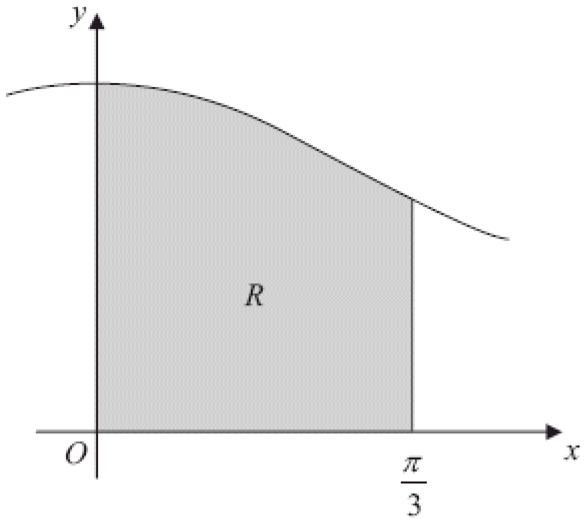
1.



The diagram above shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in the diagram, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

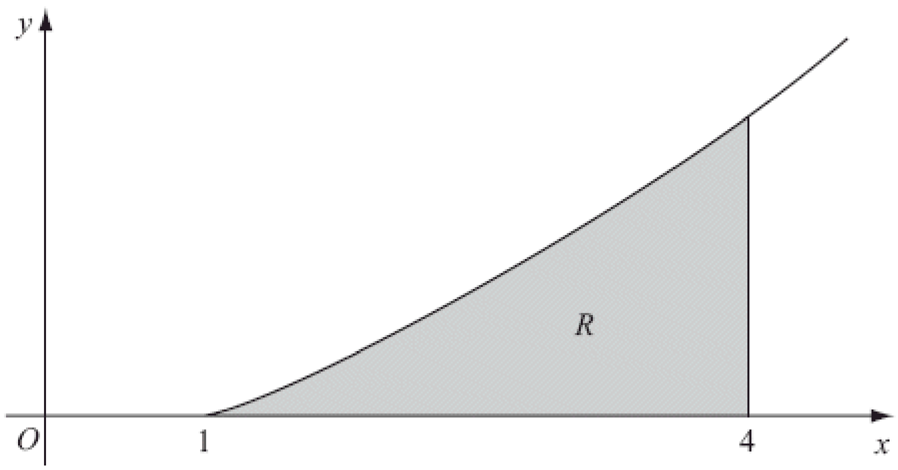
(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. to find an estimate of the area of R. Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}, x = \frac{\pi}{6}, x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)
(Total 8 marks)

2.



The diagram above shows a sketch of the curve with equation $y = x \ln x, x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

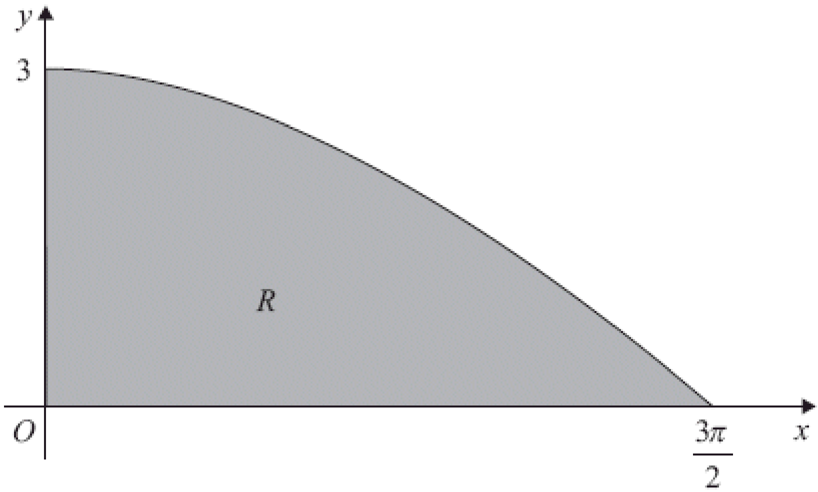
(c) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

(Total 13 marks

3.



The diagram above shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right), 0 \le x \le \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

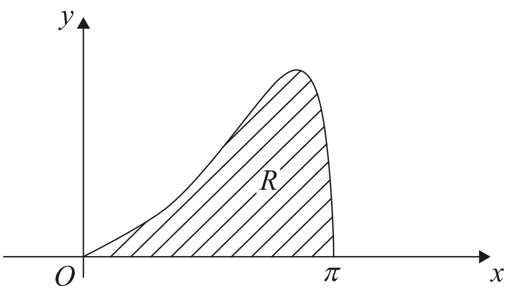
(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(c) Use integration to find the exact area of R.

(3)
(Total 8 marks)

4.



The curve shown in the diagram above has equation $y = e^x \sqrt{(\sin x)}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in the diagram.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

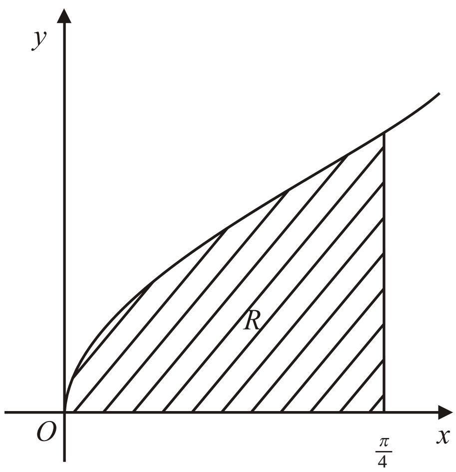
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimated for the area of the region R. Give your answers to 4 decimal places.

(4)
(Total 6 marks)

5.



The diagram above shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in the diagram.

(a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(c) Use integration to find an exact value for the volume of the solid generated.

(4)
Total 11 marks)

6.

$$I = \int_0^5 e^{\sqrt{(3x+1)}}\,dx.$$

(a) Given that $y = e^{\sqrt{(3x+1)}}$, complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.

(3)

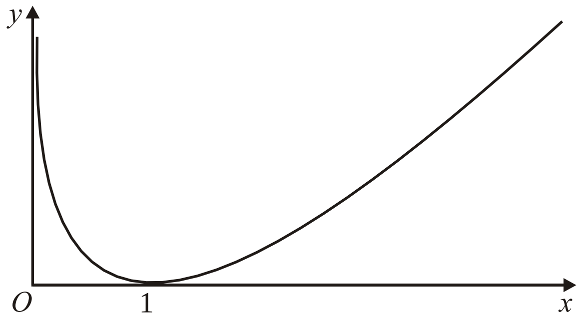
(c) Use the substitution $t = \sqrt{(3x + 1)}$ to show that I may be expressed as $\int_a^b kte^t\,dt$, giving the values of a, b and k.

