1. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)
(Total 12 marks)

2. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(a) Write down the coordinates of A.

(1)

(b) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X.

(1)

(d) Find the vector $\overrightarrow{AX}$

(2)

(e) Hence, or otherwise, show that $\left|\overrightarrow{AX}\right| = 4\sqrt{26}.$

(2)

The point y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures.

(3)
(Total 12 marks)

3. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l.

(3)

(b) Find $\left|\overrightarrow{CB}\right|$.

(2)

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(3)

(d) Find the shortest distance from the point C to the line l.

(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures.

(3)
(Total 14 marks)

4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where $\underline{\lambda}$ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)
(Total 13 marks)

5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

(b) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1.

(1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B.

(3)
(Total 12 marks)

6. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)
(Total 13 marks)

7. The points A and B have position vectors $\mathbf{i} - \mathbf{j} + p\mathbf{k}$ and $7\mathbf{i} + q\mathbf{j} + 6\mathbf{k}$ respectively, where p and q are constants.

The line l_1, passing through the points A and B, has equation

$$\mathbf{r} = 9\mathbf{i} + 7\mathbf{j} + 7\mathbf{k} + 7\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}), \text{ where } \lambda \text{ is a parameter.}$$

(a) Find the value of p and the value of q.

(4)

(b) Find a unit vector in the direction of $\overrightarrow{AB}$.

(2)

A second line l_2 has vector equation

$$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}), \text{ where } \mu \text{ is a parameter.}$$

(c) Find the cosine of the acute angle between l_1 and l_2.

(3)

(d) Find the coordinates of the point where the two lines meet.

(5)
(Total 14 marks)

8. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(a) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1

(b) find the coordinates of P.

(5)

(c) Hence find the distance OP, giving your answer as a simplified surd.

(2)
(Total 10 marks)

9. The points A and B have position vectors $5\mathbf{j} + 11\mathbf{k}$ and $c\mathbf{i} + d\mathbf{j} + 21\mathbf{k}$ respectively, where c and d are constants.

The line l, through the points A and B, has vector equation $\mathbf{r} = 5\mathbf{j} + 11\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k})$, where λ is a parameter.

(a) Find the value of c and the value of d.

(3)

The point P lies on the line l, and $\overrightarrow{OP}$ is perpendicular to l, where O is the origin.

(b) Find the position vector of P.

(6)

(c) Find the area of triangle OAB, giving your answer to 3 significant figures.

(4)
(Total 13 marks)

10. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(a) Find the coordinates of B.

(4)

(b) Find the value of $\cos\theta$, giving your answer as a simplified fraction.

(4)

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(3)

(d) Find the position vector of the point D.

(2)
(Total 13 marks)

11. Relative to a fixed origin O, the point A has position vector $5\mathbf{j} + 5\mathbf{k}$ and the point B has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(a) Find a vector equation of the line L which passes through A and B.

(2)

The point C lies on the line L and OC is perpendicular to L.

(b) Find the position vector of C.

(5)

The points O, B and A, together with the point D, lie at the vertices of parallelogram $OBAD$.

(c) Find, the position vector of D.

(2)

(d) Find the area of the parallelogram $OBAD$.

(4)
(Total 13 marks)

12. Relative to a fixed origin O, the vector equations of the two lines l_1 and l_2 are

$$l_1\colon \mathbf{r} = 9\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} + t(-8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}),$$

and

$$l_2\colon \mathbf{r} = -16\mathbf{i} + \alpha\mathbf{j} + 10\mathbf{k} + s(\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}),$$

where α is a constant.

The two lines intersect at the point A.

(a) Find the value of α.

(6)

(b) Find the position vector of the point A.

(1)

(c) Prove that the acute angle between l_1 and l_2 is 60°.

(5)

Point B lies on l_1 and point C lies on l_2. The triangle ABC is equilateral with sides of length $14\sqrt{2}$.

(d) Find one of the possible position vectors for the point B and the corresponding position vector for the point C.

(4)
(Total 16 marks)

13. The equations of the lines l_1 and l_2 are given by

$$l_1: \quad \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

$$l_2: \quad \mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$

where λ and μ are parameters.

(a) Show that l_1 and l_2 intersect and find the coordinates of Q, their point of intersection.

(6)

(b) Show that l_1 is perpendicular to l_2.

(2)

The point P with x-coordinate 3 lies on the line l_1 and the point R with x-coordinate 4 lies on the line l_2.

(c) Find, in its simplest form, the exact area of the triangle PQR.

(6)
(Total 14 marks)

14. Referred to a fixed origin O, the points A and B have position vectors $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$ and $(5\mathbf{i} - 3\mathbf{j})$ respectively.

(a) Find, in vector form, an equation of the line l_1 which passes through A and B.

(2)

The line l_2 has equation $r = (4\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$, where μ is a scalar parameter.

(b) Show that A lies on l_2.

(1)

(c) Find, in degrees, the acute angle between the lines l_1 and l_2.

(4)

The point C with position vector $(2\mathbf{i} - \mathbf{k})$ lies on l_2.

(d) Find the shortest distance from C to the line l_1.

(4)

15. The points A, B and C have position vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ and $\mathbf{i} - \mathbf{j}$ respectively, relative to a fixed origin O.

(a) Prove that the points A, B and C lie on a straight line l.

(4)

The point D has position vector $2\mathbf{i} + \mathbf{j} - 3\,\mathbf{k}$.

(b) Find the cosine of the acute angle between l and the line OD.

(3)

The point E has position vector $-3\mathbf{j} - \mathbf{k}$.

(c) Prove that E lies on l and that OE is perpendicular to OD.

(4)
(Total 11 marks)

16. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 11 \\ 5 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$, where λ is a parameter.

The line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 24 \\ 4 \\ 13 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix}$, where μ is a parameter.

(a) Show that the lines l_1 and l_2 intersect.

(4)

(b) Find the coordinates of their point of intersection.

