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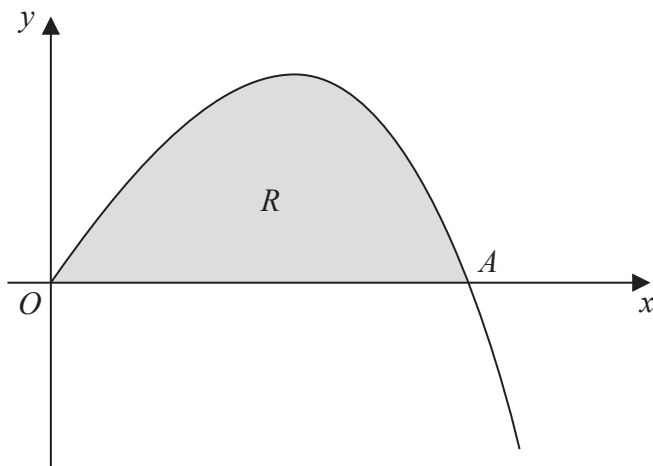


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$

The curve meets the  $x$ -axis at the origin  $O$  and cuts the  $x$ -axis at the point  $A$ .

(a) Find, in terms of  $\ln 2$ , the  $x$  coordinate of the point  $A$ . (2)

(b) Find

$$\int xe^{\frac{1}{2}x} dx$$
(3)

The finite region  $R$ , shown shaded in Figure 1, is bounded by the  $x$ -axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \quad x \geq 0$$

(c) Find, by integration, the exact value for the area of  $R$ .  
Give your answer in terms of  $\ln 2$  (3)

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5. A curve  $C$  has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of  $\frac{dy}{dx}$  at the point on  $C$  where  $t = 2$ , giving your answer as a fraction in its simplest form.

**(3)**

(b) Show that the cartesian equation of the curve  $C$  can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where  $a$  and  $b$  are integers to be determined.

**(3)**

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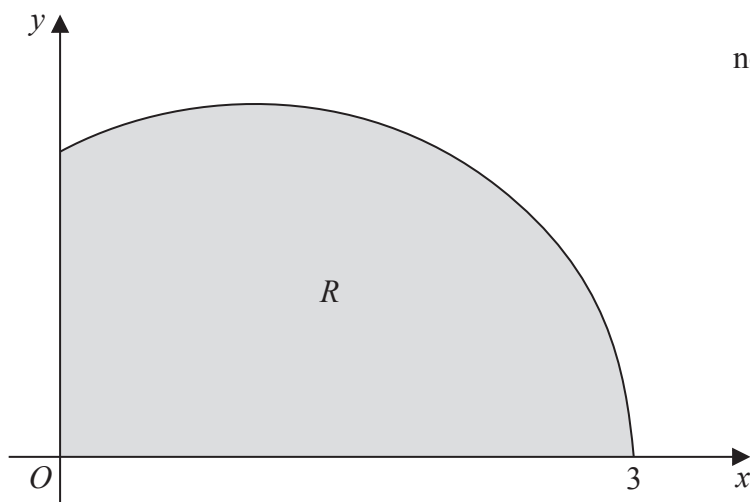
Diagram  
not to scale**Figure 2**

Figure 2 shows a sketch of the curve with equation  $y = \sqrt{(3-x)(x+1)}$ ,  $0 \leq x \leq 3$

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve, the  $x$ -axis, and the  $y$ -axis.

- (a) Use the substitution  $x = 1 + 2 \sin \theta$  to show that

$$\int_0^3 \sqrt{(3-x)(x+1)} \, dx = k \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

where  $k$  is a constant to be determined.

**(5)**

- (b) Hence find, by integration, the exact area of  $R$ .

**(3)**


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7. (a) Express  $\frac{2}{P(P - 2)}$  in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P - 2)\cos 2t, \quad t \geq 0$$

where  $P$  is the population in thousands, and  $t$  is the time measured in years since the start of the study.

Given that  $P = 3$  when  $t = 0$ ,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.

Give your answer in years to 3 significant figures.

(3)

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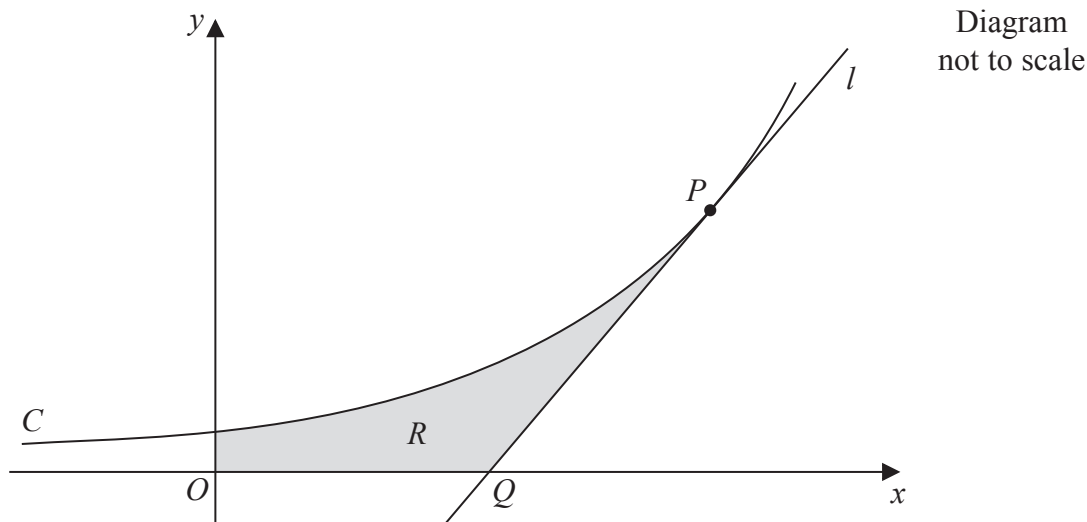


Figure 3

Figure 3 shows a sketch of part of the curve  $C$  with equation

$$y = 3^x$$

The point  $P$  lies on  $C$  and has coordinates  $(2, 9)$ .

The line  $l$  is a tangent to  $C$  at  $P$ . The line  $l$  cuts the  $x$ -axis at the point  $Q$ .

- (a) Find the exact value of the  $x$  coordinate of  $Q$ . (4)

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve  $C$ , the  $x$ -axis, the  $y$ -axis and the line  $l$ . This region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis.

- (b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form  $\frac{p}{q}$  where  $p$  and  $q$  are exact constants.

[You may assume the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a cone.] (6)

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