

Edexcel Maths C4

Past Paper Pack

2005-2013

5.

Figure 1

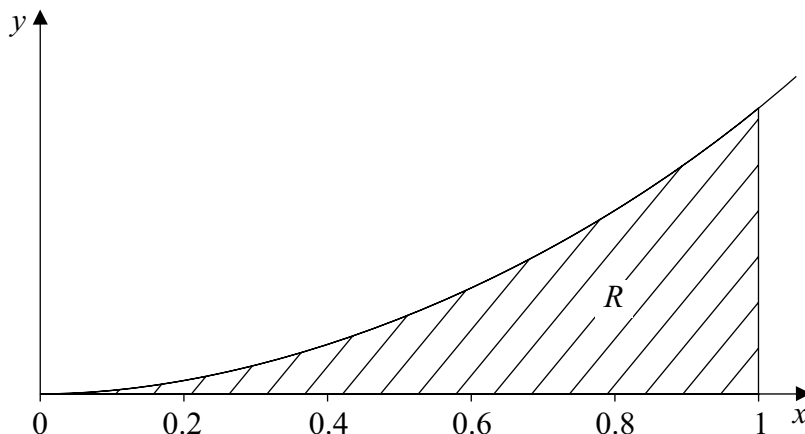


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R . (5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)



7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ .

(a) Find the coordinates of B . (4)

(b) Find the value of $\cos \theta$, giving your answer as a simplified fraction. (4)

The point A , which lies on l_1 , has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

The point C , which lies on l_2 , has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\vec{AB}| = |\vec{BC}|$. (3)

(d) Find the position vector of the point D . (2)



8. Liquid is pouring into a container at a constant rate of $20 \text{ cm}^3 \text{ s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, $V \text{ cm}^3$, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k .

(6)

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)



4.

Figure 1

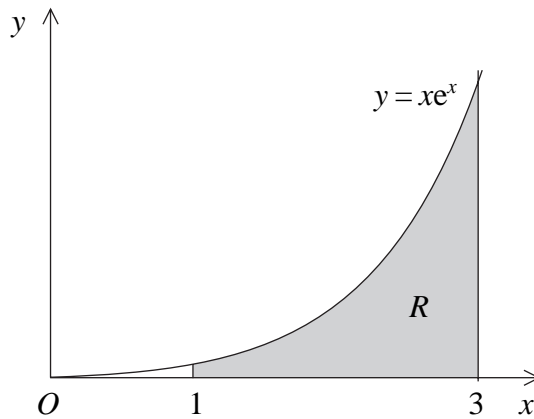


Figure 1 shows the finite shaded region, R , which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x -axis.

The region R is rotated through 360 degrees about the x -axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)



7. The volume of a spherical balloon of radius r cm is V cm³, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$. (1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{dr}{dt}$. (2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t . (4)

(d) Hence, at time $t = 5$,

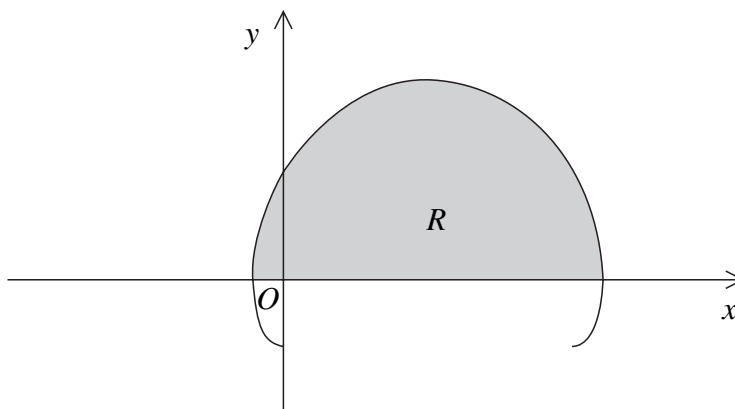
(i) find the radius of the balloon, giving your answer to 3 significant figures, (3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s⁻¹. (2)



8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2 \sin t, \quad y = 1 - 2 \cos t, \quad 0 \leq t \leq 2\pi.$$

- (a) Show that the curve crosses the x -axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$. (2)

The finite region R is enclosed by the curve and the x -axis, as shown shaded in Figure 2.

- (b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 dt. \tag{3}$$

- (c) Use this integral to find the exact value of the shaded area. (7)



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Question 8 continued

Lined area for writing the answer to Question 8.

