SUMMARY SHEET – DECISION MATHS

Algorithms

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1	D1	D1

The main ideas in this topic are

Understanding and implementing a variety of algorithms expressed as lists of instructions, flow charts or in pseudo code.

Before the exam you should know:

- The three bin packing algorithms. These are the Full-Bin Algorithm, the First-Fit Algorithm and the First-Fit Decreasing Algorithm.
- The sorting algorithms. Make sure you know which of these algorithms you need to learn by heart.
- How to count the umber of comparisons and swaps in each pass and know the maximum number of passes that are required for a list of a given length.
- The different ways algorithms are presented and make sure you practice following unfamiliar algorithms.
- What is meant by efficiency of an algorithm.

What is an algorithm?

An algorithm must have the following properties

- it is a set of precisely defined instructions.
- it has generality: it will work for all valid inputs.
- it is finite: it has a stopping condition.
- it may be an iterative process: you may need to follow the procedure a number of times in order to reach the best solution.

Presenting and Implementing Algorithms

An algorithm is a well-defined, finite sequence of instructions to solve a problem. They can be communicated in various ways, including written English, pseudo code and flowcharts. Make sure you are experienced in all possible formats.

Bin Packing

These are examples of HEURISTIC algorithms. This means that none of these algorithms necessarily lead you to the best or optimal solution of the problem.

1. Full-Bin Algorithm

Look for combinations of boxes to fill bins. Pack these boxes. For the remainder, place the next box to be packed in the first available slot that can take that box.

Note – the full bin algorithm does not always lead to the same solution of the problem. In other words, two people could apply the full bin algorithm perfectly correctly and end up with their boxes packed differently.

2. First-Fit Algorithm

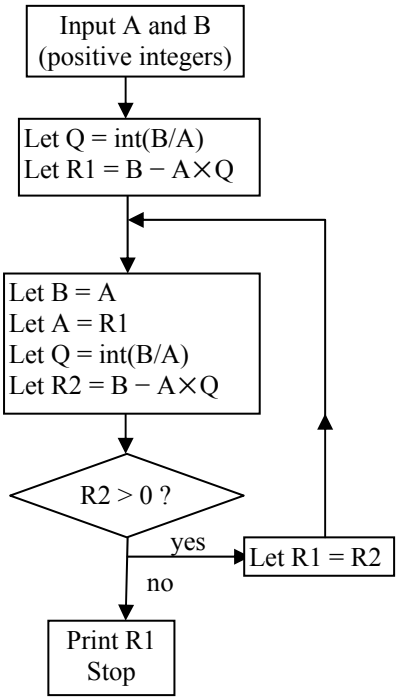
Taking the boxes in the order listed, place the next box to be packed in the first available slot that can take that box.

3. First-Fit Decreasing Algorithm

i) Re-order the boxes in order of decreasing size.

ii) Apply the First-Fit algorithm to this reordered list.

You should be able to form a judgement about the relative efficiency of these algorithms. The First-Fit Decreasing Algorithm requires a sort to be made before applying the First-Fit Algorithm so, in terms of computation, it requires more resources than the First-Fit Algorithm alone.



Example

a) What is the output of the algorithm when A = 84 and B = 660?

b) What does the algorithm achieve?

Solution

a)

A	84	72	12
B	660	84	72
Q	7	1	6
R1	72		12
R2		12	0

PRINT 12

b) It finds the highest common factor of A and B.

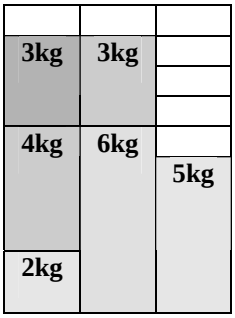
Example: Show how the following items are to be packed into boxes each of which has a capacity of 10Kg.

Item	A	B	C	D	E	F
Weight (kg)	2	4	6	3	3	5

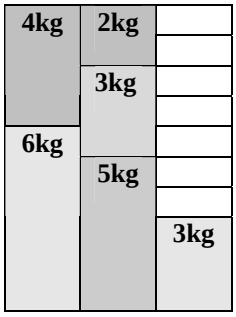
1. Full Bin

6+4=10, 5+3+2=10, 3 3 bins needed

2. First-Fit



3. First-Fit Decreasing



Notice that in this example the First-Fit Decreasing Algorithm gives the same result as the Full Bin Algorithm. This will not always be the case.

Sorting Algorithms

There are many sorting algorithms, so you must check carefully to see which, if any, you need to memorise for the examination.

Questions often ask about the relative efficiency of sorting algorithms by comparing the number of comparisons (c) and swaps that are made to sort the same list of numbers, as seen in this example:

List	1st pass	2nd pass	3rd pass
6	1	1	1
1	3	3	3
3	6	5	5
7	5	**6**	6
5	**7**	**7**	7
c	4	3	2
s	3	1	0

total number of comparisons: **9**

total number of swaps: **4**

Bubble Sort

First pass: the first number in the list is compared with the second and whichever is smaller assumes the first position. The second number is then compared with the third and the smaller is placed in the second position, and so on. At the end of the first pass, the largest number will be at the bottom. For the list of five numbers on the right, this involves 4 comparisons and 3 swaps.

Second pass: repeat first pass but exclude the last number (on the third pass the last two numbers are excluded and so on).

The list is repeatedly processed in this way until **no swaps take place in a pass**.

