OCR Maths C4

Past Paper Pack

2005-2014

June 2005

1 Find the quotient and the remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$. [4]

2 Evaluate $\int_0^{\frac{1}{2}\pi} x \cos x \, \mathrm{d}x$, giving your answer in an exact form. [5]

3 The line L_1 passes through the points $(2, -3, 1)$ and $(-1, -2, -4)$. The line L_2 passes through the point $(3, 2, -9)$ and is parallel to the vector $4\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

(i) Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Prove that L_1 and L_2 are skew. [5]

4 **(i)** Show that the substitution $x = \tan\theta$ transforms $\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x$ to $\int \cos^2\theta \, \mathrm{d}\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1+x^2)^2}\,\mathrm{d}x$. [4]

5 $ABCD$ is a parallelogram. The position vectors of A, B and C are given respectively by

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \qquad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j}, \qquad \mathbf{c} = \mathbf{i} - \mathbf{j} - 2\mathbf{k}.$$

(i) Find the position vector of D. [3]

(ii) Determine, to the nearest degree, the angle ABC. [4]

6 The equation of a curve is $xy^2 = 2x + 3y$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 - y^2}{2xy - 3}$. [5]

(ii) Show that the curve has no tangents which are parallel to the y-axis. [3]

7 A curve is given parametrically by the equations

$$x = t^2, \qquad y = \frac{1}{t}.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, giving your answer in its simplest form. [3]

(ii) Show that the equation of the tangent at the point $P\left(4, -\frac{1}{2}\right)$ is

$$x - 16y = 12.$$ [3]

(iii) Find the value of the parameter at the point where the tangent at P meets the curve again. [4]

June 2005

8 **(i)** Given that $\dfrac{3x+4}{(1+x)(2+x)^2} \equiv \dfrac{A}{1+x} + \dfrac{B}{2+x} + \dfrac{C}{(2+x)^2}$, find A, B and C. [5]

(ii) Hence or otherwise expand $\dfrac{3x+4}{(1+x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. [1]

9 Newton's law of cooling states that the rate at which the temperature of an object is falling at any instant is proportional to the difference between the temperature of the object and the temperature of its surroundings at that instant. A container of hot liquid is placed in a room which has a constant temperature of 20 °C. At time t minutes later, the temperature of the liquid is θ °C.

(i) Explain how the information above leads to the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20),$$

where k is a positive constant. [2]

(ii) The liquid is initially at a temperature of 100 °C. It takes 5 minutes for the liquid to cool from 100 °C to 68 °C. Show that

$$\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}.$$ [8]

(iii) Calculate how much longer it takes for the liquid to cool by a further 32 °C. [3]

1 Simplify $\dfrac{x^3 - 3x^2}{x^2 - 9}$. [3]

2 Given that $\sin y = xy + x^2$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]

3 **(i)** Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$. [4]

(ii) Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder. [2]

4 **(i)** Use integration by parts to find $\int x \sec^2 x \, dx$. [4]

(ii) Hence find $\int x \tan^2 x \, dx$. [3]

5 A curve is given parametrically by the equations $x = t^2$, $y = 2t$.

(i) Find $\dfrac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Show that the equation of the tangent to the curve at $(p^2, 2p)$ is

$$py = x + p^2.$$ [2]

(iii) Find the coordinates of the point where the tangent at $(9, 6)$ meets the tangent at $(25, -10)$. [4]

6 **(i)** Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\dfrac{x}{1-x}} \, dx$ to $\int 2\sin^2 \theta \, d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\dfrac{x}{1-x}} \, dx$. [5]

7 The expression $\dfrac{11 + 8x}{(2-x)(1+x)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\dfrac{A}{2-x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$, where A, B and C are constants. [5]

(ii) Given that $|x| < 1$, find the first 3 terms in the expansion of $f(x)$ in ascending powers of x. [5]

8 **(i)** Solve the differential equation

$$\frac{dy}{dx} = \frac{2-x}{y-3},$$

giving the particular solution that satisfies the condition $y = 4$ when $x = 5$. [5]

(ii) Show that this particular solution can be expressed in the form

$$(x-a)^2 + (y-b)^2 = k,$$

where the values of the constants a, b and k are to be stated. [3]

(iii) Hence sketch the graph of the particular solution, indicating clearly its main features. [3]

9 Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is a constant.

(i) Calculate the acute angle between the lines. [5]

(ii) Given that these two lines intersect, find a and the point of intersection. [8]

June 2006

1 Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$. [4]

2 **(i)** Expand $(1 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) Find the coefficient of x^2 in the expansion of $\dfrac{(1 + 2x)^2}{(1 - 3x)^2}$ in ascending powers of x. [4]

3 **(i)** Express $\dfrac{3 - 2x}{x(3 - x)}$ in partial fractions. [3]

(ii) Show that $\displaystyle\int_1^2 \frac{3 - 2x}{x(3 - x)}\,dx = 0$. [4]

(iii) What does the result of part **(ii)** indicate about the graph of $y = \dfrac{3 - 2x}{x(3 - x)}$ between $x = 1$ and $x = 2$? [1]

4 The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$

$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$

and $$\overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

(i) Find the angle between AB and AC. [6]

(ii) Find the area of triangle ABC. [2]

5 A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

(i) Write down a differential equation which models this situation. [2]

(ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]

6 **(i)** Show that the substitution $u = e^x + 1$ transforms $\displaystyle\int \frac{e^{2x}}{e^x + 1}\,dx$ to $\displaystyle\int \frac{u - 1}{u}\,du$. [3]

(ii) Hence show that $\displaystyle\int_0^1 \frac{e^{2x}}{e^x + 1}\,dx = e - 1 - \ln\left(\frac{e + 1}{2}\right)$. [5]

June 2006

7 Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where a is a constant.