(5)

(d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)
(Total 15 marks)

7.



The figure above shows a sketch of the curve with equation $y = (x - 1) \ln x$, $x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		ln 2		2ln 3

(1)

Given that $I = \int_1^3 (x-1)\ln x \, dx$

(b) use the trapezium rule

(i) with values of y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

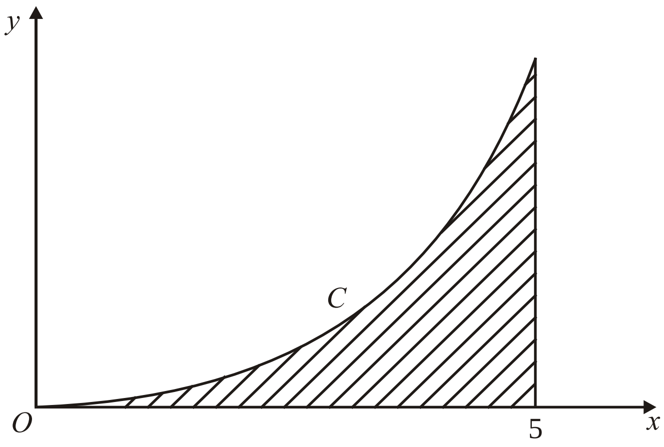
(c) Explain, with reference to the figure above, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x-1)\ln x \, dx$ is $\frac{3}{2}\ln 3$.

(6)
(Total 13 marks)

8.



The figure above shows part of the curve C with equation $y = e^{0.06x^2} - 1$. The shaded region bounded by C, the x-axis and the line with equation $x = 5$ represents the cross-section of a skateboarding ramp. The units on each axis are in metres.

(a) Complete the table, showing the height y of the ramp. Give the values of y to 3 decimal places.

x	0	1	2	3	4	5
y	0	0.062		0.716		

(3)

(b) Use the trapezium rule, with all the values from your table, to estimate the area of cross-section of the ramp.

(4)

The ramp is made of concrete and is 6 m wide.

(c) Calculate an estimate for the volume of concrete required to make the ramp.

(1)

(d) A builder makes the amount of concrete calculated in part (c). State, with a reason, whether or not there is enough concrete to make the ramp.

(2)
(Total 10 marks)

9. (a) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(b) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, dx$. Show all the steps of your working, and give your answer to 4 decimal places.

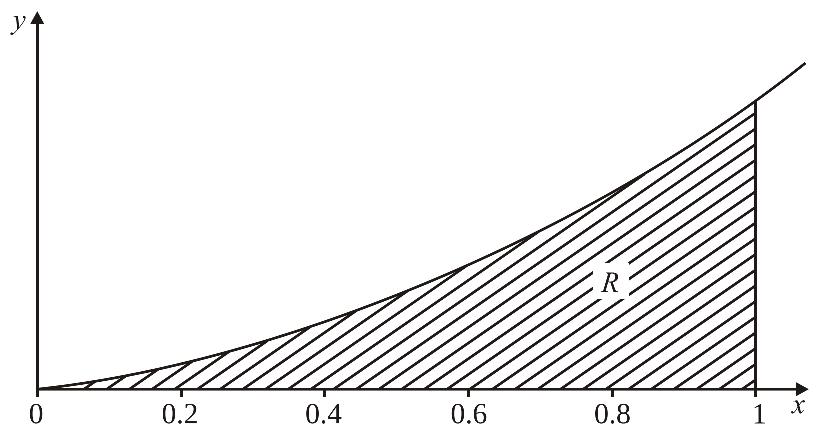
(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, dx$ is $\ln(1 + \sqrt{2})$.

(c) Calculate the % error in using the estimate you obtained in part (b).

(2)
(Total 7 marks)

10.



The diagram shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in the diagram.

(a) Use integration to find the exact value of the area for R.

(5)

(b) Complete the table with the values of y corresponding to x = 0.4 and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

11. The curve C with equation $y = k + \ln 2x$, where k is a constant, crosses the x-axis at the point $A\left(\frac{1}{2e}, 0\right)$.

(a) Show that $k = 1$.

(2)

(b) Show that an equation of the tangent to C at A is $y = 2ex - 1$.

(4)

(c) Complete the table below, giving your answers to 3 significant figures.

x	1	1.5	2	2.5	3
1 + ln 2x		2.10		2.61	2.79

(2)

(d) Use the trapezium rule, with four equal intervals, to estimate the value of

$$\int_1^3 (1 + \ln 2x) \ \ \mathrm{d}x$$
.

(4)
(Total 12 marks)

12. $$f(x) = x + \frac{e^x}{5}, \quad x \in \mathbb{R}.$$

(a) Find $f'(x)$.

(2)

The curve C, with equation $y = f(x)$, crosses the y-axis at the point A.

(b) Find an equation for the tangent to C at A.

(3)

(c) Complete the table, giving the values of $\sqrt{\left(x + \frac{e^x}{5}\right)}$ to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x + \frac{e^x}{5}\right)}$	0.45	0.91			

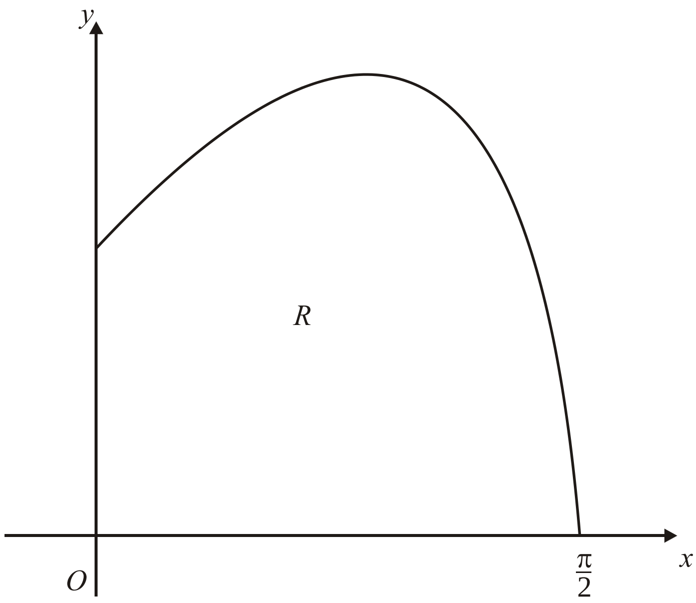
(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x + \frac{e^x}{5}\right)}\,dx$$
.