For a list of 5 numbers, the list will definitely be sorted after the 4th pass (why?), so this is the maximum number of passes. The maximum number of comparisons is 4+3+2+1=10 and the maximum number of swaps is 10. You should be able to generalise this to a list of *n* numbers.

list	1st pass	2nd pass	3rd pass	4th pass
6	**1**	**1**	**1**	**1**
1	**3**	**3**	**3**	**3**
3	6	**6**	**5**	**5**
7	**7**	5	**6**	**6**
5	5	**7**	**7**	**7**
c	4	2	1	0
s	1	1	1	0

total number of comparisons: **7**
total number of swaps: **3**

Quick Sort

Select a pivot – usually the middle item in the list

First pass: numbers are sorted into two sub lists, those smaller than the pivot element and those greater than the pivot element. The pivot element is now fixed in its correct position I the list.

Second pass: choose a pivot element in each of the two sub lists and repeat the sorting procedure.

Continue this process until all numbers are fixed and the list is sorted.

In this case the quick sort takes fewer comparisons and swaps than the bubble sort, though it does take one more pass to achieve the sort. It is worth noting that the relative efficiency of the different types of algorithm will vary depending on how "mixed up" the list is.

SUMMARY SHEET – DECISION MATHS

Graph Theory

The main ideas are covered in			
AQA	Edexcel	MEI	OCR
D1	D1	D1	D1

The main ideas in this topic are

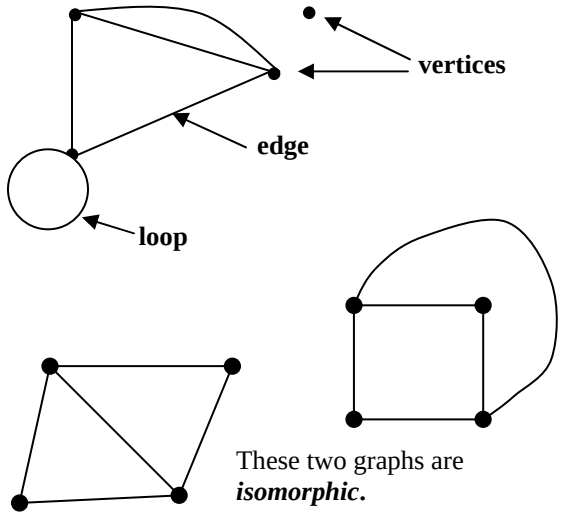
- The definition of a graph and the associated vocabulary.
- Mathematical modeling with graphs.

Before the exam you should know:

- The terms vertices (nodes), edges (arcs), digraphs, trees and paths.
- All the other vocabulary used to describe ideas in graph theory.
- How to draw a graph from an incidence matrix.
- How to model problems using graphs (e.g. Konigsberg Bridges).
- What is meant by a tree.
- How to recognise isomorphic graphs.
- What is meant by a Hamiltonian cycle.
- What is meant by an Euler cycle.

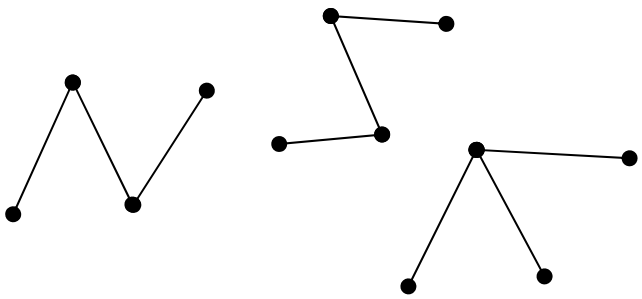
Terminology for Graph Theory

- **Graph** – collection of vertices & edges.
- **Vertex/Node –** the dots in a graph (usually where 2 or more edges meet, but not necessarily).
- **Edge/Arc** – a line between two vertices.
- **Tree** – a graph with no cycles.
- **Order (degree) of a vertex –** the number of edges starting or finishing at that vertex.
- **Simple graph** – a graph with no loops or multiple edges.
- **A path** – a route from one vertex to another which does not repeat any edge.
- **A cycle** – a route starting and finishing at the same vertex.
- **Connected graph** – a graph in which there is a route from each vertex to any other vertex (i.e. the graph is in one part).
- **Complete graph** – a simple graph in which every pair of vertices is connected by an edge.
- **Bipartite graph –** one in which the vertices are in two sets and each edge has a vertex from each set.
- **Planar graph –** one which can be drawn with no edges crossing.
- **Sub graph** – any set of edges & vertices taken from a graph is a sub-graph.
- **Hamiltonian cycle –** a cycle that visits every vertex of the graph.
- **Eulerian cycle** – a cycle that travels along every edge of the graph.
- **Eulerian graph** – a graph with no odd vertices.
- **Di-graph** – a graph in which the edges indicate direction.
- **Incidence matrix** – a matrix representing the edges in a graph.

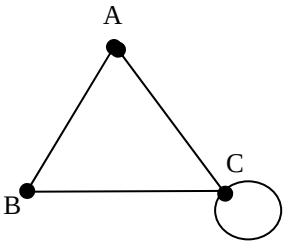


These two graphs are ***isomorphic*.**

These diagrams all show trees of the graph above

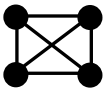


This shows a graph and its Incidence matrix.