(i) Given that the lines are skew, find the value that a cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

8 **(i)** Show that $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x \cos^2 6x \, dx$. [6]

9 A curve is given parametrically by the equations

$$x = 4\cos t, \qquad y = 3\sin t,$$

where $0 \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent at the point P, where $t = p$, is

$$3x\cos p + 4y\sin p = 12. \qquad [3]$$

(iii) The tangent at P meets the x-axis at R and the y-axis at S. O is the origin. Show that the area of triangle ORS is $\frac{12}{\sin 2p}$. [3]

(iv) Write down the least possible value of the area of triangle ORS, and give the corresponding value of p. [3]

Jan 2007

1 It is given that

$$f(x) = \frac{x^2 + 2x - 24}{x^2 - 4x} \quad \text{for } x \neq 0,\ x \neq 4.$$

Express $f(x)$ in its simplest form. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [5]

3 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(i) Find the length of AB. [3]

(ii) Use a scalar product to find angle OAB. [3]

4 Use the substitution $u = 2x - 5$ to show that $\int_{\frac{5}{2}}^{3} (4x - 8)(2x - 5)^7 \, dx = \frac{17}{72}$. [5]

5 **(i)** Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) Hence find the coefficient of x^3 in the expansion of $\left(1 - 3(x + x^3)\right)^{-\frac{1}{3}}$. [3]

6 **(i)** Express $\frac{2x + 1}{(x - 3)^2}$ in the form $\frac{A}{x - 3} + \frac{B}{(x - 3)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\int_4^{10} \frac{2x + 1}{(x - 3)^2} \, dx$, giving your answer in the form $a + b \ln c$, where a, b and c are integers. [4]

7 The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]

8 The parametric equations of a curve are $x = 2t^2$, $y = 4t$. Two points on the curve are $P\,(2p^2, 4p)$ and $Q\,(2q^2, 4q)$.

(i) Show that the gradient of the normal to the curve at P is $-p$. [2]

(ii) Show that the gradient of the chord joining the points P and Q is $\frac{2}{p + q}$. [2]

(iii) The chord PQ is the normal to the curve at P. Show that $p^2 + pq + 2 = 0$. [2]

(iv) The normal at the point $R\,(8, 8)$ meets the curve again at S. The normal at S meets the curve again at T. Find the coordinates of T. [4]

Jan 2007

9 **(i)** Find the general solution of the differential equation

$$\frac{\sec^2 y}{\cos^2(2x)}\frac{\mathrm{d}y}{\mathrm{d}x} = 2.$$ [7]

(ii) For the particular solution in which $y = \frac{1}{4}\pi$ when $x = 0$, find the value of y when $x = \frac{1}{6}\pi$. [3]

10 The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4]

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ. [3]

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]

June 2007

1 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [2]

(ii) Hence find $\mathrm{f}'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]

2 Find the exact value of $\displaystyle\int_0^1 x^2 \mathrm{e}^x \, \mathrm{d}x$. [6]

3 Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

4 **(i)** Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\dfrac{1+x^2}{(2+x)^2}$. [2]

5 A curve C has parametric equations

$$x = \cos t, \quad y = 3 + 2\cos 2t, \quad \text{where } 0 \leqslant t \leqslant \pi.$$

(i) Express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t and hence show that the gradient at any point on C cannot exceed 8. [4]

(ii) Show that all points on C satisfy the cartesian equation $y = 4x^2 + 1$. [3]

(iii) Sketch the curve $y = 4x^2 + 1$ and indicate on your sketch the part which represents C. [2]

6 The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point $(2, 3)$ on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

7 **(i)** Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part **(ii)** to find the exact value of $\displaystyle\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4} \, \mathrm{d}x$. [5]

8 The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

9 Lines L_1, L_2 and L_3 have vector equations

$$L_1:\ \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$
$$L_2:\ \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$
$$L_3:\ \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2. [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c. [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c. [5]

Jan 2008

1 Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. [4]

2 **(i)** Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x+2)}\,\mathrm{d}x$. [2]

3 When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find the values of the constants a, b and c. [5]

4 Find the equation of the normal to the curve

$$x^3 + 4x^2y + y^3 = 6$$

at the point $(1, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

5 The vector equations of two lines are

$$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = (2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} - 5\mathbf{k}).$$

Prove that the two lines are

(i) perpendicular, [3]

(ii) skew. [5]

6 **(i)** Expand $(1 + ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

7 **(i)** Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constants A and B. [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,\mathrm{d}\theta,$$

giving your answer in the form $a\pi - \ln b$. [5]

8 Water flows out of a tank through a hole in the bottom and, at time t minutes, the depth of water in the tank is x metres. At any instant, the rate at which the depth of water in the tank is decreasing is proportional to the square root of the depth of water in the tank.