(4)
(Total 11 marks)

13.



The diagram above shows part of the curve with equation

$$y = e^x \cos x, \quad 0 \leq x \leq \frac{\pi}{2}.$$

The finite region R is bounded by the curve and the coordinate axes.

(a) Calculate, to 2 decimal places, the y-coordinates of the points on the curve where $x = 0$, $\frac{\pi}{6}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

(3)

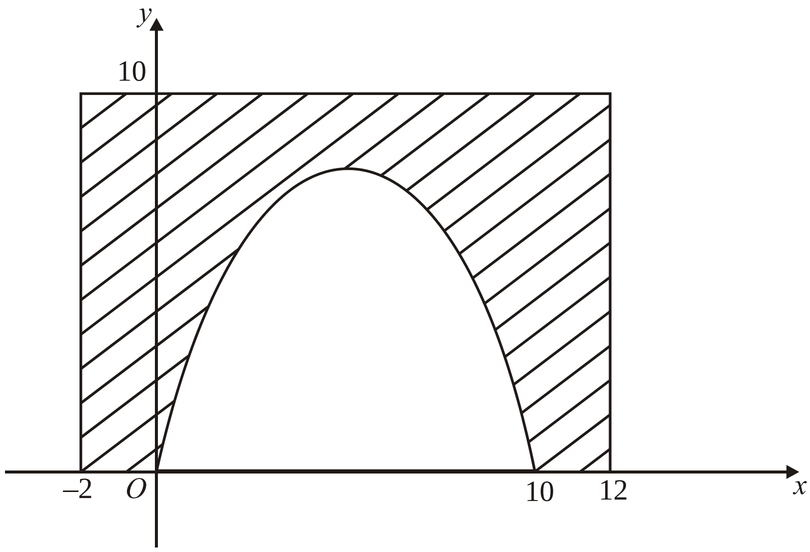
(b) Using the trapezium rule and all the values calculated in part (a), find an approximation for the area of R.

(4)

(c) State, with a reason, whether your approximation underestimates or overestimates the area of R.

(2)
(Total 9 marks)

14.



The diagram above shows the cross-section of a road tunnel and its concrete surround. The curved section of the tunnel is modelled by the curve with equation $y = 8\sqrt{\left(\sin \frac{\pi x}{10}\right)}$, in the interval $0 \le x \le 10$. The concrete surround is represented by the shaded area bounded by the curve, the x-axis and the lines $x = -2$, $x = 12$ and $y = 10$. The units on both axes are metres.

(a) Using this model, copy and complete the table below, giving the values of y to 2 decimal places.

x	0	2	4	6	8	10
y	0	6.13				0

(2)

The area of the cross-section of the tunnel is given by $\int_0^{10} y \; \mathrm{d}x$.

(b) Estimate this area, using the trapezium rule with all the values from your table.

(4)

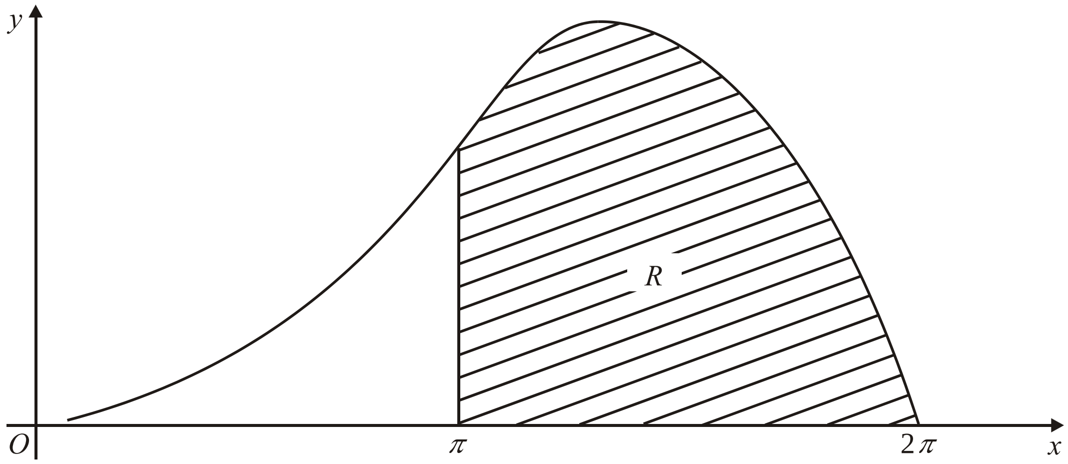
(c) Deduce an estimate of the cross-sectional area of the concrete surround.

(1)

(d) State, with a reason, whether your answer in part (c) over-estimates or under-estimates the true value.

(2)
(Total 9 marks)

15.



The diagram above shows the curve with equation

$$y = x^2 \sin\left(\tfrac{1}{2}x\right), \qquad 0 < x \leq 2\pi.$$

The finite region R bounded by the line $x = \pi$, the x-axis, and the curve is shown shaded in Fig 1.

(a) Find the exact value of the area of R, by integration. Give your answer in terms of π. **(7)**

The table shows corresponding values of x and y.

x	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y	9.8696	14.247	15.702	G	0

(b) Find the value of G. **(1)**

(c) Use the trapezium rule with values of $x^2 \sin\left(\tfrac{1}{2}x\right)$

(i) at $x = \pi$, $x = \frac{3\pi}{2}$ and $x = 2\pi$ to find an approximate value for the area R, giving your answer to 4 significant figures,

(ii) at $x = \pi$, $x = \frac{5\pi}{4}$, $x = \frac{3\pi}{2}$, $x = \frac{7\pi}{4}$ and $x = 2\pi$ to find an improved approximation for the area R, giving your answer to 4 significant figures. **(5)**

(Total 13 marks)

1.	(a)	$y\left(\frac{\pi}{6}\right)\approx 1.2247,\ y\left(\frac{\pi}{4}\right)=1.1180$	accept awrt 4 d.p.	B1 B1	2
	(b) (i)	$I\approx\left(\frac{\pi}{12}\right)(1.3229+2\times 1.2247+1)$	B1 for $\frac{\pi}{12}$	B1 M1	
		≈ 1.249	cao	A1	
	(ii)	$I\approx\left(\frac{\pi}{24}\right)(1.3229+2\times(1.2973+1.2247+1.1180)+1)$			
			B1 for $\frac{\pi}{24}$	B1 M1	
		≈ 1.257	cao	A1	6