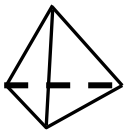


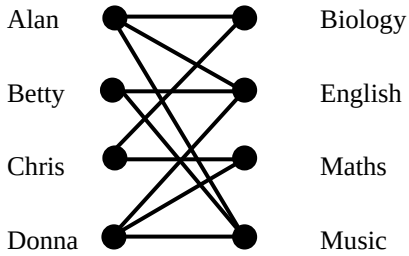
From \ To	*A*	*B*	*C*
A	–	1	1
B	1	–	1
C	1	1	2

Graphs can be used to represent many different things



This graph represents a tetrahedron





This ***bipartite graph*** shows which subjects four students study.

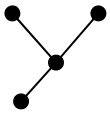
Example

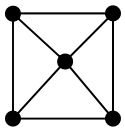
The table shows the number of vertices of degree 1, 2, 3 and 4 for three different graphs.
Draw an example of each of these graphs.

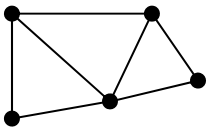
Order of vertex	**1**	**2**	**3**	**4**
Graph 1	3	0	1	0
Graph 2	0	0	4	1
Graph 3	0	2	2	1

solution

graph 1 graph 2 graph 3







Find the number of edges and the sum of the degrees of all the vertices of the graphs. What do you notice?

Graph 1: number of edges 3 sum of degrees of vertices 1+1+1+3 = 6
Graph 2: number of edges 8 sum of degrees of vertices 3+3+3+3+4=16
Graph 3: number of edges 7 sum of degrees of vertices 2+2+3+3+4 = 14

The sum of the degrees of the vertices is always twice the number of edges.

Also note that there are always an even number of odd vertices.

SUMMARY SHEET – DECISION MATHS

GRAPHICAL LINEAR PROGRAMMING

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1	D1	D1

The main ideas in this chapter are

Formulating a problem as a linear programming problem, solving a Linear Programming Problem (maximisation and minimisation) and Integer Programming.

Before the exam you should:

- Practice formulating linear programming problems. This can often be the trickiest part of the problem. Remember to be consistent with units.
- Learn the terminology – the OBJECTIVE FUNCTION is what you have to maximise or minimise subject to a number of CONSTRAINTS.
- Make sure you are able to draw straight line graphs quickly from the constraints by considering where they cross the x and y axes.
- You must be able to find the solution to problems from the graph. Make sure you can draw graphs accurately.
- Remember to shade OUT the unacceptable region to keep the feasible region clear and easy to identify.
- You must be able to find correct solutions to problems where the answer must be an integer.

Formulating a problem as a Linear Programming Problem

First: identify the variables about which a decision is to be made. These are sometimes called the decision variables. For example if your problem is to decide how many chairs to make and how many tables to make to maximise profit, begin with a statement like – let *x* be the number of chairs and let *y* be the number of tables. If your problem is to work out how many grams of wheatgerm and how grams of oat flour there should be in a new food product to meet nutritional requirements and minimise cost then let *x* be the number of grams of wheatgerm and let *y* be the number of grams of oat flour.

Next: Decide what the objective function is (this is the value you are trying to maximise or minimise) and what the constraints are as inequalities involving *x* and *y*.

Be careful to use the same units consistently. For example it's possible that some distances appearing in a problem are given in metres and some are given in centimetres. Or some times they could be given in seconds with some given in minutes. Choose one type of units and convert everything into those units.

Example:
A clothing retailer needs to order at least 200 jackets to satisfy demand over the next sales period. He stocks two types of jacket which cost him £10 and £30 to purchase. He sells them at 20 pounds and 50 pounds respectively. He has 2700 pounds to spend on jackets.

The cheaper jackets are bulky and each need 20cm of hanging space. The expensive jackets need only 10cm each. He has 40m of hanging space for jackets.

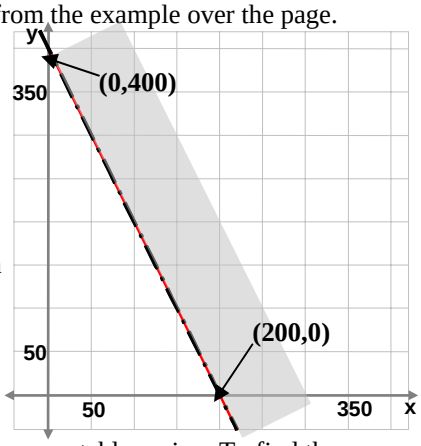
The retailer wishes to maximise profit. Assuming that all jackets will be sold, formulate a linear program, the solution of which will indicate how many jackets of each type should be ordered.

Formulation as a linear program
The decision is about how many of two types of jacket need to be ordered.
Let x = **number of cheaper jackets ordered**
Let y = **number of expensive jackets ordered**
The profit, *P*, given by selling all of these, is $P = 10x + 20y$, since the profit made on a cheaper jacket is 10 pounds and the profit made on an expensive one is 20 pounds.
The constraints are:
1. "needs to order at least 200" giving $x + y \geq 200$
2. "cost him 10 pounds and 30 pounds" and "has 2700 pounds to spend" giving $10x + 30y \leq 2700$
3. "20cm of hanging space" and "10cm" and "has 40m of hanging space" giving $0.2x + 0.1y \leq 40$

Solving a Linear Programming Problem

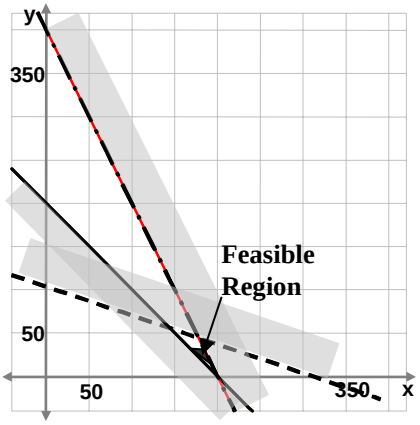
Draw a graph in which each constraint is represented by a line with shading. The unacceptable side of the line should be shaded. This leaves a "feasible region". The solution of the problem will be one of the vertices of the feasible region. These can be checked to find the best. We do this below for the example introduced over the page.