Q8

(Total 12 marks)

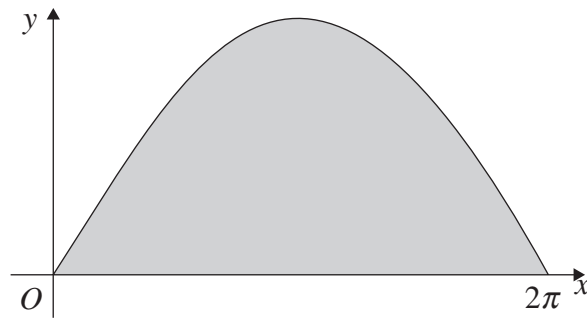
TOTAL FOR PAPER: 75 MARKS

END



3.

Figure 1



The curve with equation $y = 3 \sin \frac{x}{2}$, $0 \leq x \leq 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x -axis is shaded.

(a) Find, by integration, the area of the shaded region. (3)

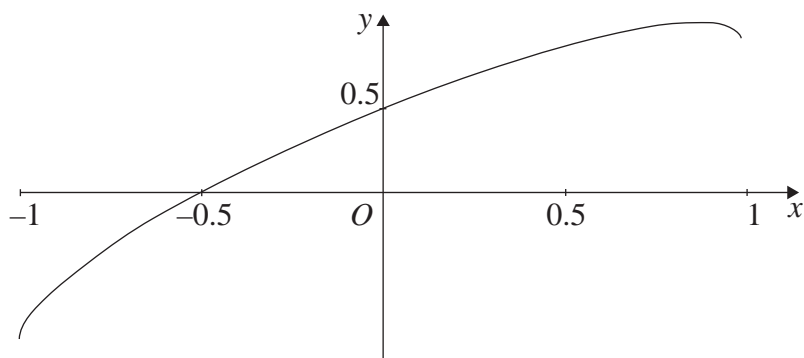
This region is rotated through 2π radians about the x -axis.

(b) Find the volume of the solid generated. (6)



4.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$. (6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}, \quad -1 < x < 1. \quad (3)$$



Leave
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5. The point A , with coordinates $(0, a, b)$ lies on the line l_1 , which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

- (a) Find the values of a and b . (3)

The point P lies on l_1 and is such that OP is perpendicular to l_1 , where O is the origin.

- (b) Find the position vector of point P . (6)

Given that B has coordinates $(5, 15, 1)$,

- (c) show that the points A, P and B are collinear and find the ratio $AP : PB$. (4)



6.

Figure 3

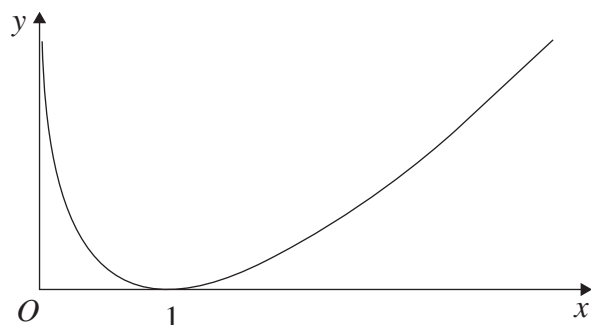


Figure 3 shows a sketch of the curve with equation $y = (x - 1) \ln x$, $x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2 \ln 3$

(1)

Given that $I = \int_1^3 (x - 1) \ln x \, dx$,

(b) use the trapezium rule

(i) with values of y at $x = 1, 2$ and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1, 1.5, 2, 2.5$ and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

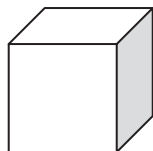
(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x - 1) \ln x \, dx$ is $\frac{3}{2} \ln 3$.

(6)



7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm², and the volume of the cube is V cm³.

The surface area of the cube is increasing at a constant rate of $8 \text{ cm}^2 \text{ s}^{-1}$.

Show that

(a) $\frac{dx}{dt} = \frac{k}{x}$, where k is a constant to be found, (4)

(b) $\frac{dV}{dt} = 2V^{\frac{1}{3}}$. (4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$. (7)



Centre No.						Paper Reference					Surname	Initial(s)		
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)
6666/01

**Edexcel GCE
 Core Mathematics C4
 Advanced Level**

Tuesday 23 January 2007 – Afternoon
 Time: 1 hour 30 minutes

<u>Materials required for examination</u>	<u>Items included with question papers</u>
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only		

Team Leader's use only		

Question Number	Leave Blank
1	
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Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.

Check that you have the correct question paper.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 8 questions in this question paper. The total mark for this paper is 75.

