Worked Solutions

Edexcel C4 Paper H

1. (*a*)

x	-1	0	1
	$\dfrac{1}{1+e^{-1}}$	$\dfrac{1}{1+e}$	$\dfrac{1}{1+1}$	$\dfrac{1}{1+\frac{1}{e}}$

$$\frac{1}{1+\frac{1}{e}} = \frac{e}{e+1}$$

$$\text{integral} \approx \frac{1}{2}\left[\frac{1}{1+e} + \frac{e}{e+1} + 2\times\frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1+e+1+e}{1+e}\right] = 1 \qquad \textbf{(4)}$$

(*b*) let $I = \displaystyle\int_{-1}^{1} \frac{1}{1+e^{-x}}\,dx$

put $u = e^x$

$\dfrac{du}{dx} = e^x$

$du = e^x \; dx$

$$= \int_{-1}^{1} \frac{e^x}{e^x+1}\,dx$$

when $x = 1, \quad u = e$

$x = -1, \quad u = e^{-1}$

$$\therefore \quad I = \int_{e^{-1}}^{e} \frac{du}{u+1} = \Big[\ln(u+1)\Big]_{e^{-1}}^{e}$$

$$= \ln(e+1) - \ln\left(1+\frac{1}{e}\right)$$

$$= \ln\left(\frac{e+1}{1+\frac{1}{e}}\right) = \ln\left[\frac{(1+e)e}{(e+1)}\right] = \ln e = 1 \qquad \textbf{(4)}$$

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2. (*a*) (i) differentiating implicitly, $1 = e^y \dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x} \qquad \textbf{(2)}$$

(ii) when $y = 0, \; x = e^0 = 1 \qquad \dfrac{dy}{dx} = 1$

equation of tangent is $y - 0 = x - 1$

$$y = x - 1 \qquad \textbf{(2)}$$

(*b*) $x = \sin y \qquad 1 = \cos y \dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}} \qquad \textbf{(3)}$$

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3. (*a*) $\dfrac{dy}{dx} = \dfrac{-2\sin\theta}{2\cos\theta} = -\dfrac{\sin\theta}{\cos\theta}$

equation of tangent is $y - (2\cos\theta + 2) = -\dfrac{\sin\theta}{\cos\theta}\Big[x - (2\sin\theta + 1)\Big]$

$$y\cos\theta - 2\cos^2\theta - 2\cos\theta = -x\sin\theta + 2\sin^2\theta + \sin\theta$$

$$x\sin\theta + y\cos\theta = 2 + 2\cos\theta + \sin\theta \qquad \textbf{(4)}$$

(*b*) when $\theta = \dfrac{\pi}{2}$ tangent is $x + 0 = 2 + 0 + 1$

$$x = 3 \qquad \textbf{(1)}$$

(*c*) $\sin\theta = \dfrac{x-1}{2}, \cos\theta = \dfrac{y-2}{2}$

$$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1 \qquad \left[\sin^2\theta + \cos^2\theta = 1\right]$$

$$(x-1)^2 + (y-2)^2 = 4 \qquad \textbf{(4)}$$

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4. (*a*) $\int (y+1)\mathrm{d}y = -\int (x-2)\mathrm{d}x$

$\frac{1}{2}(y+1)^2 = -\frac{1}{2}(x-2)^2 + k$

$(2, 2)$ lies on C, $\quad \therefore \quad \frac{1}{2}9 = -\frac{1}{2} \times 0 + k$

C is $\frac{1}{2}(y+1)^2 + \frac{1}{2}(x-2)^2 = \frac{9}{2}$

or $\quad (x-2)^2 + (y+1)^2 = 9$ **(6)**

(*b*) Circle centre $(2, -1)$, radius 3 **(2)**

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5. (*a*) $t = 0, \theta = 70 + 2 = 72$ **(1)**

(*b*) $\theta = 70\,\mathrm{e}^{-1} + 2 = 27.8$ **(2)**

(*c*) as $t \to \infty, \mathrm{e}^{-0.1t} \to 0$

$\therefore \quad \theta \to 2$ **(2)**

(*d*) $10 = 70\,\mathrm{e}^{-0.1t} + 2$

$\mathrm{e}^{-0.1t} = \frac{8}{70}$

$-0.1t = \ln\frac{8}{70}, \quad t = 21.7$ minutes **(3)**

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6. (*a*) divide each term by $\cos^2\theta$,