[8]

2.	(a)	1.386, 2.291	awrt 1.386, 2.291	B1 B1	2
	(b)	$A\approx\frac{1}{2}\times 0.5(\ldots)$		B1	
		= ... (0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)		M1	
		= 0.25(0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)	ft their (a)	A1ft	
		=0.25×29.477 ... ≈7.37	cao	A1	4
	(c) (i)	$\int x\ln x\,\mathrm{d}x=\frac{x^2}{2}\ln x-\int\frac{x^2}{2}\times\frac{1}{x}\mathrm{d}x$		M1 A1	
		$=\frac{x^2}{2}\ln x-\int\frac{x}{2}\mathrm{d}x$			
		$=\frac{x^2}{2}\ln x-\frac{x^2}{4}(+C)$		M1 A1	
	(ii)	$\left[\frac{x^2}{2}\ln x-\frac{x^2}{4}\right]_1^4=(8\ln 4-4)-\left(-\frac{1}{4}\right)$		M1	
		$=8\ln 4-\frac{15}{4}$			

$=8(2\ln 2) - \frac{15}{4}$	ln 4 = 2ln 2 seen or implied	M1	
$=\frac{1}{4}(64\ln 2 - 15)$	$a = 64, b = -15$	A1	7

[13]

3. (a) 1.14805 — awrt 1.14805 — B1 — 1

(b) $A \approx \frac{1}{2} \times \frac{3\pi}{8}(...)$ — B1

$= ... (3 + 2(2.77164 + 2.12132 + 1.14805) + 0)$ — 0 can be implied — M1

$= \frac{3\pi}{16}(3 + 2(2.77164 + 2.12132 + 1.14805))$ — ft their (a) — A1ft

$\frac{3\pi}{16} \times 15.08202 ... = 8.884$ — cao — A1 — 4

(c) $\int 3\cos\left(\frac{x}{3}\right) dx = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ — M1 A1

$= 9\sin\left(\frac{x}{3}\right)$

$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$ — cao — A1 — 3

[8]

4. (a)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1.844321332…	4.810477381…	8.87207	0

awrt 1.84432 — B1

awrt 4.81048 or 4.81047 — B1 — 2

(b) **Way 1**

Area $\approx \frac{1}{2} \times \frac{\pi}{4}; \times \underline{\{0^* + 2(1.84432 + 4.81048 + 8.87207) + 0^*\}}$

*0 can be implied

$= \frac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)

Outside brackets awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79 B1

$\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$

For structure of trapezium rule {..........................}; M1ft

Correct expression inside brackets which all must be multiplied by their "outside constant". A1ft

12.1948 A1cao 4

(b) ***Aliter***
Way 2

Area ≈ $\frac{\pi}{4} \times \left\{ \frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2} \right\}$

which is equivalent to:

Area ≈ $\frac{1}{2} \times \frac{\pi}{4}; \times \underline{\{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}}$

$= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)

$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets. B1

One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. M1ft

Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out. A1ft

12.1948 A1 cao 4

Note an expression like Area ≈ $\frac{1}{2} \times \frac{\pi}{4} 2(1.84432 + 4.81048 + 8.87207)$ would score B1M1A0A0

[6]

5. (a)

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0	0.445995927...	0.643594252...	0.817421946...	1

Enter marks into ePEN in the correct order. 3

B1 0.446 or awrt 0.44600

B1 awrt 0.64359

B1 awrt 0.81742

For $x = \frac{\pi}{16}$ writing 0.4459959... then 0.45600 gains B1 for awrt 0.44600 even though 0.45600 is incorrect.

(b) **Way 1**

Area ≈ $\frac{1}{2} \times \frac{\pi}{16}$; × <u>{0* + 2(0.44600 + 0.64359 + 0.81742) + 1}</u>

(*) 0 can be implied

= $\frac{\pi}{32}$ × 4.81402... = 0.472615308... = <u>0.4726</u> (4dp) 4

B1 Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$

M1ft <u>For structure of trapezium rule {...............}</u>;

A1ft Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.

A1 cao for seeing <u>0.4726</u>

Aliter
Way 2

$$\text{Area} \approx \frac{\pi}{16} \times \left\{ \frac{0 + 0.44600}{2} + \frac{0.44600 + 0.64359}{2} + \frac{0.64359 + 0.81742}{2} + \frac{0.81742 + 1}{2} \right\}$$

which is equivalent to:

Area ≈ $\frac{1}{2} \times \frac{\pi}{16}$; × <u>{0 + 2(0.44600 + 0.64359 + 0.81742) + 1}</u>

= $\frac{\pi}{16}$ × 2.40701... = 0.472615308... = <u>0.4726</u>

B1 $\frac{\pi}{16}$ and a divisor of 2 on all terms inside brackets.

M1ft One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.

<u>A1</u>ft Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.

A1 cao <u>0.4726</u>

Area = $\frac{1}{2} \times \frac{\pi}{20}$ × {0 + 2(0.44600 + 0.64359 + 0.81742) + 1} = 0.3781, gains B0M1A1A0

You can follow though a candidate's values from part (a) to award M1ft, A1ft

Beware: a candidate can also add up individual trapezia in this way:

$$\text{Area} \approx \frac{1}{2}\times\frac{\pi}{16}(\underline{0+0.44600}) + \frac{1}{2}.\frac{\pi}{6}(\underline{0.44600+0.64359}) + \frac{1}{2}.\frac{\pi}{16}(\underline{0.64359+0.81742}) + \frac{1}{2}.\frac{\pi}{16}(\underline{0.81742+1})$$

(c) Volume = $(\pi)\underline{\int_0^{\frac{\pi}{4}} (\sqrt{\tan x})^2 dx} = (\pi)\underline{\int_0^{\frac{\pi}{4}} \tan x dx}$

$$= (\pi)[\underline{\ln \sec x}]_0^{\frac{\pi}{4}} \text{ or } = (\pi)[\underline{-\ln \cos x}]_0^{\frac{\pi}{4}}$$

$$= (\pi)[(\ln \sec \frac{\pi}{4}) - (\ln \sec 0)]$$

$$\text{or } = (\pi)[(-\ln \cos \frac{\pi}{4}) - (\ln \cos 0)]$$

$$= \pi\left[\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right) - \ln\left(\frac{1}{1}\right)\right] = \pi[\ln\sqrt{2} - \ln 1]$$

$$\text{or } = \pi\left[-\ln\left(\frac{1}{\sqrt{2}}\right) - \ln(1)\right]$$ 4

$$= \underline{\pi \ln\sqrt{2}} \text{ or } \underline{\pi \ln \frac{2}{\sqrt{2}}} \text{ or } \underline{\frac{1}{2}\pi \ln 2} \text{ or } \underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)} \text{ or } \underline{\frac{\pi}{2}\ln\left(\frac{1}{2}\right)}$$