There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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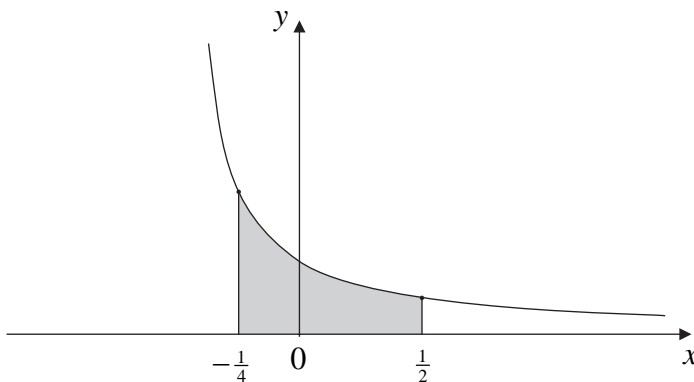
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2.

Figure 1



The curve with equation $y = \frac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x -axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x -axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

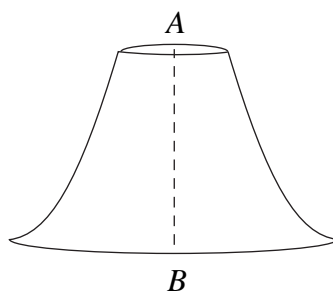


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)



8.

$$I = \int_0^5 e^{\sqrt{3x+1}} dx.$$

- (a) Given that $y = e^{\sqrt{3x+1}}$, complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I , giving your answer to 4 significant figures.

(3)

- (c) Use the substitution $t = \sqrt{3x+1}$ to show that I may be expressed as $\int_a^b kte^t dt$, giving the values of a , b and k .

(5)

- (d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)



7.

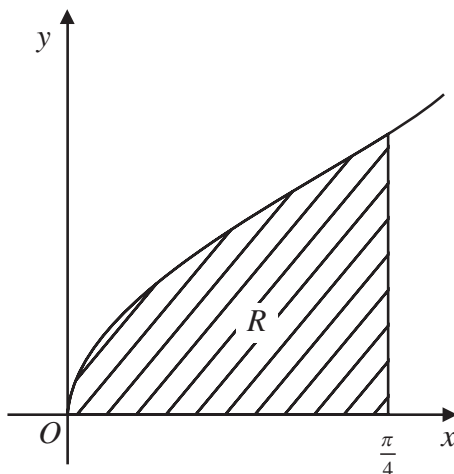


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R , which is bounded by the curve, the x -axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

- (a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

- (b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R , giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x -axis to generate a solid of revolution.

- (c) Use integration to find an exact value for the volume of the solid generated.

(4)



8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0 ,

(a) solve the differential equation, giving P in terms of P_0 , k and t . (4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$. (3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0 , λ and t . (4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model. (3)



Centre No.						Paper Reference					Surname	Initial(s)		
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced**

Tuesday 22 January 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
1	
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Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature. Check that you have the correct question paper. You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

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1.

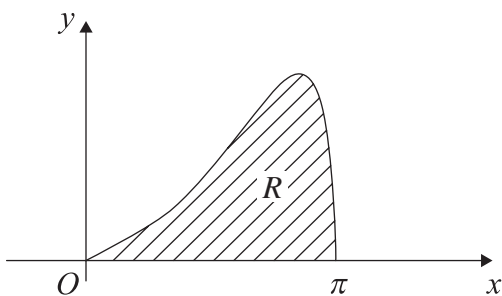


Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{\sin x}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x -axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R . Give your answer to 4 decimal places.

(4)



Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced**

Thursday 12 June 2008 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
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Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. You must write your answer for each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 8 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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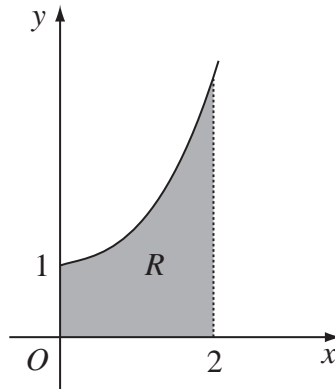


Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the y -axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R , giving your answer to 4 significant figures.

(3)



2. (a) Use integration by parts to find $\int x e^x dx$.

(3)

(b) Hence find $\int x^2 e^x dx$.

(3)



3.

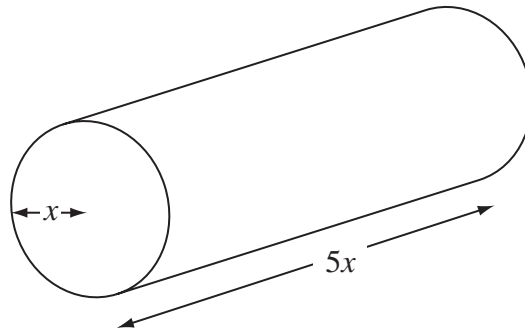


Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of $0.032 \text{ cm}^2 \text{ s}^{-1}$.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures. (4)

(b) Find the rate of increase of the volume of the rod when $x = 2$. (4)



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4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q .

