0	5	OE full set of correct solutions in radians in terms of π written in a simplified form. (A1 if correct but left unsimplified). Accept the simplification retrospectively if it appears in (b)
(b)	For both $\frac{24n\pi+7\pi}{15}$ and $\frac{24n\pi+\pi}{15}$, solns. in $0\le x\le 20\pi$ come from n=0 to n=12 inclusive.	B1F		Values for n, stated or used, ft on c's general solution
	Sum = $\sum_{n=0}^{12}\left[\frac{24n\pi+7\pi}{15}\right]+\sum_{n=0}^{12}\left[\frac{24n\pi+\pi}{15}\right]$ = $\frac{24\pi}{15}\frac{12}{2}(13)+\frac{7\pi}{15}(13)+\frac{24\pi}{15}\frac{12}{2}(13)+\frac{13\pi}{15}$ {= $\frac{\pi}{15}(1872+91+1872+13)$}	M1,A1		Method for summing; must be using <u>correct</u> general solution. PI by correct value of k .
	$=\frac{3848}{15}\pi$ (ie $k=\frac{3848}{15}$)	A1	4	OE exact value eg $256\frac{8}{15}\pi$
	Total		**9**	
(a)	Form of the answer in m1 line of soln above would score A1. If it had been simplified to $x=\frac{4}{5}\left(2n\pi+\frac{7\pi}{12}\right)$, $x=\frac{4}{5}\left(2n\pi+\frac{\pi}{12}\right)$ it would have scored A2			
(a) **(a)(b)**	Simplification requires terms of the form $a\pi+b\pi$, where a and b are numerical fractions to be combined. Full correct answer might eg be written as $x=\frac{24n\pi+7\pi}{15}$, $x=\frac{24n\pi+25\pi}{15}$ in which case for $\frac{24n\pi+25\pi}{15}$ solns in $0\le x\le 20\pi$ would come from $n=-1$ to n=11 inclusive.			
(b)	Identifying and listing all relevant solns.: (B1F as above); At least 24 of the 26 correct solns (M1 PI) $\frac{3848}{15}\pi$ (OE A2). If not A2 award A1 for **both** $\frac{1963}{15}\pi$ and $\frac{377}{3}\pi$ seen.			

Q	Solution	Mark	Total	Comment
9(a)		B1 B1	 2	Ellipse, 'centre' origin with correct values for at least two intercepts. Correct values shown for the four intercepts
(b)	$\frac{x^2}{16}+\frac{(x+k)^2}{9}=1;$ $9x^2+16(x+k)^2=16(9)$ $25x^2+32kx+16k^2-144=0$	M1 A1		Replacing y by $(x+k)$ or by $(x-k)$ OE A correct quadratic equation in the form $Ax^2+Bx+C=0$, PI by later work.
	$B^2-4AC=(32k)^2-4(25)(16k^2-144)$	M1		B^2-4AC in terms of k; ft on c's quadratic provided B and C are both in terms of k
	Roots real and different $\Rightarrow B^2-4AC>0$ $\Rightarrow (32k)^2-4(25)(16k^2-144)>0$	A1		A correct strict inequality where k is the only unknown
	$16k^2-25k^2+25(9)>0$; $9k^2<25(9)$ $k^2<25$; $-5<k<5$	A1	5	CSO AG
(c)	$\frac{(x-a)^2}{16}+\frac{(y-b)^2}{9}=1$ $9(x^2-2ax+a^2)+16(y^2-2by+b^2)=144$ $-18a=18$; $-32b=-64$; $144-9a^2-16b^2=c$ $a=-1$, $b=2$, $c=144-9-64=71$	M1 A1 m1 B2,1,0	 5	$x\to x\pm a$ and $y\to y\pm b$ Comparing non-zero coeffs to form three equations. PI B1 for two correct values.
	Altn: $9x^2+16y^2+18x-64y=c$ $9(x^2+2x)+16(y^2-4y)=c$ $9(x+1)^2+16(y-2)^2=c+9+64$ $\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=\frac{c+9+64}{144}$ $a=-1$, $b=2$, $c=144-9-64=71$	 (M1) (A1) (m1) (B2,1,0)	 (5)	 (Completing the square) $\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=\frac{c+\lambda}{144}$ (B1 for two correct values.)
(d)	Equations of tangents to E that are parallel to $y=x$ are $y=x+5$ and $y=x-5$ Tangents to translated ellipse that are parallel to $y=x$ are $y-b=x-a+5$ and $y-b=x-a-5$ $y=x+8$ and $y=x-2$	B1 M1 A1	 3	Need both equations. PI by M1 line Since '**Hence**', NMS scores 0/3
	Total		**15**	
	TOTAL		**75**	
	Condone correct coordinates in place of 'intercepts'.			