(2)

Given that θ is the acute angle between l_1 and l_2,

(c) Find the value of $\cos\theta$. Give your answer in the form $k\sqrt{3}$, where k is a simplified fraction.

(4)
(Total 10 marks)

1. (a) **j** components $3+2\lambda=9 \quad \Rightarrow \lambda=3$ $(\mu=1)$ M1 A1

Leading to $C:(5, 9, -1)$ accept vector forms A1 3

(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$ M1

$$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$$ use of scalar product M1 A1

$\angle ACB = 57.95°$ awrt $57.95°$ A1 4

Alternative method

A:(2, 3, –4) B:(–5, 9, –5) C:(5, 9, –1)

$AB^2 = 7^2 + 6^2 + 1^2 = 86$

$AC^2 = 3^2 + 6^2 + 3^2 = 54$

$BC^2 = 10^2 + 0^2 + 4^2 = 116$ Finding all three sides M1

$\cos \angle ACB = \dfrac{116+54-86}{2\sqrt{116}\sqrt{54}}$ (= 0.53066 ...) M1 A1

$\angle ACB = 57.95°$ awrt 57.95° A1 4

(c) $A:(2, 3, -4) \quad B:(-5, 9, -5)$

$$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}, \quad \overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$$

$AC^2 = 3^2+6^2+3^2 \quad \Rightarrow \quad AC = 3\sqrt{6}$ M1 A1

$BC^2 = 10^2+4^2 \quad \Rightarrow \quad BC = 2\sqrt{29}$ A1

$$\Delta ABC = \frac{1}{2}AC \times BC \sin\angle ACB$$

$= \dfrac{1}{2}3\sqrt{6}\times 2\sqrt{29}\sin\angle ACB \approx 33.5$ $15\sqrt{5}$, awrt 34 M1 A1 5

Alternative method

A:(2, 3, –4) B:(–5, 9, –5) C:(5, 9, –1)

$AB^2 = 7^2 + 6^2 + 1^2 = 86$

$AC^2 = 3^2 + 6^2 + 3^2 = 54$

$BC^2 = 10^2 + 0^2 + 4^2 = 116$	Finding all three sides	M1	
$\cos\ \angle ACB = \dfrac{116+54-86}{2\sqrt{116}\sqrt{54}}$ (= 0.53066 ...)		M1 A1	
$\angle ACB = 57.95°$	awrt 57.95°	A1	4

If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).

[12]

2. (a) A: (–6, 4,–1) — Accept vector forms — B1 — 1

(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix} = 12 + 4 + 3 =$

$\sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$ — M1 A1

$\cos\theta = \dfrac{19}{26}$ — awrt 0.73 — A1 — 3

(c) X: (10, 0,11) — Accept vector forms — B1 — 1

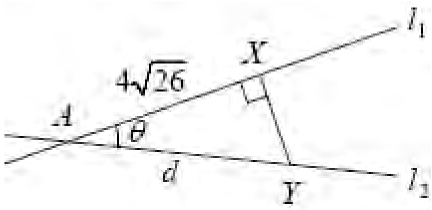
(d) $\overrightarrow{AX} = \begin{pmatrix}10\\0\\11\end{pmatrix} - \begin{pmatrix}-6\\4\\-1\end{pmatrix}$ — Either order — M1

$\begin{pmatrix}16\\-4\\12\end{pmatrix}$ — cao — A1 — 2

(e) $\left|\overrightarrow{AX}\right| = \sqrt{16^2+(-4)^2+12^2}$ — M1

$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ * — Do not penalise if consistent incorrect signs in (d) — A1 — 2

(f)



Use of correct right angled triangle M1

$\dfrac{\left|\overrightarrow{AX}\right|}{d} = \cos\theta$ M1

$d = \dfrac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9 A1 3

[12]

3. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ M1

$\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ *or* $\mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ accept equivalents M1 A1ft 3

(b) $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$

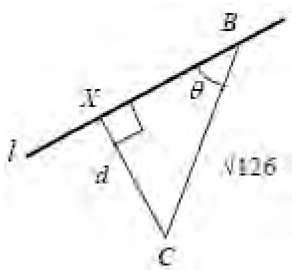
$CB = \sqrt{(1^2 + 5^2 + (-10)^2)} = \sqrt{(126)}\ (= 3\sqrt{14} \approx 11.2)$ awrt 11.2 M1 A1 2

(c) $\overrightarrow{CB}.\overrightarrow{AB} = \left|\overrightarrow{CB}\right|\left|\overrightarrow{AB}\right|\cos\theta$

$(\pm)(2 + 5 + 20) = \sqrt{126}\sqrt{9}\cos\theta$ M1 A1

$\cos\theta = \dfrac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7°$ awrt 36.7° A1 3

(d)



$\frac{d}{\sqrt{126}} = \sin\theta$ M1 A1ft

$d = 3\sqrt{5}$ (≈ 6.7) awrt 6.7 A1 3

(e) $BX^2 = BC^2 - d^2 = 126 - 45 = 81$ M1

! $CBX = \frac{1}{2} \times BX \times d = \frac{1}{2} \times 9 \times 3\sqrt{5} = \frac{27\sqrt{5}}{2}$ (≈ 30.2) awrt 30.1 or 30.2 M1 A1 3

Alternative for (e)

! $CBX = \frac{1}{2} \times d \times BC \sin \angle XCB$ M1

$= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{126} \sin(90 - 36.7)^\circ$ sine of correct angle M1

≈ 30.2 $\frac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2 A1 3

[14]

4. (a) $\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\}$ = $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$ Apply dot product calculation between two direction vectors, ie. $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$ M1

$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$

$-2q = 6 \Rightarrow \underline{q = -3}$ AGand solves to find $q = -3$ A1 cso 2

(b) Lines meet where:

$$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

First two of

i: $11 - 2\lambda = -5 + q\mu$ (1)

j: $2 + \lambda = 11 + 2\mu$ (2)

k: $17 - 4\lambda = p + 2\mu$ (3)

Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.) M1

(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$ Attempts to solve (1) and (2) to find one of either λ or μ dM1

Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$ A1

Both $\lambda = 5$ or $\underline{\mu = -2}$ A1

(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$

	(3) $\Rightarrow$ $17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1	
	$\Rightarrow$ $p = 17 - 20 + 4$ $\Rightarrow$ $p = 1$	$\underline{p = -1}$	A1 cso	6
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1	
	Intersect at $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or $\underline{(1, 7, -3)}$	$\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or $\underline{(1, 7, -3)}$	A1	2
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection			
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$	M1ft $\pm$	
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$			
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\left(\text{their } \overrightarrow{AX}\right)$	dM1ft	
	Hence, $\overrightarrow{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1	3

[13]

5. (a) Lines meet where:

$$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$$

Any two of
i: $-9 + 2\lambda = 3 + 3\mu$ (1)
j: $\lambda = 1 - \mu$ (2)
k: $10 - \lambda = 17 + 5\mu$ (3)

(1) – 2**(2)** gives: $-9 = 1 + 5\mu \Rightarrow \mu = -2$
(2) gives: $\lambda = 1 - -2 = 3$

$$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}\text{ or }\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$$

Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r} = \underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$

Either check **k:**
$\lambda = 3$: LHS $= 10 - \lambda = 10 - 3 = 7$
$\mu = -2$: RHS $= 17 + 5\mu = 17 - 10 = 7$

(As LHS = RHS then the lines intersect.)

Need any two of these correct equations seen anywhere in part (a). M1

Attempts to solve simultaneous equations to find one of either λ or μ dM1

Both $\underline{\lambda = 3}$ & $\underline{\mu = -2}$ A1

Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively**. This mark can be implied by any two correct components of (–3, 3, 7). ddM1

$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or = $\underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$ or (–3, 3, 7) A1

Either check that $\lambda = 3$, $\mu = -2$ in a third equation **or** check that $\lambda = 3$, $\mu = -2$ give the same coordinates on the other line. Conclusion not needed. B1 6

(b) $\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$

As $\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix} = \underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)} = 0$

Then l_1 is perpendicular to l_2.

Dot product calculation between the ***two direction vectors**:*
$\underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)}$ or $\underline{6 - 1 - 5}$ M1

Result '= 0' and appropriate conclusion A1 2

(c) **Way 1**

Equating **i** ; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$

$$\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$$

(= $\overrightarrow{OA}$. Hence the point A lies on l_1.)

Substitutes candidate's $\lambda = 7$ into the line l_1 and finds $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.
The conclusion on this occasion is not needed. B1 1

(d) Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection

$$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \underline{\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$$

$$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$$

Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$

Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$.

$$\overrightarrow{AX} = \pm\left(\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}\right)$$ M1ft±

$$\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\left(\text{their } \overrightarrow{AX}\right)$$ dM1ft

$\underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$ A1 3

[12]

6. (a) Equating **i**; $0 = 6 + \lambda \quad \Rightarrow \quad \lambda = -6$ B1 ⇒ d

$\underline{\lambda = -6}$

Can be implied

Using $\lambda = -6$ and

equating **j**; $a = 19 + 4(-6) = -5$ M1 ⇒ d

*For inserting **their stated** λ into either a correct j or k component*

Can be implied.

equating **k** ; $b = -1 - 2(-6) = 11$ A1 3

***a** = –5 and b = 11*

With no working...

... only one of a or b stated correctly gains the first 2 marks.

... both a and b stated correctly gains 3 marks.

(b) $\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$

direction vector or l_1 $\mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{OP} \perp \mathrm{I}_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \text{ d } = 0}$

*Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and **d** are defined as above.*

ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ (or $\underline{x + 4y - 2z = 0}$) M1

Allow either of these two <u>underlined statements</u>

$\therefore 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$ A1 oe

Correct equation

$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$ dM1

Attempt to solve the equation in λ

$21\lambda + 84 = 0 \Rightarrow \lambda = -4$ A1

$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$ M1

Substitutes their λ into an expression for $\overrightarrow{OP}$

$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

*2**i** + 3**j** + 7**k** or P(2, 3, 7)*

[6]

Aliter Way 2

(b) $\overrightarrow{OP} = (6 + \lambda)\mathbf{i} + (19 + 4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$

$\overrightarrow{AP} = (6 + \lambda - 0)\mathbf{i} + (19 + 4\lambda + 5)\mathbf{j} + (-1 - 2\lambda - 11)\mathbf{k}$

direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$

Allow this statement for M1

if $\overrightarrow{AP}$ ***and*** $\overrightarrow{OP}$ *are defined as above.*

ie $\underline{\begin{pmatrix} 6 + \lambda \\ 24 + 4\lambda \\ -12 - 2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6 + \lambda \\ 19 + 4\lambda \\ -1 - 2\lambda \end{pmatrix} = 0}$ M1

underlined statement

$\therefore (6 + \lambda)(6 + \lambda) + (24 + 4\lambda)(19 + 4\lambda) + (-12 - 2\lambda)(-1 - 2\lambda) = 0$ A1 oe

Correct equation

$36 + 12\lambda + \lambda^2 + 456 + 96\lambda + 76\lambda + 16\lambda^2 + 12 + 24\lambda + 2\lambda + 4\lambda^2 = 0$ dM1

$21\lambda^2 + 210\lambda + 504 = 0$

Attempt to solve the equation in λ

$\lambda^2 + 10\lambda + 24 = 0 \quad \Rightarrow \quad (\lambda = -6) \quad \underline{\lambda = -4}$ A1

$\overrightarrow{OP} = (6-4)\mathbf{i} + (19 + 4(-4))\mathbf{j} + (-1 - 2(-4))\mathbf{k}$ M1