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q . (6)

(b) Find the coordinates of P and Q . (3)



3.

$$f(x) = \frac{27x^2 + 32x + 16}{(3x + 2)^2(1 - x)}, \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x + 2)} + \frac{B}{(3x + 2)^2} + \frac{C}{(1 - x)},$$

- (a) find the values of B and C and show that $A = 0$. (4)

- (b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Simplify each term. (6)

- (c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures. (4)



4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$. (2)

Given further that l_1 and l_2 intersect, find

(b) the value of p , (6)

(c) the coordinates of the point of intersection. (2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2 .

Given that a circle, with centre C , cuts the line l_1 at the points A and B ,

(d) find the position vector of B . (3)



5.

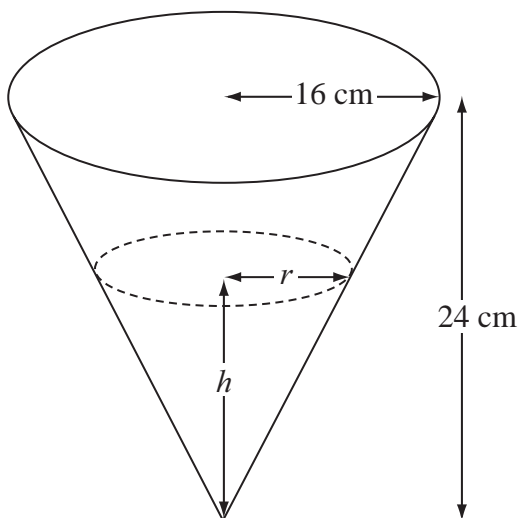


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm³.

(a) Show that $V = \frac{4\pi h^3}{27}$. (2)

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$.

(b) Find, in terms of π , the rate of change of h when $h = 12$. (5)



6. (a) Find $\int \tan^2 x \, dx$. (2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$. (4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant. (7)



7.

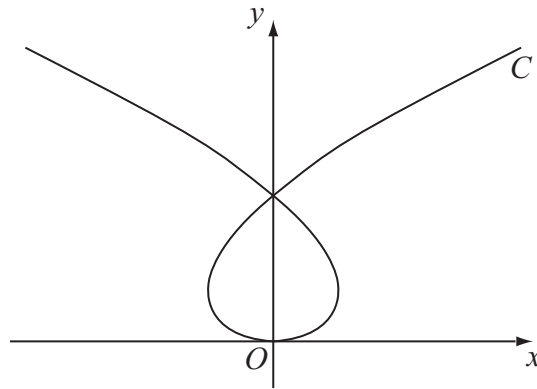


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

- (a) find the coordinates of A . **(1)**

The line l is the tangent to C at A .

- (b) Show that an equation for l is $2x - 5y - 9 = 0$. **(5)**

The line l also intersects the curve at the point B .

- (c) Find the coordinates of B . **(6)**



2.

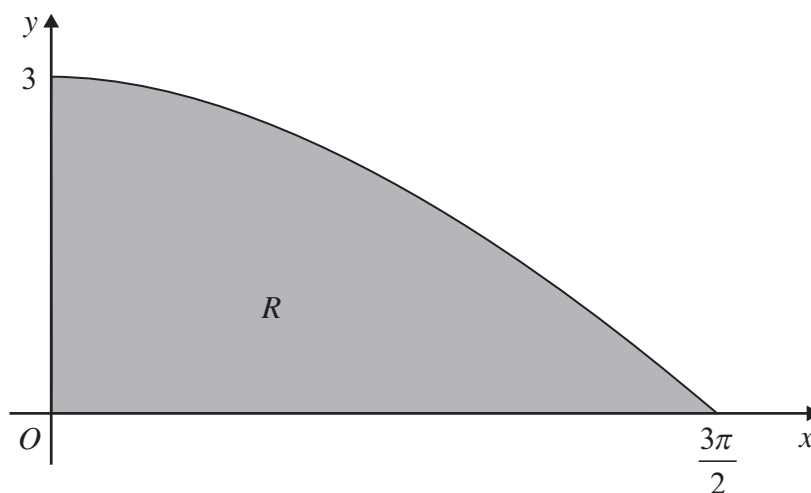


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with

equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Complete the table above giving the missing value of y to 5 decimal places. **(1)**
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. **(4)**
- (c) Use integration to find the exact area of R . **(3)**



Leave blank

Question 2 continued

Lined area for writing answers.

(Total 8 marks)

Q2



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Question 3 continued

Lined area for writing the answer to Question 3 continued.