Drawing the line representing a constraint.
As an example, take the constraint $0.2x + 0.1y \leq 40$ from the example over the page.
The initial aim is to draw the line $0.2x + 0.1y = 40$. We know this is a straight line so it's enough to find two points on the line and join them. When $x = 0$, $y = 400$ and when $y = 0$, $x = 200$. So the points (0, 400) and (200, 0) are on the line.



Then shade out, the unacceptable region. To find the unacceptable region just test a point to see if it satisfies the constraint or not. For example, in this case (10, 10) clearly satisfies the constraint and so is in the acceptable region.

The feasible region.
Once you have drawn all the constraints, the feasible region is the intersection of the acceptable regions for all of them.

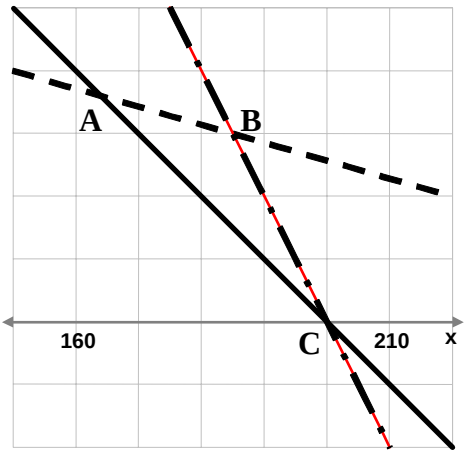


Constraint 1
$x + y \geq 200$

Constraint 2
$10x + 30y \leq 2700$

Constraint 3
$0.2x + 0.1y \leq 40$

Finding the solution
The solution of the problem will be at one of the vertices of the feasible region. You will need to solve simultaneous equations to find the co-ordinates of these vertices. Then each vertex must be checked to find the best. For example in the above we have a feasible region as in the diagram on the right. The coordinates of point A are found by solving $x + y = 200$ and $10x + 30y = 2700$ simultaneously. The solutions are $x = 165$ and $y = 35$. So the point is (165, 35) and the profit at that point is $P = 10x + 20y = 1650 + 700 = 2350$. Similarly it can be seen point B is (186, 28) giving a profit of 2420. Point C is (200, 0) giving a profit of 2000. So the best profit that can be made is by buying 165 cheap coats and 35 expensive coats.



Considering Gradients.
By calculating the gradients of each of the constraints and the gradient of the objective function, it's possible to predict in advance which vertex will give the optimal solution.

Minimisation problems are solved in exactly the same way. Just remember that this time you are looking for the vertex which makes the objective function the lowest.

Integer Programming

If the solution to the problem has to have integer values then points with integer value coordinates, close to the optimal point can be checked. This is likely to reveal the optimal solution but it is not guaranteed to. For example suppose the Objective Function is $2x + 3y$ and that this should be maximised. The optimal point may be (30.6, 40. 8) but do not assume that (30.40) will give the best solution; you must look at all the points with integer coordinates that are nearby: (31, 40), (30, 41), (30, 40) and (31, 41).

However (31, 41) and (31, 40) are not in the feasible region. You can check this by substituting in the values into the constraints. Of the two points nearby which are in the feasible region, namely (30, 41) and (30, 40), it can be seen that (30, 41) provides the best profit.

SUMMARY SHEET – DECISION MATHS

Matchings

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1		D2

The main ideas in this topic are:

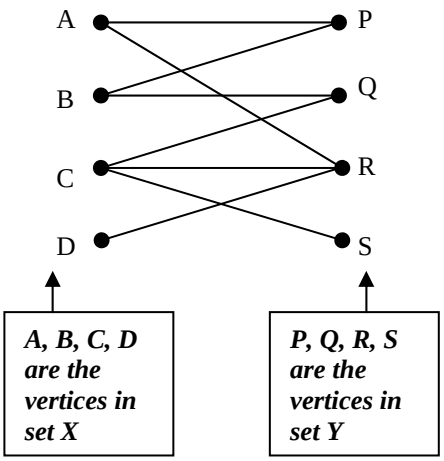
Modelling real situations using bipartite graphs.

Using the maximum matching algorithm to solve problems.

Before the exam you should know:

- What is meant by a bipartite graph.
- That a matching maps vertices in one set to vertices in a second set. No vertex may be used more than once.
- For a complete matching the two sets must have the same number of vertices.
- A complete matching pairs every vertex in the first set to one in the second set.
- A maximal matching is one where there is no solution that uses a greater number of edges.
- A complete matching is always maximal, but a maximal matching is not necessarily complete.

A **bipartite graph** has two sets of vertices, X and Y such that the edges only connect vertices in set X to those in set Y and never to vertices in the same set**.**



Example

A college has to fit French, Geography, History, Maths and Science into a single timetable slot. There are five teachers available all of whom can teach two or more of these subjects.