Substitutes their λ into an expression for $\overrightarrow{OP}$

$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ A1 6

$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ *or P(2, 3, 7)*

(c) $\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

$\overrightarrow{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$

$\overrightarrow{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$, $\overrightarrow{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$ M1;

$\overrightarrow{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$ A1ft±

Subtracting vectors to find any two of $\overrightarrow{AP}$, $\overrightarrow{PB}$ *or* $\overrightarrow{AB}$*; and both are correctly ft using candidate's* $\overrightarrow{OA}$ *and* $\overrightarrow{OP}$ *found in parts (a) and (b) respectively.*

As $\overrightarrow{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overrightarrow{PB}$ $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{PB}$

or $\overrightarrow{AB} = \frac{5}{2}\ (2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{2}\overrightarrow{AP}$

or $\overrightarrow{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\ \overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{3}\ \overrightarrow{PB}$

or $\overrightarrow{PB} = \frac{3}{2}\,(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\ \overrightarrow{AP}$ or $\overrightarrow{PB} = \frac{3}{2}\ \overrightarrow{AP}$

or $\overrightarrow{AP} = \frac{2}{5}\,(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\ \overrightarrow{AB}$ or $\overrightarrow{AP} = \frac{2}{5}\ \overrightarrow{AB}$

or $\overrightarrow{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\ \overrightarrow{AB}$ etc... or $\overrightarrow{PB} = \frac{3}{5}\ \overrightarrow{AB}$

alternatively candidates could say for example that

$\overrightarrow{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})\ \overrightarrow{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$

then the points A, P and B are collinear. A1

A, P and B are collinear
Completely correct proof.

$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2 : 3$ B1 oe 4

2 : 3 or 1 : $\frac{3}{2}$ *or* $\sqrt{84} : \sqrt{189}$ *aef*

allow SC $\frac{2}{3}$

Aliter Way 2

(c) At B; $\underline{5 = 6 + \lambda}$, $\underline{15 = 19 + 4\lambda}$ or $\underline{1 = -1 - 2\lambda}$ M1

or at B; $\lambda = -1$

Writing down any of the three underlined equations.

gives $\lambda = -1$ for all three equations.

or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$ A1

$\lambda = -1$ for all three equations or
$\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$

Hence B lies on l_1. As stated in the question both A and P lie on l_1. $\therefore$ A, P and B are collinear. A1

Must state B lies on $l_1 \Rightarrow$
A, P and B are collinear

$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2 : 3$ B1 oe

2 : 3 or aef

[13]

7.	(a)	Solves $9 + 2\lambda = 1$ *or* $7 + 2\lambda = -1$ to give $\lambda = -4$ so $p = 3$	M1 A1	
		Solves $9 + 2\lambda = 7$ or $7 + \lambda = 6$ to give $\lambda = -1$ so $q = 5$	M1 A1	4
	(b)	$\lvert 6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}\rvert = 9$ so unit vector is $\frac{1}{9}(6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$	M1 A1	2
	(c)	$\cos\theta = \dfrac{2\times2+2\times1+1\times2}{3\times3}$	M1 A1	
		$\therefore \cos\theta = \frac{8}{9}$	A1	3
	(d)	Write down two of $9 + 2\lambda = 3 + 2\mu$, $7 + 2\lambda = 2 + \mu$ or $7 + \lambda = 3 - 2\mu$	B1 B1	
		Solve to obtain $\mu = 1$ or $\lambda = -2$	M1 A1	
		Obtain coordinates (5, 3, 5)	A1	5
				[14]

8.	(a)	$\lambda = -4 \rightarrow a = 18$, $\quad\mu = 1 \rightarrow b = 9$	M1, A1, A1	3
	(b)	$\begin{pmatrix} 8+\lambda \\ 12+\lambda \\ 14-\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$	M1	
		$\therefore 8 + \lambda + 12 + \lambda - 14 + \lambda = 0$	A1	
		Solves to obtain λ $\quad(\lambda = -2)$	dM1	
		Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1	5
	(c)	$OP = \sqrt{36+100+256}$	M1	
		$(= \sqrt{392}) = 14\sqrt{2}$	A1 cao	2
				[10]

9.	(a)	$\overrightarrow{AB} = \begin{pmatrix} c \\ d-5 \\ 10 \end{pmatrix} = k\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ or $11 + 5\lambda = 21, \Rightarrow \lambda = 2,$		
		$\therefore c = 4$	M1, A1	
		$d = 7$	A1	3

(b) $\begin{pmatrix}2\\1\\5\end{pmatrix} \bullet \begin{pmatrix}2\lambda\\5+\lambda\\11+5\lambda\end{pmatrix} = 0$ M1A1

$\therefore 4\lambda + 5 + \lambda + 55 + 25\lambda = 0$ M1

$\therefore \lambda = -2$ A1

Substitutes to give the point P, $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ M1, A1 6

(Accept (–4, 3, 1)

(c) Finds the length of OA, or OB or OP or AB as

$\sqrt{146}$ or $\sqrt{506}$ or $\sqrt{26}$ or $\sqrt{120}$ resp. M1

Uses area formula- either

Area = $\frac{1}{2}|\mathbf{AB}| \times |\mathbf{OP}|$ or $= \frac{1}{2}|\mathbf{OA}| \times |\mathbf{OB}| \sin \angle AOB$

or = $\frac{1}{2}|\mathbf{OA}| \times |A\mathbf{B}| \sin\angle OAB$ or $= \frac{1}{2}|AB| \times |\mathbf{OB}| \sin \angle ABO$ M1

$= \frac{1}{2}\sqrt{120}\sqrt{26}$ or $\frac{1}{2}\sqrt{146}\sqrt{506}\sin 11.86$ M1

or $\frac{1}{2}\sqrt{146}\sqrt{120}\sin 155.04$ or $\frac{1}{2}\sqrt{120}\sqrt{506}\sin 13.10$

$= 27.9$ A1 4

[13]

10. (a) $\mathbf{k}$ component $2 + 4\lambda = -2 \Rightarrow \lambda = -1$ M1 A1