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Question 4 continued

Lined area for student response.



6. (a) Find $\int \sqrt{5-x} dx$. (2)

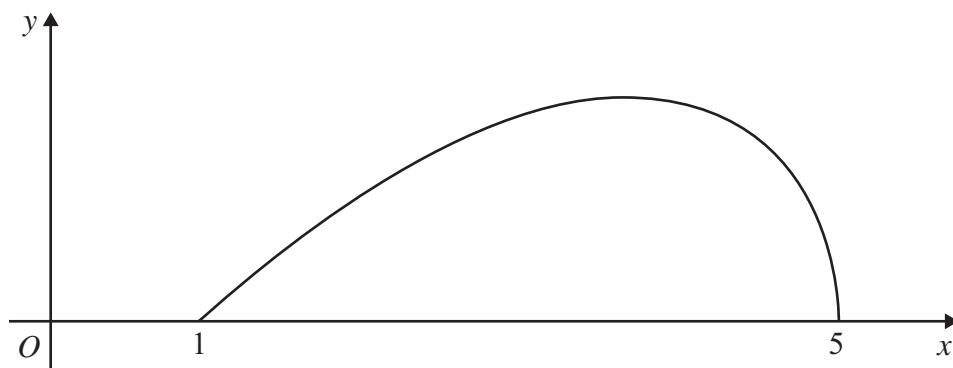


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1) \sqrt{5 - x}, \quad 1 \leq x \leq 5$$

- (b) (i) Using integration by parts, or otherwise, find

$$\int (x - 1) \sqrt{5 - x} dx \quad (4)$$

- (ii) Hence find $\int_1^5 (x - 1) \sqrt{5 - x} dx$. (2)



7. Relative to a fixed origin O , the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B .

- (a) Find a vector equation for the line l . (3)
- (b) Find $|\vec{CB}|$. (2)
- (c) Find the size of the acute angle between the line segment CB and the line l , giving your answer in degrees to 1 decimal place. (3)
- (d) Find the shortest distance from the point C to the line l . (3)

The point X lies on l . Given that the vector \vec{CX} is perpendicular to l ,

(e) find the area of the triangle CXB , giving your answer to 3 significant figures. (3)



8. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta d\theta$. (2)

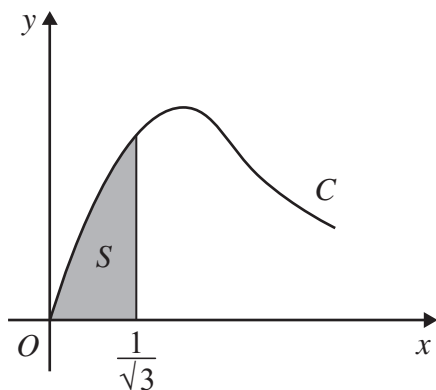


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C , the line $x = \frac{1}{\sqrt{3}}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_0^{\frac{\pi}{6}} \sin^2\theta d\theta$$

where k is a constant. (5)

- (c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants. (3)



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Question 8 continued

Lined writing area for the answer to Question 8.

Q8

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(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



Centre No.						Paper Reference					Surname	Initial(s)		
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 25 January 2010 – Morning
 Time: 1 hour 30 minutes

Examiner’s use only

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Team Leader’s use only

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Materials required for examination
 Mathematical Formulae (Pink or Green)

Items included with question papers
 Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
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Instructions to Candidates

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 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 8 questions in this question paper. The total mark for this paper is 75.
 There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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 You should show sufficient working to make your methods clear to the Examiner.
 Answers without working may not gain full credit.



Turn over

2.

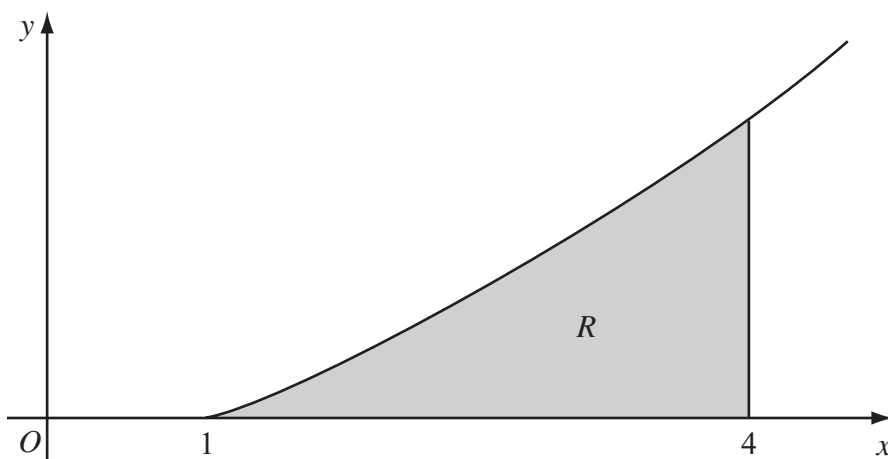


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find $\int x \ln x \, dx$.
 (ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. (7)



3. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y . **(3)**

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P . **(3)**

(c) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers. **(3)**



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Question 3 continued

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4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ .

(a) Write down the coordinates of A . (1)

(b) Find the value of $\cos \theta$. (3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X . (1)

(d) Find the vector \overrightarrow{AX} . (2)

(e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$. (2)

The point Y lies on l_2 . Given that the vector \overrightarrow{YX} is perpendicular to l_1 ,

(f) find the length of AY , giving your answer to 3 significant figures. (3)



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Question 5 continued

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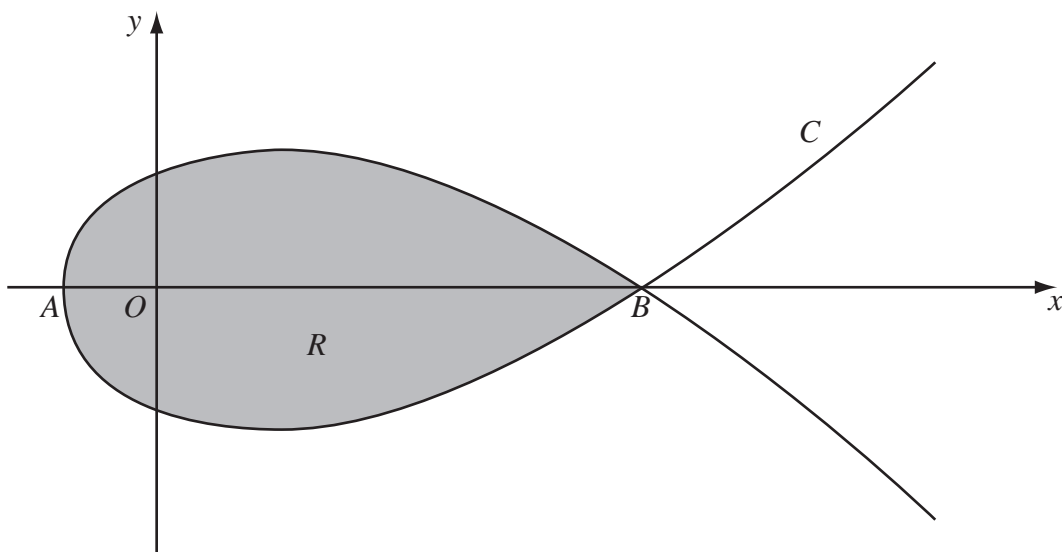


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

- (a) Find the x -coordinate at the point A and the x -coordinate at the point B . (3)

The region R , as shown shaded in Figure 2, is enclosed by the loop of the curve.

- (b) Use integration to find the area of R . (6)



8. (a) Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \tag{7}$$

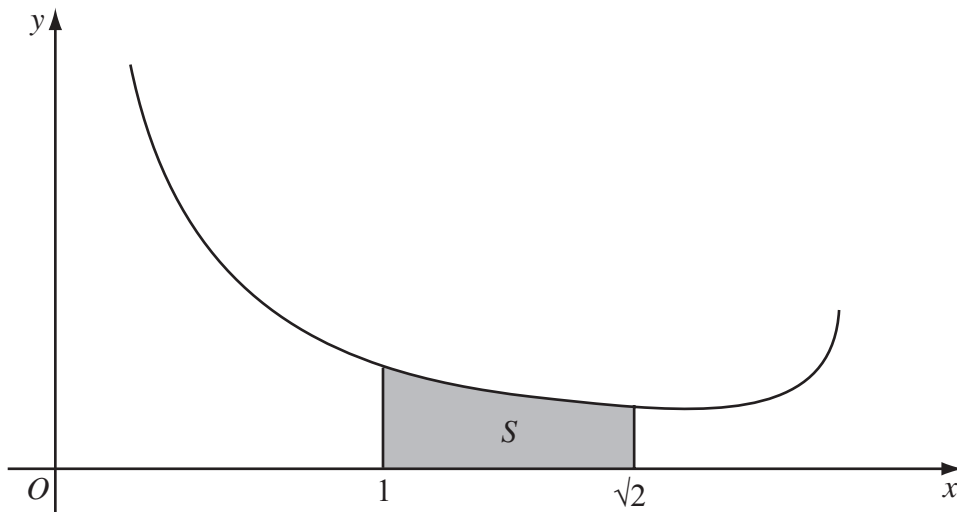


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S , shown in Figure 3, is bounded by the curve, the x -axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Using your answer to part (a), find the exact volume of the solid of revolution formed. (3)



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Question 8 continued

Q8

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(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END



1.