(i) Write down a differential equation which models this situation. [2]

(ii) When $t = 0$, $x = 2$; when $t = 5$, $x = 1$. Find t when $x = 0.5$, giving your answer correct to 1 decimal place. [6]

9 The parametric equations of a curve are $x = t^3$, $y = t^2$.

(i) Show that the equation of the tangent at the point P where $t = p$ is

$$3py - 2x = p^3.$$ [4]

(ii) Given that this tangent passes through the point $(-10, 7)$, find the coordinates of each of the three possible positions of P. [5]

10 **(i)** Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x.$$ [6]

(ii) Find the exact value of

$$\int_1^3 \frac{\ln x}{x^2}\,\mathrm{d}x.$$ [5]

June 2008

1 **(a)** Simplify $\dfrac{(2x^2-7x-4)(x+1)}{(3x^2+x-2)(x-4)}$. [2]

(b) Find the quotient and remainder when x^3+2x^2-6x-5 is divided by x^2+4x+1. [4]

2 Find the exact value of $\int_1^e x^4 \ln x \, dx$. [5]

3 The equation of a curve is $x^2y - xy^2 = 2$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y^2-2xy}{x^2-2xy}$. [3]

(ii) **(a)** Show that, if $\dfrac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

4 Relative to an origin O, the points A and B have position vectors $3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B. [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB. [5]

5 **(i)** Show that $\sqrt{\dfrac{1-x}{1+x}} \approx 1 - x + \tfrac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \tfrac{2}{7}$, show that $\sqrt{5} \approx \tfrac{111}{49}$. [3]

6 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

7 **(i)** Show that, if $y = \operatorname{cosec} x$, then $\dfrac{dy}{dx}$ can be expressed as $-\operatorname{cosec} x \cot x$. [3]

(ii) Solve the differential equation

$$\frac{dx}{dt} = -\sin x \tan x \cot t,$$

given that $x = \tfrac{1}{6}\pi$ when $t = \tfrac{1}{2}\pi$. [5]

8 **(i)** Given that $\dfrac{2t}{(t+1)^2}$ can be expressed in the form $\dfrac{A}{t+1} + \dfrac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

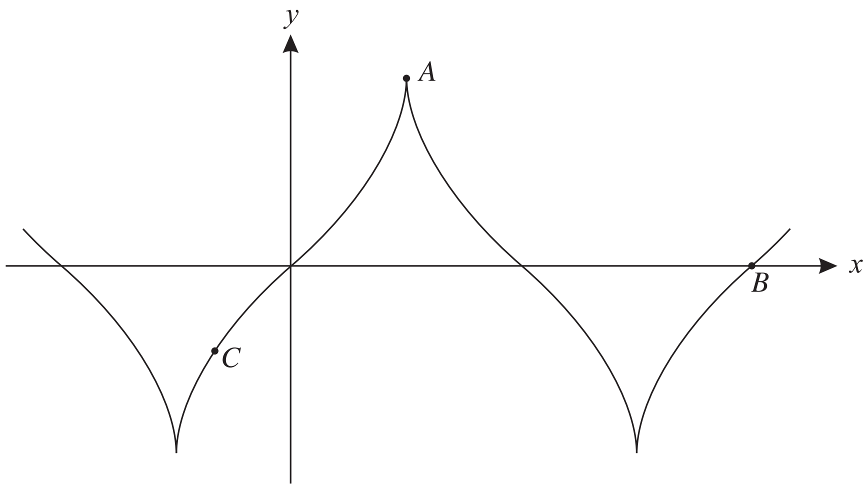
(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\displaystyle\int \frac{1}{x+\sqrt{2x-1}}\,dx$ to $\displaystyle\int \frac{2t}{(t+1)^2}\,dt$. [4]

(iii) Hence find the exact value of $\displaystyle\int_1^5 \frac{1}{x+\sqrt{2x-1}}\,dx$. [4]

9 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 4\sin\theta,$$

and part of its graph is shown below.



(i) Find the value of θ at A and the value of θ at B. [3]

(ii) Show that $\dfrac{dy}{dx} = \sec\theta$. [5]

(iii) At the point C on the curve, the gradient is 2. Find the coordinates of C, giving your answer in an exact form. [3]

1 Simplify $\dfrac{20-5x}{6x^2-24x}$. **[3]**

2 Find $\displaystyle\int x\sec^2 x \, dx$. **[4]**

3 **(i)** Expand $(1+2x)^{\frac{1}{2}}$ as a series in ascending powers of x, up to and including the term in x^3. **[3]**

(ii) Hence find the expansion of $\dfrac{(1+2x)^{\frac{1}{2}}}{(1+x)^3}$ as a series in ascending powers of x, up to and including the term in x^3. **[5]**

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. **[1]**

4 Find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} (1+\sin x)^2 \, dx$. **[6]**

5 **(i)** Show that the substitution $u=\sqrt{x}$ transforms $\displaystyle\int \frac{1}{x(1+\sqrt{x})} \, dx$ to $\displaystyle\int \frac{2}{u(1+u)} \, du$. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_1^9 \frac{1}{x(1+\sqrt{x})} \, dx$. **[5]**

6 A curve has parametric equations

$$x=t^2-6t+4, \quad y=t-3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t=2$, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. **[5]**

7 **(i)** Show that the straight line with equation $\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+t\begin{pmatrix}1\\4\\-2\end{pmatrix}$ meets the line passing through $(9, 7, 5)$ and $(7, 8, 2)$, and find the point of intersection of these lines. **[6]**

(ii) Find the acute angle between these lines. **[4]**

Jan 2009

8 The equation of a curve is $x^3 + y^3 = 6xy$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

(ii) Show that the point $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ lies on the curve and that $\frac{dy}{dx} = 0$ at this point. **[4]**

(iii) The point (a, a), where $a > 0$, lies on the curve. Find the value of a and the gradient of the curve at this point. **[4]**

9 A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $160 - \theta$, where θ °C is the temperature of the liquid at that time.

