

C4 VECTORS*Answers - Worksheet A*

1 a) $-\mathbf{p}$ b) $2\mathbf{q}$ c) $\frac{1}{2}\mathbf{p}$ d) \mathbf{p} e) $-\mathbf{q}$ f) $\mathbf{p} + \mathbf{q}$
 g) $\frac{1}{2}\mathbf{p} + 2\mathbf{q}$ h) $\mathbf{p} - \mathbf{q}$ i) $2\mathbf{q} - \mathbf{p}$ j) $-\mathbf{p} - 2\mathbf{q}$ k) $\frac{1}{2}\mathbf{p} - \mathbf{q}$ l) $-\frac{1}{2}\mathbf{p} - 2\mathbf{q}$

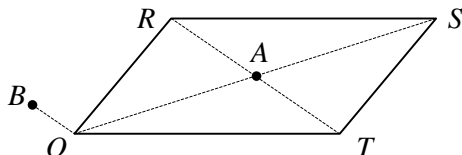
2 a) $\mathbf{u} + \mathbf{v}$ b) $\mathbf{w} - \mathbf{u}$ c) $\mathbf{u} + \mathbf{v} - \mathbf{w}$

3 a) \mathbf{q} b) $\mathbf{p} + \mathbf{r}$ c) $\mathbf{r} - \mathbf{q}$ d) $\mathbf{p} + \mathbf{q} + \mathbf{r}$ e) $-\mathbf{q} - \mathbf{r}$ f) $\mathbf{q} + \mathbf{r} - \mathbf{p}$

4 a i) $(\mathbf{a} + 2\mathbf{b}) + (\mathbf{a} - 2\mathbf{b})$
 $= 2\mathbf{a}$
 ii) $(\mathbf{a} + 2\mathbf{b}) - (\mathbf{a} - 2\mathbf{b})$
 $= 4\mathbf{b}$

b $\overrightarrow{OA} = \frac{1}{2}\overrightarrow{OS}$, $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{TR}$

\therefore



5 a i) $= \frac{1}{2}\mathbf{a}$
 ii) $= \mathbf{b} - \mathbf{a}$
 iii) $= \frac{1}{2}(\mathbf{b} - \mathbf{a})$
 iv) $= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$
 v) $= \frac{1}{2}(\mathbf{a} + \mathbf{b}) - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{b}$

b they are parallel (and the magnitude of \overrightarrow{CD} is half that of \overrightarrow{OB})

6 a parallel, $3\mathbf{p} = \frac{3}{2}(2\mathbf{p})$
 b not parallel
 c parallel, $(\mathbf{p} - \frac{1}{3}\mathbf{q}) = \frac{1}{3}(3\mathbf{p} - \mathbf{q})$
 d parallel, $(4\mathbf{q} - 2\mathbf{p}) = -2(\mathbf{p} - 2\mathbf{q})$
 e parallel, $(6\mathbf{p} + 8\mathbf{q}) = 8(\frac{3}{4}\mathbf{p} + \mathbf{q})$
 f not parallel

7 a $= (2\mathbf{m} + 3\mathbf{n}) - (4\mathbf{m} + 2\mathbf{n})$
 $= \mathbf{n} - 2\mathbf{m}$
 b $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \mathbf{m} + \frac{3}{2}\mathbf{n}$
 $\overrightarrow{AM} = (\mathbf{m} + \frac{3}{2}\mathbf{n}) - 4\mathbf{m} = \frac{3}{2}\mathbf{n} - 3\mathbf{m}$
 $\therefore \overrightarrow{AM} = \frac{3}{2}\overrightarrow{BC}$
 $\therefore AM$ is parallel to BC

8 a $\overrightarrow{OM} = \frac{1}{2} \overrightarrow{OA} = 3\mathbf{u} - 2\mathbf{v}$

$\overrightarrow{AB} = (3\mathbf{u} - \mathbf{v}) - (6\mathbf{u} - 4\mathbf{v}) = 3\mathbf{v} - 3\mathbf{u}$

$\overrightarrow{ON} = \overrightarrow{OA} + \frac{1}{3} \overrightarrow{AB} = (6\mathbf{u} - 4\mathbf{v}) + \frac{1}{3}(3\mathbf{v} - 3\mathbf{u}) = 5\mathbf{u} - 3\mathbf{v}$

b $\overrightarrow{CM} = (3\mathbf{u} - 2\mathbf{v}) - (\mathbf{v} - 3\mathbf{u}) = 6\mathbf{u} - 3\mathbf{v}$

$\overrightarrow{CN} = (5\mathbf{u} - 3\mathbf{v}) - (\mathbf{v} - 3\mathbf{u}) = 8\mathbf{u} - 4\mathbf{v}$

$\therefore \overrightarrow{CN} = \frac{4}{3} \overrightarrow{CM}$

$\therefore \overrightarrow{CN}$ and \overrightarrow{CM} are parallel

common point $C \therefore C, M$ and N are collinear

9 a $a = 5, b = 3$

b $2 + b = 0$ and $a - 4 = 0$

$\therefore a = 4, b = -2$

c $-1 = b$ and $4a = -2$

d $2a + 6 = 0$ and $b - a = 0$

$\therefore a = -\frac{1}{2}, b = -1$

$\therefore a = -3, b = -3$

10 a $\overrightarrow{OC} = \frac{1}{2} \mathbf{a}$

$\overrightarrow{CB} = \mathbf{b} - \frac{1}{2} \mathbf{a}$

$\overrightarrow{OD} = \frac{1}{2} \mathbf{a} + \frac{1}{2} (\mathbf{b} - \frac{1}{2} \mathbf{a}) = \frac{1}{4} \mathbf{a} + \frac{1}{2} \mathbf{b}$

b $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{OE} = \overrightarrow{OA} + k \overrightarrow{AB}$

$\therefore \overrightarrow{OE} = \mathbf{a} + k(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OE} = l \overrightarrow{OD}$

$\therefore \mathbf{a} + k(\mathbf{b} - \mathbf{a}) = l(\frac{1}{4} \mathbf{a} + \frac{1}{2} \mathbf{b})$

$\therefore 1 - k = \frac{1}{4} l$

and $k = \frac{1}{2} l$

adding $1 = \frac{3}{4} l$

$l = \frac{4}{3}$

$\therefore \overrightarrow{OE} = \frac{4}{3} (\frac{1}{4} \mathbf{a} + \frac{1}{2} \mathbf{b}) = \frac{1}{3} \mathbf{a} + \frac{2}{3} \mathbf{b}$

d $k = \frac{1}{2} l = \frac{2}{3}$

$\therefore \overrightarrow{AE} = \frac{2}{3} \overrightarrow{AB}$

$\therefore AE : EB = 2 : 1$