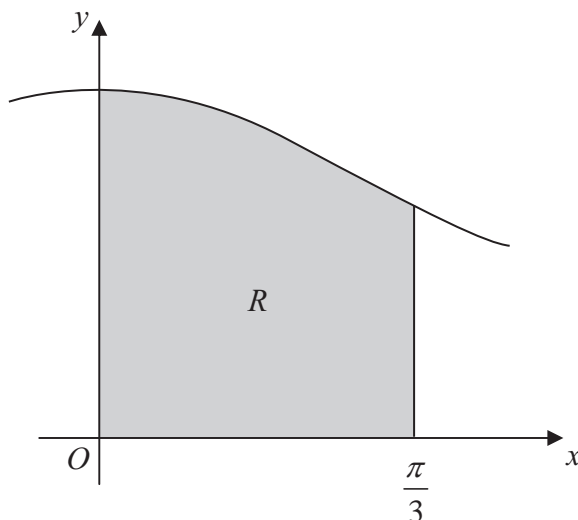


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{0.75 + \cos^2 x}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the y -axis, the x -axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R .

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a

further estimate of the area of R . Give your answer to 3 decimal places.

(6)



4. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2 \tan t, \quad 0 \leq t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t .

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x -axis at the point P .

(b) Find the x -coordinate of P .

(6)



7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C , find

(a) the coordinates of C . (3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB . Give your answer in degrees to 2 decimal places. (4)

(c) Hence, or otherwise, find the area of the triangle ABC . (5)



8.

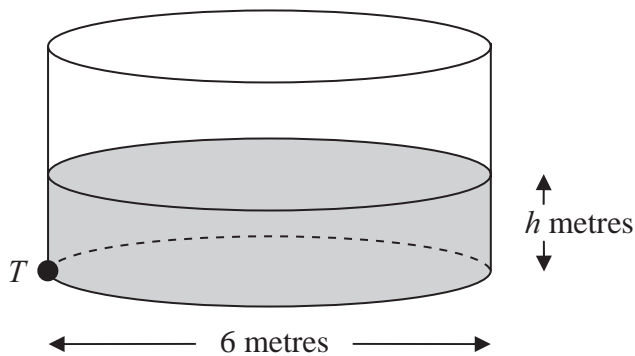


Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of $0.48\pi \text{ m}^3 \text{ min}^{-1}$. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h \text{ m}^3 \text{ min}^{-1}$.

(a) Show that t minutes after the tap has been opened

$$75 \frac{dh}{dt} = (4 - 5h) \tag{5}$$

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$ (6)



Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.					6 6 6 6 / 0 1	Signature		

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced**

**Wednesday 26 January 2011 – Afternoon
Time: 1 hour 30 minutes**

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
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Information for Candidates

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Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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Answers without working may not gain full credit.

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H 3 5 4 0 5 A 0 1 2 4

Turn over

4. Relative to a fixed origin O , the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l .

(a) Find \vec{AB} . (2)

(b) Find a vector equation of l . (2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O , where p is a constant. Given that AC is perpendicular to l , find

(c) the value of p , (4)

(d) the distance AC . (2)



5. (a) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a + bx}{(2 - 3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x , the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) the value of a and the value of b ,

(5)

(c) the coefficient of x^3 , giving your answer as a simplified fraction.

(3)



6. The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$, **(6)**

(b) a cartesian equation of C . **(3)**

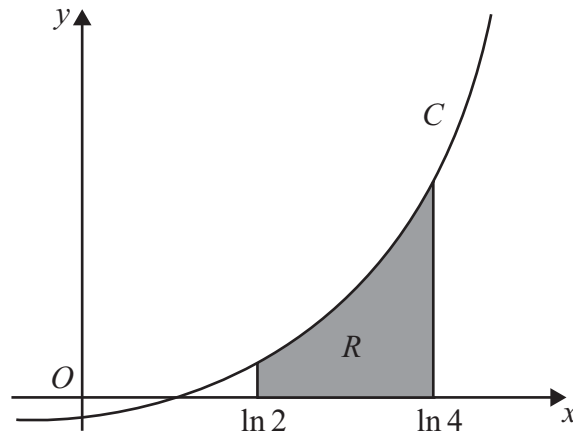


Figure 1

The finite area R , shown in Figure 1, is bounded by C , the x -axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x -axis.