(i) Write down a differential equation connecting θ and t. **[2]**

When the liquid was placed in the oven, its temperature was 20 °C and 5 minutes later its temperature had risen to 65 °C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. **[9]**

June 2009

1 Find the quotient and the remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$. **[4]**

2 Use the substitution $x = \tan\theta$ to find the exact value of

$$\int_1^{\sqrt{3}} \frac{1 - x^2}{1 + x^2}\,dx.$$ [7]

3 **(i)** Expand $(a + x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(ii) When $(1 - x)(a + x)^{-2}$ is expanded, the coefficient of x^2 is 0. Find the value of a. **[3]**

4 **(i)** Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. **[4]**

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} e^x \sin 2x\,dx$. **[3]**

5 A curve has parametric equations

$$x = 2t + t^2, \quad y = 2t^2 + t^3.$$

(i) Express $\frac{dy}{dx}$ in terms of t and find the gradient of the curve at the point $(3, -9)$. **[5]**

(ii) By considering $\frac{y}{x}$, find a cartesian equation of the curve, giving your answer in a form not involving fractions. **[4]**

6 The expression $\frac{4x}{(x - 5)(x - 3)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\frac{A}{x - 5} + \frac{B}{x - 3} + \frac{C}{(x - 3)^2}$, where A, B and C are constants. **[4]**

(ii) Hence find the exact value of $\int_1^2 f(x)\,dx$. **[5]**

7 **(i)** The vector $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is perpendicular to the vector $4\mathbf{i} + \mathbf{k}$ and to the vector $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. Find the values of b and c, and show that $\mathbf{u}$ is a unit vector. **[6]**

(ii) Calculate, to the nearest degree, the angle between the vectors $4\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$. **[3]**

June 2009

8 **(i)** Given that $14x^2 - 7xy + y^2 = 2$, show that $\dfrac{dy}{dx} = \dfrac{28x - 7y}{7x - 2y}$. **[4]**

(ii) The points L and M on the curve $14x^2 - 7xy + y^2 = 2$ each have x-coordinate 1. The tangents to the curve at L and M meet at N. Find the coordinates of N. **[6]**

9 A tank contains water which is heated by an electric water heater working under the action of a thermostat. The temperature of the water, θ °C, may be modelled as follows. When the water heater is first switched on, $\theta = 40$. The heater causes the temperature to increase at a rate k_1 °C per second, where k_1 is a constant, until $\theta = 60$. The heater then switches off.

(i) Write down, in terms of k_1, how long it takes for the temperature to increase from 40 °C to 60 °C. **[1]**

The temperature of the water then immediately starts to decrease at a variable rate $k_2(\theta - 20)$ °C per second, where k_2 is a constant, until $\theta = 40$.

(ii) Write down a differential equation to represent the situation as the temperature is decreasing. **[1]**

(iii) Find the total length of time for the temperature to increase from 40 °C to 60 °C and then decrease to 40 °C. Give your answer in terms of k_1 and k_2. **[8]**

1 Find the quotient and the remainder when $x^4 + 11x^3 + 28x^2 + 3x + 1$ is divided by $x^2 + 5x + 2$. **[4]**

2 Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. **[4]**

(ii) Given instead that ABC is a straight line, find the value of p. **[2]**

3 By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x}\,dx$. **[5]**

4 Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^{e} \frac{1}{t(2 + \ln t)^2}\,dt.$$ **[6]**

5 **(i)** Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[2]**

(ii) **(a)** Hence, or otherwise, expand $(8 + 16x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[4]**

(b) State the set of values of x for which the expansion in part **(ii) (a)** is valid. **[1]**

6 A curve has parametric equations

$$x = 9t - \ln(9t), \quad y = t^3 - \ln(t^3).$$

Show that there is only one value of t for which $\dfrac{dy}{dx} = 3$ and state that value. **[6]**

7 Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[8]**

8 **(i)** State the derivative of $e^{\cos x}$. **[1]**

(ii) Hence use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}\pi} \cos x \sin x\, e^{\cos x}\,dx.$$ **[6]**

<u>Jan 2010</u>

9 The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. **[4]**

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. **[4]**

(iii) Find the length of OQ. **[2]**

10 **(i)** Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions. **[2]**

(ii) In a chemical reaction, the amount x grams of a substance at time t seconds is related to the rate at which x is changing by the equation

$$\frac{dx}{dt} = k(3-x)(6-x),$$

where k is a constant. When $t = 0$, $x = 0$ and when $t = 1$, $x = 1$.

(a) Show that $k = \frac{1}{3}\ln\frac{5}{4}$. **[7]**

(b) Find the value of x when $t = 2$. **[4]**

1 Expand $(1+3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

2 Given that $y = \dfrac{\cos x}{1-\sin x}$, find $\dfrac{dy}{dx}$, simplifying your answer. **[4]**

3 Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions. **[5]**

4 Use the substitution $u = \sqrt{x+2}$ to find the exact value of

$$\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,dx.$$ **[7]**

5 Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ **[7]**

6 Lines l_1 and l_2 have vector equations

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

respectively, where t and s are parameters and a is a constant.