Ann can teach French and Geography.
Bob can teach French, Maths and Science.
Carol can teach Geography and History.
David can teach Geography, Maths and Science.
Elaine can teach History Maths and Science.

How should the college allocate the staff so that all subjects are covered?

The algorithm for finding a maximum matching

1. Always start with an initial matching.

2. If the matching is not maximal it can be improved by finding an ***alternating path.***

An alternating path:

- Starts on an unmatched vertex on the right hand side.
- Consists of edges alternately *not in* and *in* the matching.
- Finishes on an unmatched vertex in the second set.

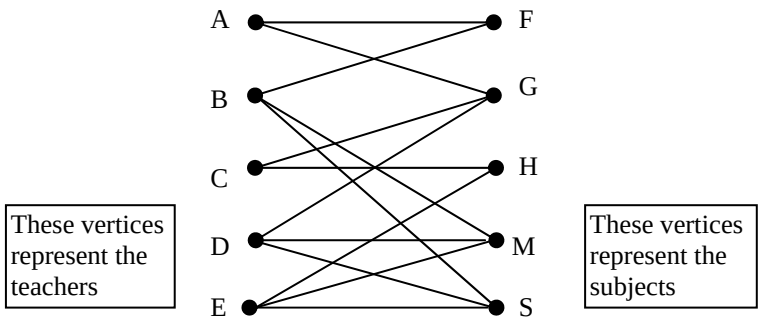
3. If every vertex is now matched so we have a complete matching. If it is not, then repeat step 2.

4. The solution consists of:

- Edges ***in*** the alternating path but ***not in*** the initial matching.
- Edges ***in*** the initial matching but ***not in*** the alternating path.

Solution

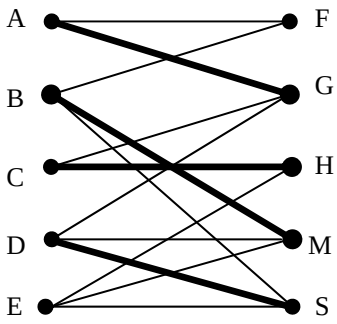
Start by drawing a bipartite graph to model the situation



Start with an initial matching:

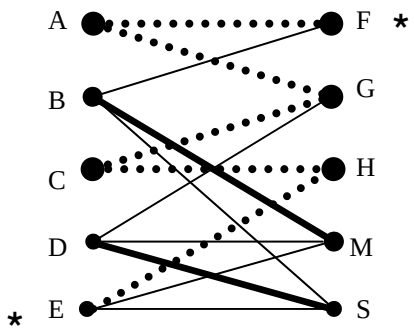
A – G, B – M, C – H, D – S

This is not a maximum matching since Elaine has not been allocated a subject and there is no-one to teach French.



We must try to find an ***alternating path***

Start on an unmatched vertex on the right hand side (F)
Choose an edge which is not in the initial matching (FA)
Choose an edge which is in the initial matching (AG)
Choose an edge which is not in the initial matching (GC)
Choose an edge which is in the initial matching (CH)
Choose an edge which is not in the initial matching (HE)



We have now reached E which was not in the initial matching so we have a ***breakthrough***.

The laternating path is F – A – G – C – H – E

The solution consists of:

- Edges ***in*** the alternating path but ***not in*** the initial matching: AF, CG, EH
- Edges ***in*** the initial matching but ***not in*** the alternating path: BM, DS

So the solution is:

Ann teaches French
Bob teaches Maths
Carol teaches Geography
David teaches Science
Elaine teaches History

SUMMARY SHEET – DECISION MATHS 1

NETWORKS – Minimum spanning tree and shortest path

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1	D1	D1

The main ideas in this topic are

- Appling Kruskal's and Prim's Algorithms to find the minimum spanning tree of a network.
- Applying Dijkstra's Algorithm to find the shortest (or least value path from one vertex to any other vertex in the network.

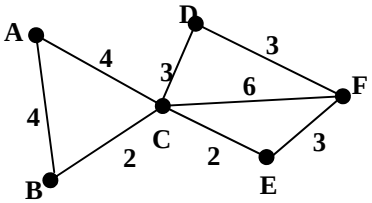
Before the exam you should know:

- How to show all the working clearly, there are more marks for the working than for getting the right answer.
- The distinction between Kruskal's and Prim's algorithms.
- How to apply Prim's algorithm to both a network and a table correctly.
- That Prim's and Kruskal's algorithms will usually give the same MST but often select the edges in a different order. Make sure you show sufficient working so that the examiner can see which algorithm you have used.
- How to work with networks or tables and be able to convert between the two.
- That you must always show all the working values as well as the permanent labels when using Dijkstra's algorithm.

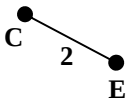
Minimum Spanning Tree

The minimum connector problem is to make a selection of the available edges so that any one vertex can be reached from any other, and the total length of the chosen edges is as small as possible. A connected set of edges with no loops is called a *tree* and the set which solves the minimum connector problem is the *minimum spanning tree* for the network.