M1 $\underline{\int (\sqrt{\tan x})^2 dx}$ or $\underline{\int \tan x dx}$

Can be implied. Ignore limits and (π)

A1 $\tan x \to$ <u>ln sec x</u> or $\tan x \to$ <u>–ln cos x</u>

dM1 The correct use of limits on a function other than tan x; ie $x = \frac{\pi}{4}$ 'minus' $x = 0$. ln(sec 0) = 0 may be implied. Ignore (π)

A1 aef $\underline{\pi \ln\sqrt{2}}$ or $\underline{\pi \ln \frac{2}{\sqrt{2}}}$ or $\underline{\frac{1}{2}\pi \ln 2}$ or $\underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)}$ or $\underline{\frac{\pi}{2}\ln\left(\frac{1}{2}\right)}$

must be exact.

If a candidate gives the correct exact answer and then writes 1.088779..., then such a candidate can be awarded A1 (aef). The subsequent working would then be ignored. (isw)

Beware: the factor of π is not needed for the first three marks.

[11]

6. (a)

x	0	1	2	3	4	5
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4
or y	2.71828…	7.38906…	14.09403…	23.62434…	36.80197…	54.59815…

Either $e^{\sqrt{7}}, e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's)

At least two correct B1

All three correct B1 2

(b) $I \approx \frac{1}{2} \times 1; \times \underline{\left\{e^1 + 2(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}) + e^4\right\}}$

$= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6}$ (4sf)

Outside brackets $\frac{1}{2} \times 1$ B1;

For structure of trapezium rule {...............}. M1ft

110.6 A1 **cao** 3

Beware: Candidates can add up the individual trapezia:

$I \approx \frac{1}{2}.1(\underline{e^1 + e^2}) + \frac{1}{2}.1(\underline{e^2 + e^{\sqrt{7}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{13}} + e^4})$

(c) $t = (3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$

... or $t^2 = 3x + 1 \Rightarrow \underline{2t\frac{dt}{dx} = 3}$

so $\frac{dt}{dx} = \frac{3}{2.(3x+1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$

$\therefore I = \int e^{\sqrt{(3x+1)}} dx = \int e^t \frac{dx}{dt}.dt = \int e^t . \frac{2t}{3}.dt$

$\therefore I = \underline{\int \frac{2}{3} te^t dt}$

change limits:

When $x = 0$, $t = 1$ & when $x = 5$, $t = 4$

Hence $I = \int_1^4 \frac{2}{3} te^t dt$; where $a = 1$, $b = 4$, $k = \frac{2}{3}$

$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{\mathrm{d}t}{\mathrm{d}x} = A$ M1

$\frac{3}{2}(3x+1)^{\frac{1}{2}}$ or $2\mathrm{t}\frac{\mathrm{d}t}{\mathrm{d}x} = 3$ A1

Candidate obtains either $\frac{\mathrm{d}t}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of t …

… and moves on to substitute this into I to convert an integral wrt x to an integral wrt t. dM1

$\int \frac{2}{3} te^t$ A1

changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$ B1 5

(d) $\left\{\begin{array}{l} u = t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = 1 \\ \frac{\mathrm{d}v}{\mathrm{dt}} = \mathrm{e}^t \Rightarrow v = \mathrm{e}^t \end{array}\right\}$

$k\int t\mathrm{e}^t \,\mathrm{d}t = k(t\mathrm{e}^t - \int \mathrm{e}^t.1\mathrm{d}t)$

$= k(\underline{t\mathrm{e}^t - \mathrm{e}^t}) + c$

$\therefore \int_1^4 \frac{2}{3} t\mathrm{e}^t \mathrm{d}t = \frac{2}{3}\left\{\left(4\mathrm{e}^4 - \mathrm{e}^4\right) - \left(\mathrm{e}^1 - \mathrm{e}^1\right)\right\}$

$= \frac{2}{3}(3\mathrm{e}^4) = \underline{2\mathrm{e}^4} = 109.1963...$

Let k be any constant for the first three marks of this part.

Use of 'integration by parts' formula in the correct direction. M1

Correct expression with a constant factor k. A1

Correct integration with/without a constant factor k A1

Substitutes their changed limits into the integrand and subtracts oe. dM1 oe.

either $2\mathrm{e}^4$ or awrt 109.2 A1 5

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.

[15]

7. (a)

x	1	1.5	2	2.5	3
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3
or y	0	0.2027325541 ...	ln2	1.374436098 ...	2 ln 3

Either 0.5 ln 1.5 and 1.5 ln 2.5 B1
or awrt 0.20 and 1.37
(or mixture of decimals and ln's)

(b) (ii) $I_1 \approx \frac{1}{2} \times 1 \times \left\{\underline{0 + 2(\ln 2) + 2\ln 3}\right\}$ M1;

For structure of trapezium rule {…………};

$\frac{1}{2} \times$ 3.583518938... = 1.791759... = 1.792 (4sf) A1 cao

(ii) $I_2 \approx \frac{1}{2} \times 0.5; \times \{\underline{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3}\}$

Outside brackets $\frac{1}{2} \times 0.5$ B1

For structure of trapezium rule {…………..}; M1 ft

$= \frac{1}{4} \times$ 6.737856242....=1.684464 A1 5

awrt 1.684

(c) With increasing ordinates, the line segments at the top of the trapezia are closer to the curve. B1 1

Reason or an appropriate diagram elaborating the correct reason.