(i) Given that l_1 and l_2 are perpendicular, find the value of a. **[3]**

(ii) Given instead that l_1 and l_2 intersect, find

(a) the value of a, **[4]**

(b) the angle between the lines. **[3]**

7 The parametric equations of a curve are $x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

(i) Show that $\dfrac{dy}{dx} > 0$. **[6]**

(ii) Find the cartesian equation of the curve, giving your answer in a form not involving fractions. **[5]**

8 **(i)** Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. **[3]**

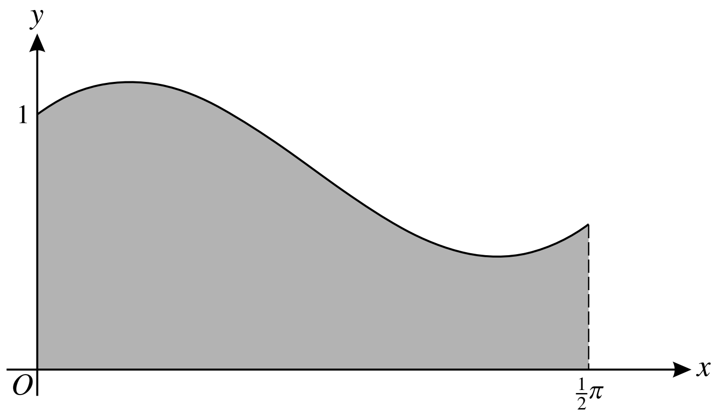
(ii) **(a)** Find the general solution of the differential equation

$$\left(\frac{x-1}{x^2-5x+6}\right)\frac{dy}{dx} = y - 5.$$ **[3]**

(b) Given that $y = 7$ when $x = 8$, find y when $x = 6$. **[4]**

9 **(i)** Find $\int (x + \cos 2x)^2 \, dx$. **[9]**

(ii)



The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form. **[4]**

Jan 2011

1 **(i)** Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^2. **[3]**

(ii) Hence expand $\left(1-2y+4y^2\right)^{\frac{1}{2}}$ in ascending powers of y as far as the term in y^2. **[3]**

2 **(i)** Express $\dfrac{7-2x}{(x-2)^2}$ in the form $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$, where A and B are constants. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_4^5 \frac{7-2x}{(x-2)^2}\,dx$. **[4]**

3 **(i)** Show that the derivative of $\sec x$ can be written as $\sec x \tan x$. **[4]**

(ii) Find $\displaystyle\int \frac{\tan x}{\sqrt{1+\cos 2x}}\,dx$. **[4]**

4 A curve has parametric equations

$$x = 2 + t^2, \qquad y = 4t.$$

(i) Find $\dfrac{dy}{dx}$ in terms of t. **[2]**

(ii) Find the equation of the normal at the point where $t = 4$, giving your answer in the form $y = mx + c$. **[3]**

(iii) Find a cartesian equation of the curve. **[2]**

5 In this question, I denotes the definite integral $\displaystyle\int_2^5 \frac{5-x}{2+\sqrt{x-1}}\,dx$. The value of I is to be found using two different methods.

(i) Show that the substitution $u = \sqrt{x-1}$ transforms I to $\displaystyle\int_1^2 (4u-2u^2)\,du$ and hence find the exact value of I. **[5]**

(ii) **(a)** Simplify $\left(2+\sqrt{x-1}\right)\left(2-\sqrt{x-1}\right)$. **[1]**

(b) By first multiplying the numerator and denominator of $\dfrac{5-x}{2+\sqrt{x-1}}$ by $2-\sqrt{x-1}$, find the exact value of I. **[3]**

Jan 2011

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$.

(i) Find the acute angle between l_1 and l_2. **[4]**

(ii) Show that l_1 and l_2 are skew. **[4]**

(iii) One of the numbers in the equation of line l_1 is changed so that the equation becomes $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ a \end{pmatrix} + s\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$. Given that l_1 and l_2 now intersect, find a. **[2]**

7 Show that $\int_0^{\pi} (x^2 + 5x + 7)\sin x \, dx = \pi^2 + 5\pi + 10$. **[7]**

8 The points P and Q lie on the curve with equation

$$2x^2 - 5xy + y^2 + 9 = 0.$$

The tangents to the curve at P and Q are parallel, each having gradient $\frac{3}{8}$.

(i) Show that the x- and y-coordinates of P and Q are such that $x = 2y$. **[5]**

(ii) Hence find the coordinates of P and Q. **[3]**

9 Paraffin is stored in a tank with a horizontal base. At time t minutes, the depth of paraffin in the tank is x cm. When $t = 0$, $x = 72$. There is a tap in the side of the tank through which the paraffin can flow. When the tap is opened, the flow of the paraffin is modelled by the differential equation

$$\frac{dx}{dt} = -4(x - 8)^{\frac{1}{3}}.$$

(i) How long does it take for the level of paraffin to fall from a depth of 72 cm to a depth of 35 cm? **[7]**

(ii) The tank is filled again to its original depth of 72 cm of paraffin and the tap is then opened. The paraffin flows out until it stops. How long does this take? **[3]**

June 2011

1 Simplify $\dfrac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. **[4]**

2 Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. **[3]**

3 **(i)** Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x. **[4]**

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\,\mathrm{d}x.$$ **[4]**

4 Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{\left(1 - 9x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x.$$ **[6]**