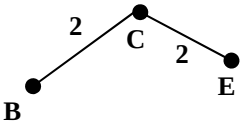
Kruskal's Algorithm



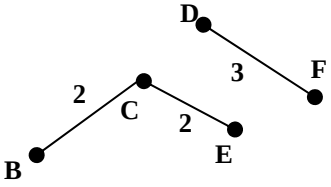
1. Choose the shortest edge (if there is more than one, choose any of the shortest)…



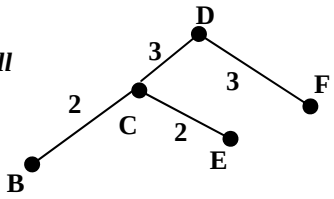
2. Choose the next shortest edge in the network (it doesn't have to be joined to the edges already…

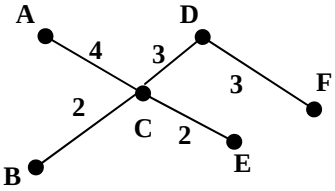


3. Choose the next shortest edge which does not create a cycle and add it…



4. Repeat step 3 until all the vertices are connected then stop.





Length of minimum spanning tree: 14

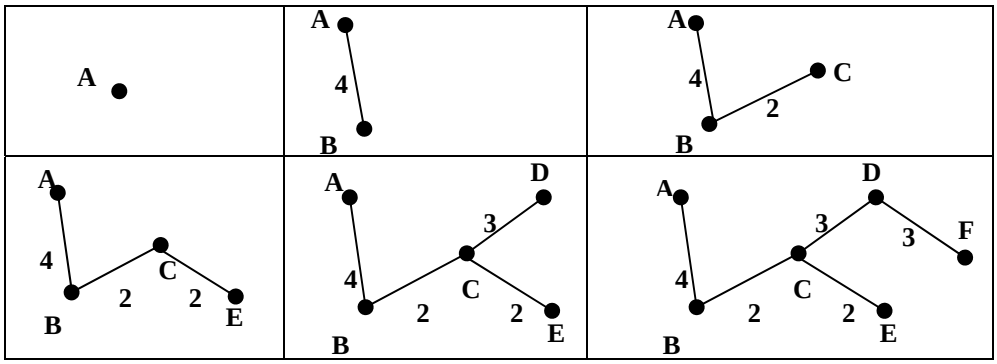
Prim's Algorithm on a network

1. Choose a vertex…

2. Choose the shortest edge from this vertex to any vertex connected directly to it…

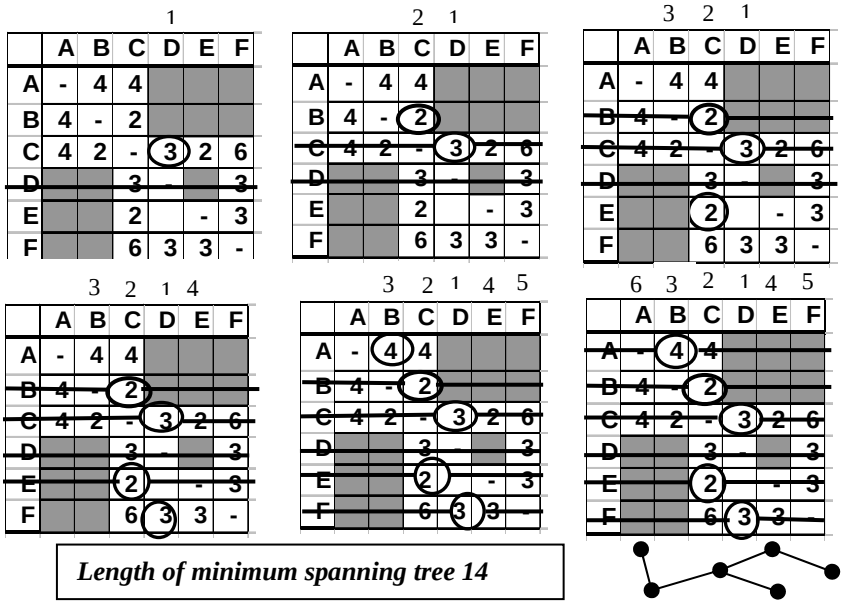
3. Choose the nearest vertex not yet in the solution which is connected to any vertex which is in the solution and which does not create a cycle…

4. Repeat step 3 until all the vertices are connected then stop.



Prim's Algorithm on a Table

1. *Choose a column and cross out its row. Here D has been chosen. Delete row D.*
2. *Choose the smallest number in the column D and circle it. If there is a choice, choose either.*
3. *For the number you have just circled, cross out its row and put an arrow above its row at the top of the table.*
4. *Choose the smallest number not already crossed out from the arrowed columns and circle it.*
5. *For the number you have just circled, cross out its row*
6. *and put an arrow above it's row at the top of the table.*
7. *Continue till all vertices have been included in the tree.*

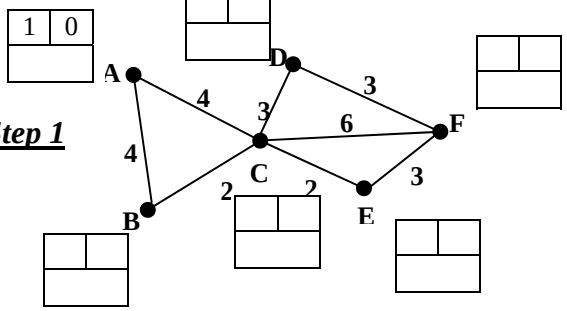


Length of minimum spanning tree 14

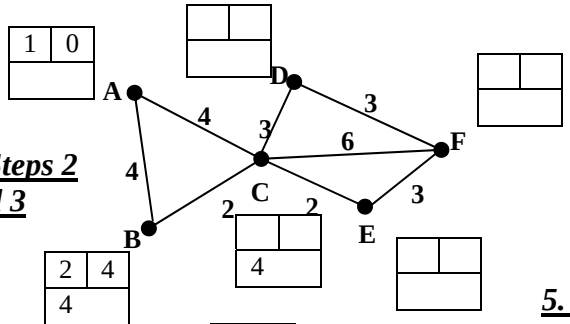
Dijkstra's Algorithm for the shortest path