(d) $\left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow v = \frac{x^2}{2} - x \end{array}\right\}$ M1

Use of 'integration by parts' formula in the correct direction

$$\text{I} = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$$ A1

Correct expression

$$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int \left(\frac{x}{2} - x\right) dx}$$

An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...

$$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)}(+c)$$

... integrate; M1

<u>correct integration</u> A1

$$\therefore \text{I} = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$$

$$= \left(\tfrac{3}{2}\ln 3 - \tfrac{9}{4} + 3\right) - \left(-\tfrac{1}{2}\ln 1 - \tfrac{1}{4} + 1\right)$$ ddM1

Substitutes limits of 3 and 1 and subtracts.

$$= \tfrac{3}{2}\ln 3 + \tfrac{3}{4} + 0 - \tfrac{3}{4} = \underline{\tfrac{3}{2}\ln 3}$$ **AG** A1 cso 6

Aliter Way 2

(d) $\int (x-1)\ln x\, dx = \int x \ln x\, dx - \int \ln x\, dx$

$$\int x \ln x = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right) dx \text{ J}$$ M1

Correct application of 'by parts'

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4}(+c)$$ A1

Correct integration

$\int \ln x \, dx = x\ln x - \int x.\left(\frac{1}{x}\right) dx$ M1

Correct application of 'by parts''

$= x\ln x - x \, (+c)$ A1

Correct integration

$\therefore \int_1^3 (x-1)\ln x \, dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ **AG**

Substitutes limits of 3 and 1 into both integrands and subtracts. dd M1

$\frac{3}{2}\ln 3$ A1 cso 6

Aliter Way 3

(d) $\left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x-1 & \Rightarrow v = \frac{(x-1)^2}{2} \end{array}\right\}$ M1

Use of 'integration by parts' formula in the correct direction

$\text{I} = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x} dx$ A1

Correct expression

$= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2 - 2x + 1}{2x} dx$

$= \frac{(x-1)^2}{2}\ln x - \int \left(\frac{1}{2}x - 1 + \frac{1}{2x}\right) dx$

Candidate multiplies out numerator to obtain three terms...
... multiplies at least one term through by $\frac{1}{x}$ and then attempts to ...
... integrate the result; M1
correct integration A1

$= \frac{(x-1)^2}{2}\ln x - \left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right) (+c)$

$\therefore \text{I} = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$

$= (2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3) - (0 - \frac{1}{4} + 1 - 0)$ ddM1

Substitutes limits of 3 and 1 and subtracts.

$= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \ = \underline{\frac{3}{2}\ln 3}$ **AG** A1 cso 6

Aliter Way 4

(d) By substitution

$u = \ln x \qquad \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\mathrm{I} - \int (e^u - 1).ue^u \, \mathrm{d}u$

Correct expression

$= \int u\,(e^{2u} - e)\,\mathrm{d}u$ M1

Use of 'integration by parts' formula in the correct direction

$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)}\mathrm{d}x$ A1

Correct expression

$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)}(+c)$

Attempt to <u>integrate</u>; M1;

<u>correct integration</u> A1

$\therefore l = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$

$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - (0 - 0 - \frac{1}{4} + 1)$ ddM1

Substitutes limits of ln3 and ln1 and subtracts.

$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \; = \underline{\frac{3}{2}\ln 3}$ **AG** A1 cso 6

$\frac{3}{2}\ln 3$

[13]

8. (a)

0	1	2	3	4	5
0	0.062	0.271	0.716	1.612	3.482

B1, B1, B1 3

(b) $1 \times \frac{1}{2}(0 + 3.482 + 2) \times (0.062 + 0.271 + 0.716 + 1.612)$ B1, M1, A1ft

$= 4.402 \text{ m}^2$ A1 4

(c) $6 \times 4.402 = 26.4\text{m}^3$ B1ft 1

trapezium rule overestimates $\therefore$ will be enough B1 B1 2

[10]

9. (a)

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$		
y	1	**1.01959**	**1.08239**	1.20269	**1.41421**	M1 A1	2

M1 for one correct, A1 for all correct

(b) Integral = $\frac{1}{2} \times \frac{\pi}{16} \times \{1 + 1.4142 + 2(1.01959 + ...+ 1.20269)\}$ M1 A1ft

$\left(= \frac{\pi}{2} \times 9.02355\right)$ $= 0.8859$ A1 cao 3

(c) Percentage error = $\frac{approx - 0.88137}{0.88137} \times 100 = 0.51\%$ M1 A1 2 7

(allow 0.5% to 0.54% for A1)

M1 gained for (±) $\frac{approx - \text{1n}\,(1+\sqrt{2})}{\text{1n}(1+\sqrt{2})}$

[7]

10. (a) $\int x\text{e}^{2x}\,\text{d}x = \frac{1}{2}x\text{e}^{2x} - \frac{1}{2}\int \text{e}^{2x}\text{d}x$ M1 A1

Attempting parts in the right direction

$= \frac{1}{2}x\text{e}^{2x} - \frac{1}{4}\text{e}^{2x}$ A1

$\left[\frac{1}{2}x\text{e}^{2x} - \frac{1}{4}\text{e}^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}\text{e}^2$ M1 A1 5

(b) $x = 0.4 \Rightarrow y \approx 0.89022$

$x = 0.8 \Rightarrow y \approx 3.96243$ B1 1

Both are required to 5.d.p.

(c) $I \approx \frac{1}{2} \times 0.2 \times [\ldots]$ B1

$\approx \ldots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$ M1 A1ft

ft their answers to (b)

		$\approx 0.1 \times 21.67522$		
		≈ 2.168	cao A1	4
		Note: $\frac{1}{4} + \frac{1}{4}e^2 \approx 2.097\ldots$		

[10]

11.	(a)	$0 = k + \ln 2\left(\frac{1}{2e}\right) \Rightarrow 0 = k - 1 \Rightarrow k = 1$ (*)	M1 A1	2
		(Allow also substituting k = 1and x = $\frac{1}{2e}$ *into equation and showing y = 0 and substituting k = 1 and y = 0 and showing x =* $\frac{1}{2e}$*.)*		
	(b)	$\frac{dy}{dx} = \frac{1}{x}$	B1	
		At A gradient of tangent is $\frac{1}{1/2e} = 2e$	M1	
		Equations of tangent: $y = 2e\left(x - \frac{1}{2e}\right)$	M1	
		Simplifying to $y = 2ex - 1$ (*) cso	A1	4
	(c)	$y_1 = 1.69$, $y_2 = 2.39$	B1, B1	2
	(d)	$\int_1^3 (1 + \ln 2x)dx \approx \frac{1}{2} \times \frac{1}{2} \times (\ldots)$	B1	
		$\approx \ldots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c)	M1 A1ft	
		≈ 4.7	A1	4
		accept 4.67		

[12]

12.	(a)	Differentiating; $\mathbf{f'(x) = 1 + \frac{e^x}{5}}$	M1;A1	2
	(b)	A: $\left(0, \frac{1}{5}\right)$	B1	
		Attempt at $y - f(0) = f'(0)x$;	M1	
		$\boldsymbol{y - \frac{1}{5} = \frac{6}{5}x}$ or equivalent "one line" 3 termed equation	A1 ft	3