Note: $\mu = 2$

Substituting their λ (or μ) into equation of line and obtaining B M1

B: (2, 2, –2) A1 4

Accept vector forms

(b)

$\left|\begin{pmatrix}1\\-1\\4\end{pmatrix}\right| = \sqrt{18}; \left|\begin{pmatrix}1\\-1\\0\end{pmatrix} = \sqrt{2}\right|$ both B1

$\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\0\end{pmatrix} = 1 + 1 + 0\ (= 2)$ B1

$\cos\theta = \frac{2}{\sqrt{18}\sqrt{2}} = \frac{1}{3}$ cao M1 A1 4

(c) $\overrightarrow{AB} = -\mathbf{I} + \mathbf{j} - 4\mathbf{k} \Rightarrow \left|\overrightarrow{AB}\right|^2 = 18$ or $\left|\overrightarrow{AB}\right| = \sqrt{18}$ M1

ignore direction of vector

$\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} \Rightarrow \left|\overrightarrow{BC}\right|^2 = 18$ or $\left|\overrightarrow{BC}\right| = \sqrt{18}$ M1

ignore direction of vector

Hence $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$ (*) A1 3

(d) $\overrightarrow{OD} = 6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ B1 B1 2

Allow first B1 for any two correct
Accept column form or coordinates

[13]

11. (a) $\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$; $\therefore$ Equation of L M1; A1 2

$$\underline{r} = \underline{\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}} \quad \textit{(o.e.)}$$

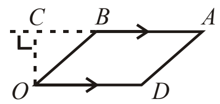
(b) $\begin{pmatrix} 3t \\ 5-3t \\ 5-6t \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} = 0$ M1M1M1

$\Rightarrow 9t - 15 + 9t - 30 + 36t = 0$ A1

<u>i.e.</u> $t = \dfrac{5}{6}$ A1 5

$$\therefore \underline{\overrightarrow{OC} = \begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}}$$

(c)



$\overrightarrow{OD} = \overrightarrow{BA} = \underline{a} - \underline{b}$ or $-\overrightarrow{AB}$g $= \underline{\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}}$ M1, A1 2

(d) $\left|\overrightarrow{OC}\right| = 2.5\sqrt{2}$; $\left|\overrightarrow{OD}\right| = 3\sqrt{1^2 + 1^2 + 2^2} = 3\sqrt{6}$ M1A1

Area = $\left|\overrightarrow{OC}\right| \times \left|\overrightarrow{OD}\right|$ (o.e.), = $7.5\sqrt{12}$ or $\underline{15\sqrt{3}\text{ or AWRT }26.0}$ M1, A1 4

[13]

12. (a) $9 - 8t = -16 + s$ M1

Attempt a correct equation

$4 + 5t = 10 + 9s$ A1

Both correct

Sub. $s = 25 - 8t \Rightarrow 5t = 6 + 225 - 72t$ M1

Solving either

$77t = 231$ or $t = 3$, $s = 1$ A1

Sub. into 'j' $2 - 3t = \alpha - 4s$ Use of 3rd equation M1

$\Rightarrow \underline{\alpha = -3}$ A1 6

(b) $\underline{\overrightarrow{OA} = \begin{pmatrix} -15 \\ -7 \\ 19 \end{pmatrix}}$ B1 1

(c) $\begin{pmatrix} -8 \\ -3 \\ 5 \end{pmatrix} . \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} = -8 + 12 + 45\ (= 49)$ Attempt correct scalar product M1

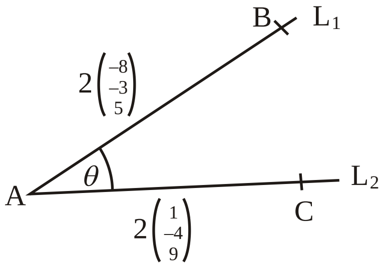
$\cos\theta = \dfrac{49}{\sqrt{8^2 + 3^2 + 5^2}\sqrt{1^2 + 4^2 + 9^2}} = \dfrac{49}{\sqrt{98}\sqrt{98}} = \dfrac{1}{2}$ M1, M1

Use of $\frac{\mathbf{a.b}}{|\mathbf{a}||\mathbf{b}|}$, $|\boldsymbol{a}|$ *or* $|\boldsymbol{b}|$

$\cos\theta = \dfrac{1}{2}$ A1

$\underline{\theta = 60°}$ (*) A1 cso 5

(d)



$14\sqrt{2} = 2 \times 7\sqrt{2} = 2\sqrt{98}$ M1

$\overrightarrow{OB} = \overrightarrow{OA} \pm 2\begin{pmatrix}-8\\-3\\5\end{pmatrix} = \begin{pmatrix}-31\\-13\\29\end{pmatrix}$ or $\begin{pmatrix}1\\-1\\9\end{pmatrix}$ M1, A1

M1: $\mathbf{a} \pm 2(\,)$*, A1: any one*

$\overrightarrow{OC} = \overrightarrow{OA} \pm 2\begin{pmatrix}1\\-4\\9\end{pmatrix} = \begin{pmatrix}-13\\-15\\37\end{pmatrix}$ or $\begin{pmatrix}-17\\1\\1\end{pmatrix}$ any correct pair A1 4

[16]

13. (a) Any two of $1 + \lambda = -2 + 2\mu$ M1

$3 + 2\lambda = 3 + \mu$

$5 - \lambda = -4 + 4\mu$

Solve simultaneous equations to obtain $\mu = 2$, **or** $\lambda = 1$ M1 A1

$\therefore$ intersect at (2, 5, 4) M1 A1

Check in the third equation or on second line B1 6

(b) $1 \times 2 + 2 \times 1 + (-1) \times 4 = 0 \;\therefore$ perpendicular M1 A1 2

(c) P is the point (3, 7, 3) [i.e. $\lambda = 2$] M1 A1

and R is the point (4, 6, 8) [i.e. $\mu = 3$]