1. *Label the start vertex with permanent label 0 and order label 1.*
2. *Assign temporary labels to all the vertices that can be reached directly from the start.*
3. *Select the vertex with the smallest temporary label and make its label permanent. Add the correct order label.*
4. *Put temporary labels on each vertex that can be reached directly from the vertex you have just made permanent. The temporary label must be equal to the sum of the permanent label and the direct distance from it. If there is an existing temporary label at a vertex, it should be replaced only if the new sum is smaller.*
5. *Repeat steps 3 and 4 until the finishing vertex has a permanent label.*
6. *To find the shortest paths(s), trace back from the end vertex to the start vertex. Write the route forwards and state the length.*

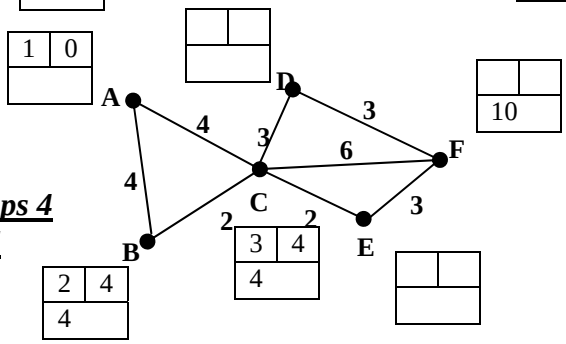
1. Step 1



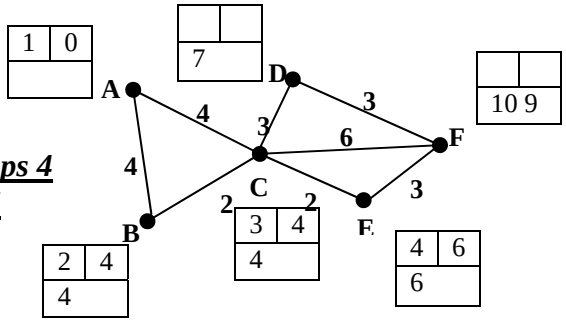
2. Steps 2 and 3



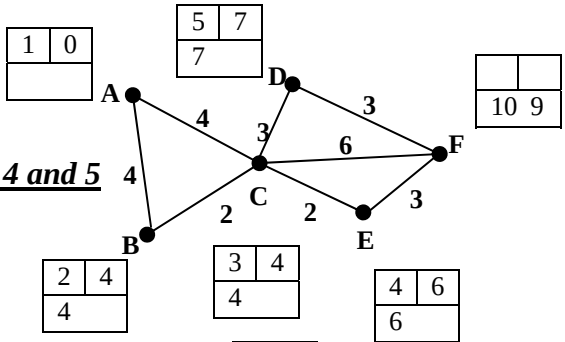
3. Steps 4 and 5



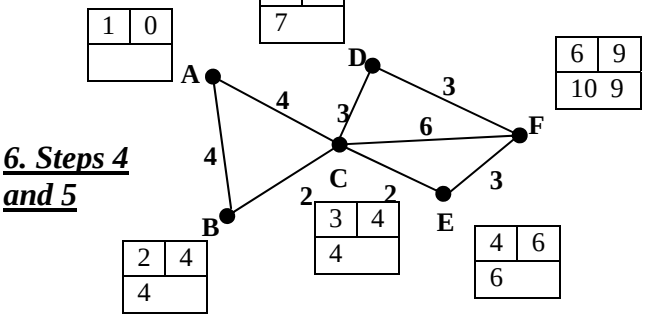
4. Steps 4 and 5



5. Steps 4 and 5



6. Steps 4 and 5



7. Step 6

Solution:
Shortest path ACEF
Length 9

REVISION SHEET – DECISION MATHS

Networks: Travelling Salesperson and Route Inspection

The main ideas are covered in

AQA	Edexcel	MEI	OCR
D1	D1	D2	D1

The main ideas in this topic are:

Finding bounds within which the solution to the Travelling Salesperson problem lies.

Applying the Nearest Neighbour algorithm to find an upper bound for the solution.

Apply the Chinese Postman Algorithm to obtain the closed trail of minimum weight.

Before the exam you should know:

- That a Hamiltonian cycle is a tour which contains every vertex (node) precisely once.
- That an Euler cycle is a tour which travels along every edge of a network.
- That the Nearest Neighbour algorithm is used for finding upper bounds for the TSP.
- That the Nearest Neighbour algorithm will always produce a tour but it may not be the optimal solution.
- How to find a lower bound to TSP by deleting a vertex.
- That it may be possible to improve the tour by interchanging the order in which two nodes are visited.
- The meaning of: order of a vertex (node), traversable graph and Eulerian graph.
- That the direct route is not always the shortest.
- That you need to identify ALL the odd vertices in the route inspection problem.