5 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. **[3]**

(ii) Find the acute angle between l_1 and l_2. **[4]**

(iii) The point A lies on l_1 and OA is perpendicular to l_1, where O is the origin. Find the position vector of A. **[3]**

6 Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1 + ax}{4 - x}},$$

giving your answer in terms of a. **[8]**

7 The gradient of a curve at the point (x, y), where $x > -2$, is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x + 2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q, giving your answer correct to 3 significant figures. **[7]**

8 A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. **[2]**

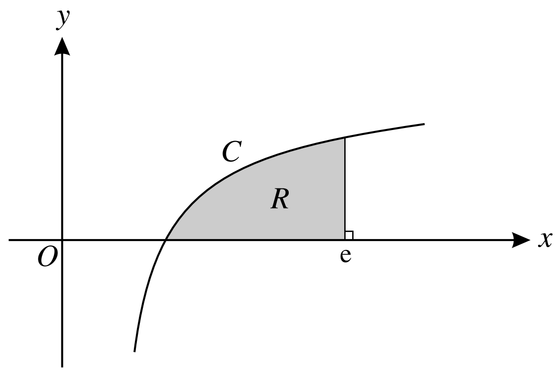
(ii) Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

(iii) Find the value of t at the point where this normal meets the curve again. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = \mathrm{f}(x)$. **[3]**

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$. **[3]**

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C, the x-axis and the line $x = \mathrm{e}$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x-axis. **[6]**

(b) The region R is rotated completely about the y-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi \mathrm{e}^2 - \pi \int_0^1 \mathrm{e}^{2y} \, \mathrm{d}y,$$

and find this volume. **[4]**

1 When the polynomial $f(x)$ is divided by $x^2 + 1$, the quotient is $x^2 + 4x + 2$ and the remainder is $x - 1$. Find $f(x)$, simplifying your answer. **[3]**

2 **(i)** Find, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line l through the points $(4, 2, 7)$ and $(5, -4, -1)$. **[3]**

(ii) Find the acute angle between the line l and a line in the direction of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. **[4]**

3 The equation of a curve C is $(x + 3)(y + 4) = x^2 + y^2$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. **[4]**

(ii) The line $2y = x + 3$ meets C at two points. What can be said about the tangents to C at these points? Justify your answer. **[2]**

(iii) Find the equation of the tangent at the point $(6, 0)$, giving your answer in the form $ax + by = c$, where a, b and c are integers. **[2]**

4 **(i)** Expand $(1 - 4x)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

(ii) The term of lowest degree in the expansion of

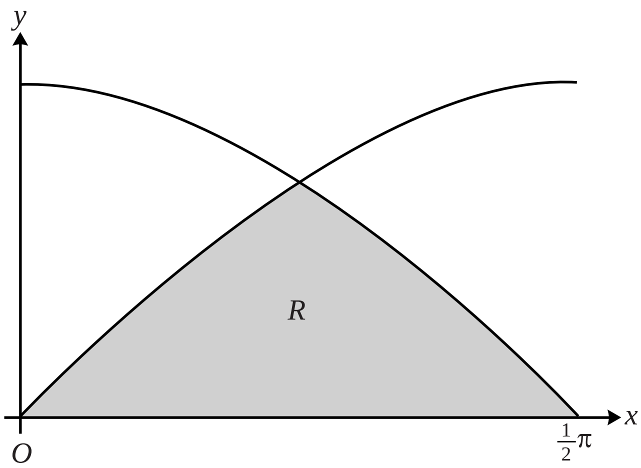
$$(1 + ax)(1 + bx^2)^7 - (1 - 4x)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3. Find the values of the constants a and b. **[4]**

5 Use the substitution $u = \cos x$ to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, dx .$$ **[6]**

6



The diagram shows the curves $y = \cos x$ and $y = \sin x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The region R is bounded by the curves and the x-axis. Find the volume of the solid of revolution formed when R is rotated completely about the x-axis, giving your answer in terms of π. **[7]**

7 The equation of a straight line l is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

O is the origin.

(i) Find the position vector of the point P on l such that OP is perpendicular to l. **[3]**

(ii) A point Q on l is such that the length of OQ is 3 units. Find the two possible position vectors of Q. **[3]**

8 A curve is defined by the parametric equations

$$x = \sin^2 \theta, \quad y = 4\sin\theta - \sin^3\theta,$$

where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{4 - 3\sin^2\theta}{2\sin\theta}$. **[3]**

(ii) Find the coordinates of the point on the curve at which the gradient is 2. **[3]**

(iii) Show that the curve has no stationary points. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$. **[2]**

[Questions 9 and 10 are printed overleaf.]