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$ **(2)**

(*b*) $I = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{1+x^2}\mathrm{d}x$ $\qquad$ let $x = \tan\theta$

$\mathrm{d}x = \sec^2\theta \;\; \mathrm{d}\theta$

$x = 1, \theta = \frac{\pi}{4}$

$x = \frac{1}{\sqrt{3}}, \theta = \frac{\pi}{6}$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2\theta}{\sec^2\theta}\mathrm{d}\theta = \left[\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{\pi}{12}$ **(6)**

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7. (*a*) $\frac{9x}{(1-2x)(1+x)^2} \equiv \frac{A}{(1-2x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$

$9x \equiv A(1+x)^2 + B(1-2x)(1+x) + C(1-2x)$

$x = \frac{1}{2}$: $\qquad \frac{9}{2} = A \cdot \left(\frac{3}{2}\right)^2 \Rightarrow A = 2$

$x = -1$: $\qquad -9 = C(1+2) \Rightarrow C = -3$

constants: $\qquad 0 = A + B + C \Rightarrow B = 1$

$\therefore$ expression is $\frac{2}{1-2x} + \frac{1}{1+x} - \frac{3}{(1+x)^2}$ **(4)**

(*b*) $2(1-2x)^{-1} + (1+x)^{-1} - 3(1+x)^{-2}$

$= 2\left[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}\left(-2x\right)^2 + \frac{(-1)(-2)(-3)}{3.2}\left(-2x\right)^3\right]$

$+\left[1 - x + \frac{(-1)(-2)}{2}\left(x^2\right) + \frac{(-1)(-2)(-3)}{3.2}x^3\right]$

$-3\left[1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3.2}x^3\right]$

$= (2 + 4x + 8x^2 + 16x^3) + (1 - x + x^2 - x^3) - 3(1 - 2x + 3x^2 - 4x^3)$

$= 9x + 27x^3$ **(5)**

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8. (*a*) area $= \int_1^3 \left(2 + \frac{1}{x}\right) \mathrm{d}x = \left[2x + \ln x\right]_1^3 = 4 + \ln 3$ **(3)**

(*b*) volume $= \pi \int_1^3 \frac{4x^2 + 4x + 1}{x^2}\ \mathrm{d}x$

$$= \pi \int_1^3 \left(4 + \frac{4}{x} + x^{-2}\right) \mathrm{d}x$$

$$= \pi \left[4x + 4\ln x - \frac{1}{x}\right]_1^3$$

$$= \pi \left[12 + 4\ln 3 - \frac{1}{3} - (4 + 0 - 1)\right] = \pi \left[\frac{26}{3} + 4\ln 3\right]$$ **(6)**

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9. (*a*) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, line through AB is $r = \begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ **(3)**

(*b*) $\overrightarrow{AO} = \begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix}$ $\quad |\overrightarrow{AO}| = \sqrt{49 + 64} = \sqrt{113}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ $\quad |\overrightarrow{AB}| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$\begin{pmatrix} -7 \\ -8 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \sqrt{113}\ \sqrt{14}\cos\theta$, where $\theta =$ angle required

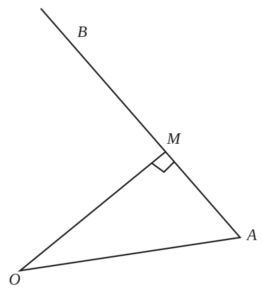
$$-14 + 8 = \sqrt{113}\ \sqrt{14}\cos\theta$$

$$\theta = 98.7°$$

acute angle between OA and AB is 81° (nearest degree)

(*c*) M lies on line AB

$\therefore\ \overrightarrow{OM}$ is $\begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 0 + 3\lambda \end{pmatrix}$



$\overrightarrow{OM} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} 7 + 2\lambda \\ 8 - \lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$= 14 + 4\lambda - 8 + \lambda + 9\lambda = 0$

$\lambda = \frac{-3}{7}$

position vector of M is $\begin{pmatrix} 7 - \frac{6}{7} \\ 8 + \frac{3}{7} \\ \frac{-9}{7} \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 43 \\ 59 \\ -9 \end{pmatrix}$ **(4)**