	(c)	**1.24, 1.55, 1.86**	B2(1,0)	2
	(d)	Estimate = $\frac{0.5}{2}$; (×) [(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)]	B1 M1 A1 ft	
		= **2.4275** $\left(\begin{matrix}\mathbf{2.428} \\ \mathbf{2.429}\end{matrix}, \quad \mathbf{2.43}\right)$	A1	4

[11]

13	(a)	x 0 $\frac{\pi}{6}$ $\frac{\pi}{3}$ $\frac{\pi}{2}$	1,0	B1	
		y 1 1.46 1.42 0	1.46, 1.42	B1 B1	3
		NB. Not giving 2 d.p. loses a maximum of one mark			
	(b)	$I \approx \frac{1}{2}\left(\frac{\pi}{6}\right)\ldots$		B1	
		$\approx \ldots(1 + 2(1.46 + 1.42) + 0)$		M1 A1 ft	
		ft their ys			
		≈ 1.8		A1	4
		accept 1.77			
	(c)	underestimates		B1	
		diagram or explanation		B1	2
		NB. Exact answer is $\frac{1}{2}\left(e^{\frac{\pi}{2}} - 1\right) \approx 1.905\ldots$			

[9]

14 (a)

Distance from one side (m)	0	2	4	6	8	10
Height (m)	0	6.13	**7.80**	**7.80**	**6.13**	0

		"y" = 7.80 when "x" = 4 or 6	B1	
		Symmetry	B1 ft	2
	(b)	Estimate area = $\frac{2}{2}$[0 + 2(6.13 + 7.80 + 7.80 + 6.13)]	B1 M1 A1 ft	
		= 55.7 m^2	A1	4
	(c)	140 – (b) = 84.3 m^2	A1 ft	1
	(d)	Over-estimate;	B1	
		reason, e.g. area under curve is under-estimate (due to curvature)	B1	2

[9]

15. (a) $R = \int_{\pi}^{2\pi} x^2 \sin\left(\frac{1}{2}x\right) dx = -2x^2 \cos\left(\frac{1}{2}x\right) + \int 4x \cos\left(\frac{1}{2}x\right) dx$ M1 A1

$= -2x^2 \cos\left(\frac{1}{2}x\right) + 8x \sin\left(\frac{1}{2}x\right) - \int 8\sin\left(\tfrac{1}{2}x\right)$ M1 A1

$= -2x^2 \cos\left(\frac{1}{2}x\right) + 8x \sin\left(\frac{1}{2}x\right) + 16\cos\left(\tfrac{1}{2}x\right)$ A1

Use limits to obtain $[8\pi^2 - 16] - [8\pi]$ M1 A1 7

(b) Requires 11.567 B1 1

(c) (i) Area = $\frac{\pi}{4}$, [9.8696 + 0 + 2 × 15.702]

(B1 for $\frac{\pi}{4}$ *in (i) or* $\frac{\pi}{8}$ *in (ii))* B1, M1

= 32.42 A1

(ii) Area = $\frac{\pi}{8}$ [9.8696 + 0 + 2(14.247 + 15.702 + 11.567)] M1

= 36.48 A1 5

[13]

1. This question was a good starting question and over 60% of the candidates gained full marks. A few candidates used a wrong angle mode when calculating the values in part (a). In part (b), the majority knew the structure of the trapezium rule. The most common errors were to miscalculate the interval width using, for example, $\frac{\pi}{9}$ and $\frac{\pi}{15}$ in place of $\frac{\pi}{12}$ and $\frac{\pi}{24}$. Some were unable to adapt to the situation in which they did not need all the information given in the question to solve part of it and either used the same interval width for (b)(i) and (b)(ii) or answered b(ii) only. A few answered b(ii) only and proceeded to attempt to find an exact answer using analytic calculus, which in this case is impossible. These candidates were apparently answering the question that they expected to be set rather than the one which had actually been set. In Mathematics, as in all other subjects, carefully reading and answering the question as set are necessary examination skills.

2. Nearly all candidates gained both marks in part (a). As is usual, the main error seen in part (b) was finding the width of the trapezium incorrectly. There were fewer errors in bracketing than had been noted in some recent examinations and nearly all candidates gave the answer to the specified accuracy. The integration by parts in part (c) was well done and the majority of candidates had been well prepared for this topic.

Some failed to simplify $\int \frac{x^2}{2} \times \frac{1}{x} \mathrm{d}x$ to $\int \frac{x}{2} \mathrm{d}x$ and either gave up or produced $\frac{\frac{1}{3}x^3}{x^2}$.

In evaluating the definite integral some either overlooked the requirement to give the answer in the form $\frac{1}{4}(a1n2+b)$or were unable to use the appropriate rule of logarithms correctly.

3. Most candidates could gain the mark in part (a) although 2.99937, which arises from the incorrect angle mode, was seen occasionally. The main error seen in part (b) was finding the width of the trapezium incorrectly, $\frac{3\pi}{10}$ being commonly seen instead of $\frac{3\pi}{8}$. This resulted from confusing the number of values of the ordinate, 5, with the number of strips, 4. Nearly all candidates gave the answer to the specified accuracy. In part (c), the great majority of candidates recognised that they needed to find $\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x$ and most could integrate correctly. However $\sin x$, $9\sin x$, $3\sin\left(\frac{x}{3}\right)$, $-9\sin\left(\frac{x}{3}\right)$, $-\sin\left(\frac{x}{3}\right)$ and $-3\sin\left(\frac{x}{3}\right)$ were all seen from time to time. Candidates did not seem concerned if their answers to part (b) and part (c) were quite different, possibly not connecting the parts of the question. Despite these difficulties, full marks were common and, generally, the work on these topics was sound.

4. A significant majority of candidates were able to score full marks on this question. In part (a), some candidates struggled to find either one or both of the y-ordinates required. A few of these candidates did not change their calculator to radian mode. In part (b), some candidates incorrectly stated the width of each of the trapezia as either $\frac{1}{4}$ or $\frac{\pi}{5}$. Nearly all answers were given to 4 decimal places as requested in the question.

5. Part (a) was generally well answered as was part (b). In part (a), there were a significant number of candidates, however, who struggled with evaluating $\tan\left(\frac{\pi}{16}\right)$ and $\tan\left(\frac{\pi}{8}\right)$ decimal places and a few other candidates did not change their calculator to radian mode. In part (b), some candidates incorrectly stated the width of each of the trapezia as either 1 or $\frac{\pi}{20}$.
Nearly all answers were given to 4 decimal places as requested in the question.