9 Find the exact value of $\int_0^1 (x^2 + 1)e^{2x}\,dx$. **[7]**

10 **(i)** Write down the derivative of $\sqrt{y^2 + 1}$ with respect to y. **[1]**

(ii) Given that $\frac{dy}{dx} = \frac{(x-1)\sqrt{y^2+1}}{xy}$ and that $y = \sqrt{e^2 - 2e}$ when $x = e$,

find a relationship between x and y. **[8]**

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS

4724 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4732470612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Simplify

(i) $\dfrac{1-x}{x^2-3x+2}$, **[2]**

(ii) $\dfrac{(x+1)}{(x-1)(x-3)} - \dfrac{(x-5)}{(x-3)(x-4)}$. **[4]**

2 Use integration by parts to find $\int \ln(x+2)\,\mathrm{d}x$. **[5]**

3 **(i)** Expand $\dfrac{1+x^2}{\sqrt{1+4x}}$ in ascending powers of x, up to and including the term in x^3. **[6]**

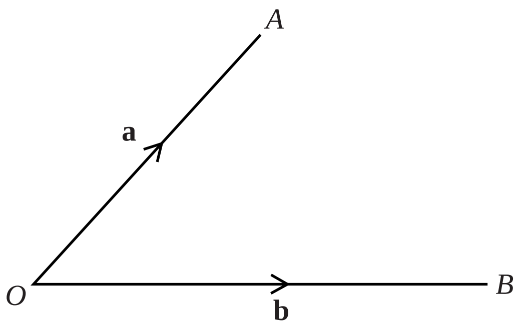
(ii) State the set of values of x for which this expansion is valid. **[1]**

4 Solve the differential equation

$$e^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + \tan x = 0,$$

given that $x = 0$ when $y = 0$. Give your answer in the form $y = \mathrm{f}(x)$. **[6]**

5



In the diagram the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to the origin O. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $\mathbf{a}.\mathbf{b} = 6$, find

(i) the angle AOB, **[2]**

(ii) $|\mathbf{a} - \mathbf{b}|$. **[3]**

6 Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_4^9 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x = 2 + 2\ln\tfrac{3}{4}.$$ **[7]**

June 2012

7 Find the exact value of $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)^2 \, dx$. **[7]**

8 **(a)** Find the gradient of the curve $x^2 + xy + y^2 = 3$ at the point $(-1, -1)$. **[4]**

(b) A curve C has parametric equations

$$x = 2t^2 - 1, \quad y = t^3 + t.$$

(i) Find the coordinates of the point on C at which the tangent is parallel to the y-axis. **[3]**

(ii) Find the values of t for which x and y have the same rate of change with respect to t. **[3]**

9 **(i)** Express $\dfrac{x^2 - x - 11}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Find the exact value of $\int_3^4 \dfrac{x^2 - x - 11}{(x+1)(x-2)^2} \, dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. **[4]**

10 Lines l_1 and l_2 have vector equations

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} + 9\mathbf{j} - 4\mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$$

respectively. The point A has coordinates $(-3, 0, 6)$ relative to the origin O.

(i) Show that A lies on l_1 and that OA is perpendicular to l_1. **[3]**

(ii) Show that the line through O and A intersects l_2. **[4]**

(iii) Given that the point of intersection in part **(ii)** is B, find the ratio $|\overrightarrow{OA}| : |\overrightarrow{BA}|$. **[3]**

Jan 2013

1 Find $\int x\cos 3x\,dx$. **[4]**

2 Find the first three terms in the expansion of $(9-16x)^{\frac{3}{2}}$ in ascending powers of x, and state the set of values for which this expansion is valid. **[5]**

3 The equation of a curve is $xy^2 = x^2 + 1$. Find $\frac{dy}{dx}$ in terms of x and y, and hence find the coordinates of the stationary points on the curve. **[7]**

4 The equations of two lines are

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}).$$

(i) Show that these lines meet, and find the coordinates of the point of intersection. **[5]**

(ii) Find the acute angle between these lines. **[3]**

5 The parametric equations of a curve are

$$x = 2 + 3\sin\theta \quad \text{and} \quad y = 1 - 2\cos\theta \quad \text{for} \quad 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. **[5]**

(ii) Find the cartesian equation of the curve. **[2]**

6 Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5}\,dx$. **[7]**

7 **(i)** Given that $y = \ln(1+\sin x) - \ln(\cos x)$, show that $\frac{dy}{dx} = \frac{1}{\cos x}$. **[4]**

(ii) Using this result, evaluate $\int_0^{\frac{1}{3}\pi} \sec x\,dx$, giving your answer as a single logarithm. **[3]**

8 The points $A\ (3, 2, 1)$, $B\ (5, 4, -3)$, $C\ (3, 17, -4)$ and $D\ (1, 6, 3)$ form a quadrilateral $ABCD$.

(i) Show that $AB = AD$. **[2]**

(ii) Find a vector equation of the line through A and the mid-point of BD. **[3]**

(iii) Show that C lies on the line found in part **(ii)**. **[1]**

(iv) What type of quadrilateral is $ABCD$? **[1]**

9 The temperature of a freezer is –20 °C. A container of a liquid is placed in the freezer. The rate at which the temperature, θ °C, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

(i) Express θ in terms of t, k and an arbitrary constant. **[3]**

Initially the temperature of the liquid in the container is 40 °C and, at this instant, the liquid is cooling at a rate of 3 °C per minute. The liquid freezes at 0 °C.

(ii) Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze. **[5]**

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90 °C. After 19 minutes its temperature is 0 °C.