$PQ = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$

$RQ = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21}$ M1 A1

$PR = \sqrt{27}$

Need two of these for M1

The area of the triangle = $\frac{1}{2} \times \sqrt{6} \times \sqrt{21} = \frac{3\sqrt{14}}{2}$ M1 A1 6

Or area = $\frac{1}{2} \times \sqrt{6} \times \sqrt{27}$ sin P where sin P = $\frac{\sqrt{7}}{3}$ = $\frac{3\sqrt{14}}{2}$

Or area = $\frac{1}{2} \times \sqrt{21} \times \sqrt{27}$ sin R where sin $R = \frac{\sqrt{2}}{3}$ = $\frac{3\sqrt{14}}{2}$ *(must be simplified)*

[14]

14. (a) $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$
(or any equivalent vector equation) M1 A1 2

(b) Show that $\mu = -3$ B1 1

(c) Using $\cos\theta = \dfrac{(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})}{\sqrt{(4^2 + 5^2 + 3^2)}\sqrt{(1^2 + 2^2 + 2^2)}}$ M1

$= \dfrac{20}{15\sqrt{2}} = \dfrac{4}{3\sqrt{2}}$ (ft on 4i – 5j + 3k) A1 ft A1 ft

num, denom.

$\theta = 19.5°$ (allow 19 or 20 if no wrong working is seen) A1 4

(d) Shortest distance = AC sin θ M1

AC = $\sqrt{((a-1)^2 + 2^2 + (b+3)^2}$ (= 3) M1 A1

Shortest distance = 1 unit A1 4

Alternatives

Since $X = (1 + 4\lambda, 2 - 5\lambda, -3 + 3\lambda)$
$\mathbf{CX} = (-1 + 4\lambda)\mathbf{I} + (2 - 5\lambda)\mathbf{j} + (-2 + 3\lambda)\mathbf{k}$ M1
Use Scalar product $\mathbf{CX}.(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 0$ OR differentiate $|\mathbf{CX}|$ or $|\mathbf{CX}|^2$ M1
and equate to zero,

to obtain $\lambda = 0.4$ A1
and thus $|\mathbf{CX}| = 1$ A1 4

[15]

15. (a) $\overrightarrow{AB} = 3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ B1
$\overrightarrow{AC} = (-\mathbf{i} - 2\mathbf{j} - \mathbf{k})$ B1
$= -\frac{1}{3}(3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) = -\frac{1}{3}\overrightarrow{AB}$ M1
Hence A, B and C are collinear A1 4

(b) $\cos\theta = \dfrac{(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}).(\mathbf{i} + 2\mathbf{j} + \mathbf{k})}{\sqrt{14} \times \sqrt{6}}$ M1 A1

$= \dfrac{1}{\sqrt{84}}$ A1 3

	(c)	$\overrightarrow{AE} = (-2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = -\frac{2}{3}\overrightarrow{AB}$	M1	
		so E is on l	A1	
		$\overrightarrow{OE}.\overrightarrow{OD} = (-3\mathbf{j} - \mathbf{k}).(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	M1	
		$= -3 + 3 = 0$		
		So $\overrightarrow{OE}$ and $\overrightarrow{OD}$ are perpendicular	A1	4

[11]

16.	(a)	$11 + 4\lambda = 24 + 7\mu$		
		$5 + 2\lambda = 4 + \mu$		
		$6 + 4\lambda = 13 + 5\mu$		
		Give 2 of these equations and eliminate variable to find λ or μ, find other	M1	
		$5 = 11 + 2\mu$		
		$\therefore \mu = -3; \quad \lambda = -2$	A1 A1	
		<u>Check</u> in 3rd equation	B1	4
	(b)	Use $\mu = -3$ or $\lambda = -2$ to obtain (3, 1, –2)	M1 A1	2
	(c)	$\cos\theta = \dfrac{4\times7+2\times1+4\times5}{\sqrt{4^2+2^2+4^2}\sqrt{7^2+1^2+5^2}} = \dfrac{50}{\sqrt{36}\sqrt{75}}$	M1 A1	
		$\therefore \cos\theta = \dfrac{50}{6\times5\sqrt{3}} = \dfrac{50\sqrt{3}}{90} = \dfrac{5\sqrt{3}}{9}$	M1 A1	4

[10]

1. Part (a) was fully correct in the great majority of cases but the solutions were often unnecessarily long and nearly two pages of working were not unusual. The simplest method is to equate the j components. This gives one equation in λ, leading to $\lambda = 3$, which can be substituted into the equation of l_1 to give the coordinates of C. In practice, the majority of candidates found both λ and μ and many proved that the lines were coincident at C. However the question gave the information that the lines meet at C and candidates had not been asked to prove this. This appeared to be another case where candidates answered the question that they had expected to be set, rather than the one that actually had been.

The great majority of candidates demonstrated, in part (b), that they knew how to find the angle between two vectors using a scalar product. However the use of the position vectors of A and B, instead of vectors in the directions of the lines was common. Candidates could have used either the vectors $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}5\\0\\2\end{pmatrix}$, given in the question, or $\overrightarrow{AC}$ and $\overrightarrow{BC}$. The latter was much the commoner choice but many made errors in signs. Comparatively few chose to use the cosine rule. In part (c), many continued with the position vectors they had used incorrectly in part (b) and so found the area of the triangle OAB rather than triangle ABC. The easiest method of completing part (c) was usually to use the formula Area = $\frac{1}{2}ab\sin C$ and most chose this. Attempts to use Area = $\frac{1}{2}$base × height were usually fallacious and often assumed that the triangle was isosceles. A few complicated attempts were seen which used vectors to find the coordinates of the foot of a perpendicular from a vertex to the opposite side. In principle, this is possible but, in this case, the calculations proved too difficult to carry out correctly under examination conditions.