Part (c) proved more demanding but it was still pleasing to see many correct solutions. Many candidates who attempted this part were able to integrate tan x correctly (given in formula booklet) although this was sometimes erroneously given as $\sec^2 x$. There were also a few candidates who attempted to integrate $\sqrt{\tan x}$. The substitution of limits caused little difficulty but sometimes a rounded answer was given instead of the required exact answer. Whilst most candidates used $\pi\int \tan x\mathrm{d}x$, 2π was occasionally seen in place of π and more often π was omitted.

6. Part (a) was invariably well answered as was part (b). In part (b), some candidates incorrectly stated the width of each of the trapezia as $\frac{5}{6}$ whilst a few candidates did not give their answer to 4 significant figures.
The most successful approach in part (c) was for candidates to rearrange the given substitution to make x the subject. The expression for x was differentiated to give $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2t}{3}$ and then substituted into the original integral to give the required integral in terms of t. Weaker candidates, who instead found $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3}{2}(3x+1)^{-\frac{1}{2}}$, then struggled to achieve the required integral in terms of t. Most candidates were able to correctly find the changed limits although a sizeable number of candidates obtained the incorrect limits of $t = 2$ and $t = 4$.

Those candidates, who had written down a form of the required integral in part (c), were usually able to apply the method of integration by parts and integrate kte^t with respect to t and use their correct changed limits to find the correct answer of 109.2. Some candidates incorrectly used 'unchanged' limits of $t = 0$ and $t = 5$.

7. In part (a), the first mark of the question was usually gratefully received, although for x 1.5 it was not uncommon to see $\frac{1}{2}\ln(\frac{1}{2})$.

In part (b), it was not unusual to see completely correct solutions but common errors included candidates either stating the wrong width of the trapezia or candidates not stating their final answer correct to four significant figures.

Answers to part (c) were variable and often the mark in this part was not gained.

In part (d) all four most popular ways detailed in the mark scheme were seen. For weaker candidates this proved a testing part. For many candidates the method of integration by parts provided the way forward although some candidates applied this formula in the 'wrong direction' and incorrectly stated that $\frac{dv}{dx}$ =ln x implied v=$\underline{1}$. Sign errors were common in this part, eg: the incorrect statement of $\int\left(\frac{x}{2}-1\right)dx=-\frac{x^2}{4}-x$, and as usual, where final answers have to be derived, the last few steps of the solution were often not convincing.

In summary, this question proved to be a good source of marks for stronger candidates, with 12 or 13 marks quite common for such candidates; a loss of one mark was likely to have been in part (c).

8. Most candidates applied the given formula correctly to obtain values for y, which were not always given to the required degree of accuracy.

The majority of candidates then went on to apply the trapezium rule correctly, although some had difficulties with the interval width (5/6 was quite a popular alternative), and in such a familiar question it is disappointing to see so many candidates misapplying the formula as $\frac{1}{2}(0+3.482)+2(0.062+0.271+0.716+1.612)$. In most of these cases it is not an error in the way in which they use their calculators, they simply express the formula incorrectly from the outset. Some candidates who set the working out like this do come up with the correct numerical result because they have used poor presentation and "invisible brackets".

Most candidates realised that to find the volume of the section they simply needed to multiply their area by 6, but several came up with complicated false alternatives, often involving formulae for cone, cylinder or sphere.

For the final two marks, some candidates gave answers relating to the complexity of forming this shape from concrete, and missed the point that the trapezium rule over-estimates this area. Others were more concerned about the rounding errors due to working to 3 decimal places.

9. For a large number of candidates this proved to be a very good question, and there were many full marks awarded. However, some candidates had little appreciation of the degree of accuracy they should use, and in both parts (b) and (c) there were some common and serious errors seen.

Although few candidates had trouble with the method of the trapezium rule itself, a common error in part (b) was in the miscalculation of "*h*". Many candidates divided the *x* interval $\left(\frac{\pi}{4}\right)$ by 5 instead of 4, even though the first two given *x*-values were 0 and $\frac{\pi}{16}$, but a more alarming error was in the use of 180 for π , so that "*h*" became 11.25 or 9.

In part (c) a very common error, resulting in the loss of both marks, was to use the answer to part (b), rather than the true value of ln(1 + $\sqrt{2}$), as the denominator in calculating the percentage error.

10. Those who recognised that integration by parts was needed in part (a), and these were the great majority, usually made excellent attempts at this part and, in most cases, the indefinite integral was carried out correctly. Many had difficulty with the evaluating the definite integral. There were many errors of sign and the error $e^0 = 0$ was common. The trapezium rule was well known, although the error of thinking that 6 ordinates gave rise to 6 strips, rather than 5, was often seen and some candidates lost the final mark by not giving the answer to the specified accuracy.

11. In part (a), the log working was often unclear and part (b) also gave many difficulty. The differentiation was often incorrect. $\frac{1}{2x}$ was not unexpected but expressions like $x+\frac{1}{x}$ were also seen. Many then failed to substitute $x=\frac{1}{2e}$ into their $\frac{dy}{dx}$ and produced a non-linear tangent. Parts (c) and (d) were well done. A few did, however, give their answers to an inappropriate accuracy. As the table is given to 2 decimal places, the answer should not be given to a greater accuracy.

12. For many candidates this was a good source of marks. Even weaker candidates often scored well in parts (c) and (d). In part (a) there were still some candidates who were confused by the notation, f' often interpreted as f^{-1}, and common wrong answers to the differentiation were $\frac{e^x}{5}$ and 1 + e^x . The most serious error, which occurred far too frequently, in part (b) was to have a variable gradient, so that equations such as $y-\frac{1}{5} = \left(1+\frac{e^x}{5}\right)x$ were common. The normal, rather than the tangent, was also a common offering.

13. The majority of candidates knew the techniques needed to solve this question well and full marks were common. In part (a) a substantial minority used their calculators wrongly. The use of the wrong angle mode seemed commoner than in some recent examinations and errors deriving from wrong bracketing were not infrequent. The majority of candidates gave their answers to the degree of accuracy requested. Nearly all gained marks in part (b) as follow through marks were given from answers in part (a). Part (c) was well done. The majority understood the issue involved and many illustrated their answer with a convincing diagram.

14. This proved to be a popular confidence boosting question and high marks were gained by the majority of candidates. Completion of the table caused little problem with relatively few candidates making the mistake of using x in degrees. The trapezium rule was generally handled very well with few errors occurring, and the only errors in part (c) tended to be in using 100 – (b) or 120 – (b). Explanations in part (d) sometimes showed confused thinking but some candidates were clearly giving a reason why the tunnel, and not the surround, had been over or under-estimated.

15. No Report available for this question.