The traveling Salesperson Problem

A Hamiltonian cycle is defined as a tour which contains every vertex precisely once. In a simple case it is easy to list all the Hamiltonian cycles but as the number of nodes increases, the number of Hamiltonian cycles tends to increase very rapidly. There is no algorithm for finding the optimal solution to the **travelling salesperson** problem. The method used finds a reasonably good solution by establishing upper and lower bounds.

Lower Bounds

1. Delete a vertex and the edges incident on it to form a reduced matrix.
2. Find a minimum spanning tree for the remaining network.
3. Reconnect the deleted vertex by the two shortest edges.

Repeat for all vertices. Greatest lower bound is the best lower bound

Upper bounds: Nearest Neighbour Algorithm (NNA)

Before you can apply the nearest neighbour algorithm you need to make a complete matrix of all shortest distance between pairs of vertices:

1. Choose any starting node.
2. Consider the edges which join the previously chosen vertex to not-yet-chosen vertex and choose the one with minimum weight.
3. Repeat Step 2 until all nodes have been chosen.
4. Then add the arc that joins the last-chosen node to the first-chosen node.

For a full solution, NNA should be repeated starting at each vertex in turn. The shortest tour will be the least upper bound.

Note the similarity between the nearest neighbour method and Prim's algorithm. Do not confuse the two: with Prim, choose the least weight arc from *all the nodes* in the tree. In the nearest neighbour, choose the least weight arc from the *current node only*.

The Route Inspection Problem

The problem is to find a route of minimum length which goes along each edge in the network once and returns to the starting point. This type of problem arises in contexts such as a rail safety expert needing to inspect every piece of track in a railway system, or a postman needing to walk along every street to deliver mail in the most efficient way possible, hence it is often called the **Chinese Postman problem** because a Chinese mathematician developed the algorithm.
For a network to be traversable it must be Eulerian (no odd nodes) or semi-Eulerian (two odd nodes). A network will always have an even number of odd nodes (handshaking theorem). If the network is Eulerian (every vertex is of even order) there are many equal optimum solutions.

The Algorithm can be stated as follows:

1. Identify the odd vertices in the network.
2. Consider all the routes joining pairs of odd vertices and select the one with the least weight.
3. Find the sum of the weights on all the edges.
4. Shortest distance is the sum of the weights plus the extra that must be traveled.
5. Find a tour which repeats the edges found in step 2.

Example: For the network shown below

(a) Find the length of the shortest closed trail that covers every edge on the network below and write down a suitable route

(b) Find an upper bound for the Traveling Salesperson problem, starting at vertex A.

(c) Find a lower bound for the Traveling Salesperson by deleting vertex A.

(d) Give a suitable route

Solution:

(a) Odd vertices are A, C, D and E.

Consider all the possible pairings of odd vertices:

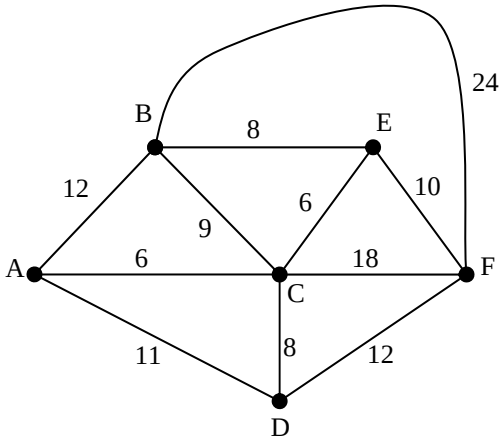
AC = 6	and	DE = 14	total = 20
AD = 11	and	CE = 6	total = 17
AE = 12	and	CD = 8	total = 20

The pairing of least weight is AD and CE = 17.

The sum of the weights in the network is 124.

Repeating AD and CE gives a total weight = 124 + 17 = 141.

A suitable route is A – B – E – F – D – A – C – B – F – C – E – C – D – A.



(b) Nearest Neighbour algorithm

Upper bound has length

6 + 17 + 6 + 12 + 8 + 22 = 71

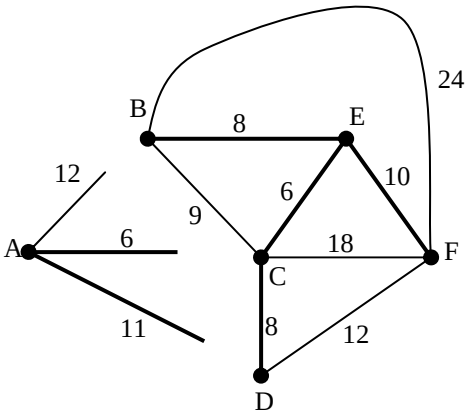
Route: A – C – E – B – D – F – A

Interpreted this is:

A – C – E – B – C – D – F – E – C – A

	1	**4**	**2**	**5**	**3**	**6**
	A	**B**	**C**	**D**	**E**	**F**
A	-	12	6	11	12	22
B	12	-	9	17	8	18
C	6	9	-	8	6	16
D	11	17	8	-	14	12
E	12	8	6	14	-	10
F	22	18	16	12	10	-

(c) delete vertex A



Minimum connector for the remaining network: 6 + 8 + 8 + 10 = 32

Minimum distance to reconnect A: 6 + 11 = 17

Lower bound = 32 + 17 = 49

(d) 49 ≤ the length of the route ≤ 71

A suitable route would be A – B – E – F – D – C - A

Length 12 + 8 + 10 + 12 + 8 + 6 = 56