2. The majority of candidates made good attempts at parts (a) to (e) of this question. Many, however, wasted a good deal of time in part (a), proving correctly that $\lambda = \mu = 0$ before obtaining the correct answer. When a question starts "Write down ….", then candidates should realise that no working is needed to obtain the answer. The majority of candidates knew how to use the scalar product to find the cosine of the angle and chose the correct directions for the lines. Parts (c) and (d) were well done. In part (e) , as in Q1(b), the working needed to establish the printed result was often incomplete. In showing that the printed result is correct, it is insufficient to proceed from $\sqrt{416}$ to $4\sqrt{23}$ without stating $416 = 16 \times 26$ or $4^2 \times 26$. Drawing a sketch, which many candidates seem reluctant to do, shows that part (f) can be solved by simple trigonometry, using the results of parts (b) and (e). Many made no attempt at this part and the majority of those who did opted for a method using a zero scalar product. Even correctly carried out, this is very complicated $\left(\mu = \frac{104}{19}\right)$ and it was impressive to see some fully correct solutions. Much valuable time, however, had been wasted.

3. This proved the most demanding question on the paper. Nearly all candidates could make some progress with the first three parts but, although there were many, often lengthy attempts, success with part (d) and (e) was uncommon. Part (a) was quite well answered, most finding $\overrightarrow{AB}$ or $\overrightarrow{BA}$ and writing down $\overrightarrow{OA} + \lambda\overrightarrow{AB}$, or an equivalent. An equation does, however need an equals sign and a subject and many lost the final A mark in this part by omitting the "**r** =" from, say, **r** = 8**i** + 13**j** – 2**k** + λ (2**i** + **j** – 2**k**). In part (b), those who realised that a magnitude or length was required were usually successful. In part (c), nearly all candidates knew how to evaluate a scalar product and obtain an equation in cos θ, and so gain the method marks, but the vectors chosen were not always the right ones and a few candidates gave the obtuse angle. Few made any real progress with parts (d) and (e). As has been stated in previous reports, a clear diagram helps a candidate to appraise the situation and choose a suitable method. In this case, given the earlier parts of the question, vector methods, although possible, are not really appropriate to these parts, which are best solved using elementary trigonometry and Pythagoras' theorem. Those who did attempt vector methods were often very unclear which vectors were perpendicular to each other and, even the minority who were successful, often wasted valuable time which sometimes led to poor attempts at question 8. It was particularly surprising to see quite a large number of solutions attempting to find a vector, $\overrightarrow{CX}$ say, perpendicular to l, which never used the coordinates or the position vector of C.

4. The majority of candidates identified the need for some form of dot product calculation in part (a). Taking the dot product $l_1.l_2$, was common among candidates who did not correctly proceed, while others did not make any attempt at a calculation, being unable to identify the vectors required. A number of candidates attempted to equate l_1 and l_2 at this stage. The majority of candidates, however, were able to show that q = –3.

In part (b), the majority of candidates correctly equated the **i**, **j** and **k** components of l_1 and l_2, and although some candidates made algebraic errors in solving the resulting simultaneous equations, most correctly found λ and μ. In almost all such cases the value of p and the point of intersection in part (c) was then correctly determined.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that λ = 1 at A, λ = 5 at the point of intersection and so λ = 9 at B. So substitution of λ = 9 into l_1 yields the correct position vector –7**i** + 11**j** – 19**k**. A few candidates, by deducing that the intersection point is the midpoint of A and B were able to write down $\frac{9+x}{2} = 1, \frac{3+x}{2} = 7 \text{ and } \frac{13+z}{2} = -3,$ in order to find the position vector of B.

5. In part (a), most candidates were able to set up and solve the three equations correctly. Some candidates either did not realise that they needed to perform a check for consistency or performed this check incorrectly. A surprising number of candidates did not follow the instruction in the question to find the position vector of the point of intersection. A few candidates were unable to successfully negotiate the absence of the **j** term in (–9**i** + 10*k*) for l_1 and so formed incorrect simultaneous equations.

In part (b), a majority of candidates realised that they needed to apply the dot product formula on the direction vectors of l_1 and l_2. Many of these candidates performed a correct dot product calculation but not all of them wrote a conclusion.

In part (c), a majority of candidates were able to prove that A lies on l_1, either by substituting λ = 7 into l_1 or by checking that substituting (5, 7, 3) into l_1 gave $\lambda = 7$ for all three components.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. The most common error of those who attempted this part was to write down B as –5**i** – 7**j** – 3**k**. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that λ=7 at A, λ= 3 at the point of intersection and so $\lambda = -1$ at B. So substitution of $\lambda = -1$ into l_1 yields the correct position vector –11**i** – **j** + 11**k**. A few candidates, by deducing that the intersection point is the midpoint of A and B were able to write down $\frac{x+5}{2} = -3, \frac{y+7}{2} = 3$ and $\frac{z+3}{2} = 7$, in order to find the position vector of B.

6. The vast majority of candidates could answer part (a), mostly gaining all three marks available. Many were able to find that λ=–6 and substituted this into their y-component to find the correct value of a. A few, however, used the same parameter to incorrectly find that b = -13.
In part (b), the many candidates realised that **a.b** = 0 could be used but had very little idea of what **a** and **b** represented. Some candidates could quote $x + 4y - 2z$=0, but many of them could get no further than this. On the other hand, those who could get beyond this point mostly arrived at the correct position vector of P. It was not uncommon, however, to see some candidates who had correctly written the correct equation 21λ+81 = 0to go on to solve this incorrectly to find that λ=4. There were a few correct ‘non-standard’ methods seen by examiners that gained full credit. They included some candidates who either began their solutions by solving the equation $\overrightarrow{AP} \bullet \overrightarrow{OP}$ =0 or finding the value of λ that minimises an expression for OP^2.
In part (c), there were two main approaches used by candidates in proving that the points A, P and B were collinear. The most popular approach was for candidates to find any two of the vectors $\overrightarrow{AP}, \overrightarrow{PB}$ or $\overrightarrow{AB}$ and then go on to prove that one of these vectors was a multiple of the other. The second most popular approach was for candidates to show that B lay on the line l1 when λ=– 1. Some candidates were able to state the correct ratio, but it was not uncommon to see the square of the ratio AP:BP instead of the ratio itself.

