C4 Algebra

1. June 2010 qu. 1

Expand $(1+3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term in x^3. [5]

2. June 2010 qu. 3

Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions. [5]

3. June 2010 qu. 8

(i) Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. [3]

4. Jan 2010 qu. 1

Find the quotient and the remainder when $x^4 + 11x^3 + 28x^2 + 3x + 1$ is divided by $x^2 + 5x + 2$. [4]

5. Jan 2010 qu. 5

(i) Expand $(1+x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. [2]

(ii) (a) Hence, or otherwise, expand $(8+16x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. [4]

(b) State the set of values of x for which the expansion in part (ii) (a) is valid. [1]

6. Jan 2010 qu. 10

(i) Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions. [2]

7. June 2009 qu. 1

Find the quotient and the remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$. [4]

8. June 2009 qu. 3

(i) Expand $(a + x)^{-2}$ in ascending powers of x up to and including the term in x^2. [4]

(ii) When $(1 - x)(a + x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a. [3]

9. June 2009 qu. 6

The expression $\dfrac{4x}{(x-5)(x-3)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\dfrac{A}{x-5}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$, where A, B and C are constants. [4]

(ii) Hence find the exact value of $\int_1^2 f(x)\,dx$. [5]

10. Jan 2009 qu. 1

Simplify $\dfrac{20-5x}{6x^2-24x}$.

[3]

11. Jan 2009 qu. 3

(i) Expand $(1+2x)^{\frac{1}{2}}$ as a series in ascending powers of x, up to and including the term in x^3. [3]

(ii) Hence find the expansion of $\dfrac{(1+2x)^{\frac{1}{2}}}{(1+x)^3}$ as a series in ascending powers of x, up to and including the term in x^3. [5]

(iii) State the set of values of x for which the expansion in part (ii) is valid. [1]

12. June 2008 qu. 1

(a) Simplify $\dfrac{(2x^2-7x-4)(x+1)}{(3x^2+x-2)(x-4)}$. [2]

(b) Find the quotient and remainder when $x^3 + 2x^2 - 6x - 5$ is divided by $x^2 + 4x + 1$. [4]

13. June 2008 qu. 5

(i) Show that $\sqrt{\dfrac{1-x}{1+x}} \approx 1-x+\tfrac{1}{2}x^2$, for $|x|<1$. [5]

(ii) By taking $x=\tfrac{2}{7}$, show that $\sqrt{5} \approx \tfrac{111}{49}$. [3]

14. June 2008 qu. 8

(i) Given that $\dfrac{2t}{(t+1)^2}$ can be expressed in the form $\dfrac{A}{t+1}+\dfrac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

15. Jan 2008 qu. 2

(i) Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x+2)}\,dx$. [2]

16. Jan 2008 qu. 3

When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find the values of the constants a, b and c. [5]

17. Jan 2008 qu. 6

(i) Expand $(1 + ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

18. June 2007 qu. 1

The equation of a curve is $y = f(x)$, where $f(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $f(x)$ in partial fractions. [2]

(ii) Hence find $f'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]

19. June 2007 qu. 1

(i) Expand $(2 + x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\dfrac{1+x^2}{(2+x)^2}$. [2]

20. June 2007 qu. 7

(i) Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express in the form $\dfrac{2x^3+3x^2+9x+12}{x^2+4}$ in the form $Ax + B + \dfrac{Cx+D}{x^2+4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part (ii) to find the exact value of $\displaystyle\int_1^3 \frac{2x^3+3x^2+9x+12}{x^2+4}\,dx$. [5]

21. Jan 2007 qu. 1

It is given that $f(x) = \dfrac{x^2+2x-24}{x^2-4x}$ for $x \neq 0$, $x \neq 4$. Express f(x) in its simplest form. [3]

22. Jan 2007 qu. 5

(i) Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) Hence find the coefficient of x^3 in the expansion of $\left(1-3(x+x^3)\right)^{-\frac{1}{3}}$. [3]

23. Jan 2007 qu. 6

(i) Express $\dfrac{2x+1}{(x-3)^2}$ in the form $\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\displaystyle\int_4^{10} \frac{2x+1}{(x-3)^2}\,dx$, giving your answer in the form $a + b \ln c$, where a, b and c are integers. [4]

24. June 2006 qu. 2

(i) Expand $(1-3x)^{-2}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) Find the coefficient of x^2 in the expansion of $\dfrac{(1+2x)^2}{(1-3x)^2}$ in ascending powers of x. [4]

25. June 2006 qu. 3

(i) Express $\dfrac{3-2x}{x(3-x)}$ in partial fractions. [3]

(ii) Show that $\displaystyle\int_1^2 \frac{3-2x}{x(3-x)}\,dx = 0$. [4]

(iii) What does the result of part (ii) indicate about the graph of $y = \dfrac{3-2x}{x(3-x)}$ between $x = 1$ and $x = 2$? [1]

26. Jan 2006 qu. 1

Simplify $\dfrac{x^3-3x^2}{x^2-9}$. [3]

27. Jan 2006 qu. 3

(i) Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$. [4]

(ii) Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder. [2]

28. Jan 2006 qu. 7

The expression $\dfrac{11+8x}{(2-x)(1+x)^2}$ is denoted by f (x).

(i) Express f(x) in the form $\dfrac{A}{2-x}+\dfrac{B}{1+x}+\dfrac{C}{(1+x)^2}$, where A, B and C are constants. [5]

(ii) Given that $|x| < 1$, find the first 3 terms in the expansion of f(x) in ascending powers of x. [5]

29. June 2005 qu. 1

Find the quotient and the remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$. [4]

30. June 2005 qu. 8

(i) Given that $\dfrac{3x+4}{(1+x)(2+x)^2} \equiv \dfrac{A}{1+x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$ find A, B and C. [5]

(ii) Hence or otherwise expand $\dfrac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

(iii) State the set of values of x for which the expansion in part (ii) is valid. [1]