(iii) Without any further calculation, explain what you can deduce about the value of k in this case. **[1]**

10 **(i)** Use algebraic division to express $\dfrac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. **[4]**

(ii) Hence find $\displaystyle\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx$, giving your answer in the form $a + \ln b$. **[7]**

1 Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. **[5]**

2 Find $\int x^8 \ln(3x)\,dx$. **[5]**

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1+2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3+5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu-1)\mathbf{i} + (5-\mu)\mathbf{j} + (2-5\mu)\mathbf{k}$$

are parallel, intersect or are skew. **[6]**

4 The equation of a curve is $y = \cos 2x + 2\sin x$. Find $\dfrac{dy}{dx}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. **[6]**

5 **(i)** Show that $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x}\right) dx$, giving your answer in the form $a \ln b$. **[5]**

6 Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \frac{\ln x}{x(1+\ln x)^2}\,dx$. **[6]**

7 Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. **[4]**

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[3]**

(iii) Calculate the volume of the pyramid $ABCD$. **[3]**

[The volume of the pyramid is $V = \frac{1}{3} \times$ base area $\times$ perpendicular height.]

8 At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at $1.08\ \text{cm s}^{-1}$.

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t. **[7]**

(ii) How much air is in the balloon initially? **[2]**

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

9 A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

(i) Find $\frac{dy}{dx}$ in terms of t, simplifying your answer. **[3]**

(ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. **[4]**

(iii) Find a cartesian equation of the curve. **[2]**

10 **(i)** Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x. **[2]**

(ii) Use this result, together with a suitable value of x, to obtain a decimal estimate of the value of $\frac{100}{729}$. **[2]**

(iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1 - \frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. **[4]**

(iv) Comment on the suitability of substituting the same value of x as used in part **(ii)** in the expansion in part **(iii)** to estimate the value of $\frac{100}{729}$. **[1]**

1 Express $x+\dfrac{1}{1-x}+\dfrac{2}{1+x}$ as a single fraction, simplifying your answer. **[3]**

2 The points $O(0,\ 0,\ 0)$, $A(2,\ 8,\ 2)$, $B(5,\ 5,\ 8)$ and $C(3,\ -3,\ 6)$ form a parallelogram $OABC$. Use a scalar product to find the acute angle between the diagonals of this parallelogram. **[5]**

3 **(i)** Find the first three terms in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, where $|x|<\frac{1}{2}$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $\dfrac{x+3}{\sqrt{1-2x}}$. **[2]**

4 Show that $\displaystyle\int_0^{\frac{1}{4}\pi}\frac{1-2\sin^2 x}{1+2\sin x\cos x}\,dx=\frac{1}{2}\ln 2$. **[5]**

5 The equations of three lines are as follows.

Line A: $\mathbf{r}=\mathbf{i}+4\mathbf{j}+\mathbf{k}+s(-\mathbf{i}+2\mathbf{j}+2\mathbf{k})$

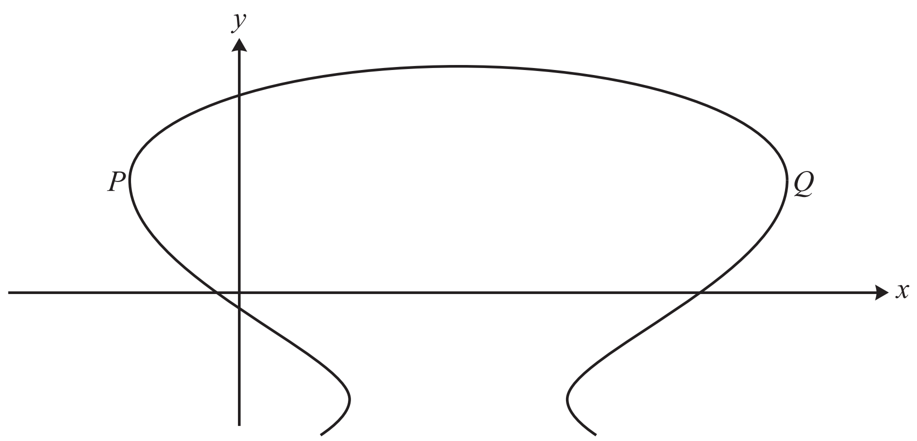
Line B: $\mathbf{r}=2\mathbf{i}+8\mathbf{j}+2\mathbf{k}+t(\mathbf{i}+3\mathbf{j}+5\mathbf{k})$

Line C: $\mathbf{r}=-\mathbf{i}+19\mathbf{j}+15\mathbf{k}+u(2\mathbf{i}-4\mathbf{j}-4\mathbf{k})$

(i) Show that lines A and B are skew. **[4]**

(ii) Determine, giving reasons, the geometrical relationship between lines A and C. **[2]**

6



The diagram shows the curve with equation $x^2+y^3-8x-12y=4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**

7 A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. **[5]**

(ii) Find the cartesian equation of the curve. **[3]**

(iii) State the set of values that x can take and hence sketch the curve. **[3]**

8 **(i)** Use division to show that $\dfrac{t^3}{t+2} \equiv t^2 - 2t + 4 - \dfrac{8}{t+2}$. **[3]**

(ii) Find $\displaystyle\int_1^2 6t^2 \ln(t+2)\,dt$. Give your answer in the form $A + B\ln 3 + C\ln 4$. **[6]**

9 Express $\dfrac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\displaystyle\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,dx = \frac{1}{2}\ln\frac{3}{2} + \frac{1}{3}$. **[9]**

10 A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of $0.01\,\text{m}^3$ per second. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]

(i) Show that $h^2\dfrac{dh}{dt} = -\dfrac{9}{400\pi}$. **[5]**

(ii) Express h in terms of t. **[4]**

(iii) Find the time it takes to empty the container, giving your answer to the nearest minute. **[2]**

END OF QUESTION PAPER