7. Most candidates found the correct values of p and q in part (a). However, inaccurate arithmetic was again seen in evaluating $q - (-1)$. Candidates who made errors were either unable to distinguish the position vector of a point on the line from a direction vector or falsely assumed that $\overrightarrow{AB} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

The method for part (b) was unfamiliar to many candidates.
Many correct solutions were seen for part (c), although a few candidates lost a mark by finding θ but never actually giving the value of cos θ as requested in the question.

Part (d) proved a good source of marks but, although $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ was accepted on this occasion, it should be noted that the question asked for coordinates not a position vector.

8. Although this seemed a very fair test of vector work, and was well answered by good candidates, it was the poorest answered question, with a high proportion of candidates unable to gain more than the odd mark in parts (b) and (c). In part (a) many candidates found one correct value, usually $a = 18$, having found $\lambda = -4$ but then used the
same parameter to find b, so that b = 8 + (–4) = 4 was common.

In part (b) the majority of candidates were clutching at straws and even some who knew that **a.b** = 0 could be used, had very little idea what **a** and **b** represented.

A generous method mark in part (c) was often as much as many candidates gained after part (a).

$= 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

The method for part (b) was unfamiliar to many candidates.
Many correct
solutionσ ωερε σεεν φορ παρτ (χ), αλτηουγη α φεω χανδιδατεσ λοστ α μαρκ βψ φινδινγ ⎝ βυτ νεϖερ αχτυαλλψ γιϖινγ τηε ϖαλυε οφ χοσ ⎛ as requested in the question.
Part (d) proved a good source of marks but, although $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ was accepted on this occasion, it should be noted that the question asked for coordinates not a position vector.

9 More success was achieved by those writing their vectors as column vectors rather than by those whose solution remained in terms of i, j and k. The common false statement $\begin{pmatrix} c \\ d \\ 21 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ was clearly wrong as the k component did not match. Those who instead wrote $\begin{pmatrix} c \\ d \\ 21 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$, and found λ =2 then easily obtained c = 4 and d = 7.

(b) A variation of this question has been set several times over the past few years, but there were still a large proportion of candidates who could not get beyond a.b=0, or beyond $2x + y + 5z = 0$.

(c) Even those who had found P frequently did not use the simple formula for the area, preferring instead ½*ab*sinC, but not specifying their *a*, *b*, or C. Again the use of 2i + j + 5k for the vector AB led to incorrect answers. Some students did not give their answers to the requested 3 s.f.

10. This vector question proved an excellent source of marks for candidates and fully correct solutions to both parts (a) and (b) were common, although many spent time fruitlessly trying to solve simultaneous equations before realising that the **k** component gave λ directly. Part (c) was also generally well done but there were candidates who clearly expected $\overrightarrow{AB}$ and $\overrightarrow{BC}$ to be equal, or at least parallel, and gave up when they were not. Part (d) proved difficult even for the strongest candidates. Given a well-drawn diagram, the answer can be written down (the examiners do not insist upon working in such cases) and the idea is one that has appeared on GCSE papers but it remains inaccessible to many candidates and it was not unusual to see 2 or more pages of complex algebra, involving distance formulae which the candidate was unable to solve.

11. Those well versed in vectors produced excellent solutions with a clear understanding at all stages. Most candidates attempted only parts (a) and (c). It was disappointing so many were unaware that an equation requires an equal sign; statements of the form $5\mathbf{i} + 5\mathbf{j} + \lambda(3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$, omitting “r =” were frequently seen.

In part (b) the majority of candidates did not appreciate $\overrightarrow{OC}.\overrightarrow{AB} = 0$ with $\overrightarrow{OC}$ being the vector equation found in part (a) and $\overrightarrow{AB}$ the vector direction of *L*. All too often candidates set up $\overrightarrow{OC} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ to be perpendicular to the vector equation of the line not the direction of the line. At this stage most moved on, few attempting part (d). Of these, many treated the parallelogram as a rectangle. Successful candidates who did not realise *OC* was perpendicular to *AB* had far more work to do in finding an appropriate angle but nevertheless often made good progress towards a correct area.

12. The examiners were impressed at the level of competence shown in parts (a), (b) and (c). The vast majority scored high marks here with only the occasional student making an arithmetic slip. Some candidates did not use the direction vectors of the lines in part (c) and a few did not appreciate that this was a "show that" question and missed out the last couple of stages of working. Part (d) was hard and most candidates had little idea of how to proceed beyond drawing a correct diagram. The very best spotted that $14\sqrt{2} = 2\sqrt{98}$ and then realised that $\overrightarrow{AB} = 2(-8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ and $\overrightarrow{AC} = 2(\mathbf{i} - 4\mathbf{j} + 9\mathbf{k})$. The most popular successful approach to this question involved finding parametric equations for $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and then solving equations such as $(24-8t)^2 + (9-3t)^2 + (5t-15)^2 = 392$. This was much longer and required care in selecting the correct pair of solutions, but a number of fully correct solutions were seen.

13. This vector question was answered well and many even included the check in part (a) and the statement that a scalar product of zero implied perpendicular lines in part (b). More errors were seen in part (c), as candidates became short of time. However a large number understood the required methods and there were many completely correct responses.

14. The vector equation of a line was understood well by most candidates but there are still a number of answers which are not equations. Candidates penalise themselves by not writing **r** = ….., before their vector expression.

Parts (b) and (c) were answered well with most realising that only the direction vectors should be used in the scalar product to give the angle between two lines. Part (d) was found difficult by many candidates, and proved to be a good discriminator. A good diagram was useful and the neatest method was to use AC sin θ.

15. No Report available for this question.

16. No Report available for this question.