(c) Use calculus to find the exact volume of the solid generated. **(6)**



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7.

$$I = \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx$$

(a) Given that $y = \frac{1}{4 + \sqrt{x-1}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I , giving your answer to 3 decimal places.

(4)

(c) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I .

(8)



3.

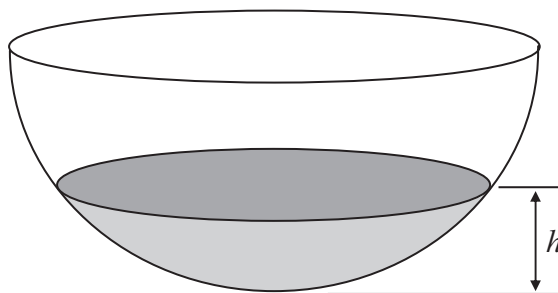


Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume V m³ is given by

$$V = \frac{1}{12} \pi h^2 (3 - 4h), \quad 0 \leq h \leq 0.25$$

- (a) Find, in terms of π , $\frac{dV}{dh}$ when $h = 0.1$ (4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m³s⁻¹.

- (b) Find the rate of change of h , in ms⁻¹, when $h = 0.1$ (2)



4.

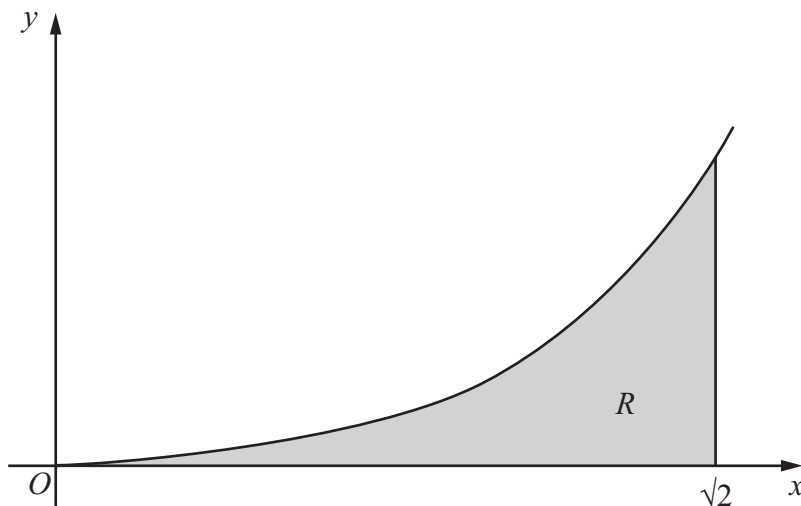


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$. The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \tag{4}$$

(d) Hence, or otherwise, find the exact area of R . (6)



5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0, y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)



6. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A . (6)

(b) Find, to the nearest 0.1° , the acute angle between l_1 and l_2 . (3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1 . (1)

(d) Find the shortest distance from B to the line l_2 , giving your answer to 3 significant figures. (4)



7.

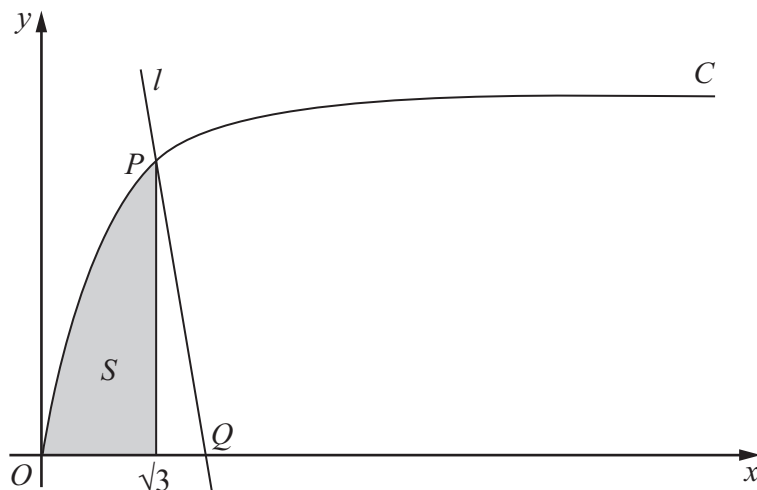


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P . (2)

The line l is a normal to C at P . The normal cuts the x -axis at the point Q .

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k . (6)

The finite shaded region S shown in Figure 3 is bounded by the curve C , the line $x = \sqrt{3}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants. (7)



Centre No.						Paper Reference					Surname	Initial(s)	
Candidate No.					6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 25 January 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
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Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

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Advice to Candidates

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3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x , up to and including the term in x^2 , giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}$, $|x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

(b) find the value of the constant k ,

(2)

(c) find the value of the constant A .

(2)



4.

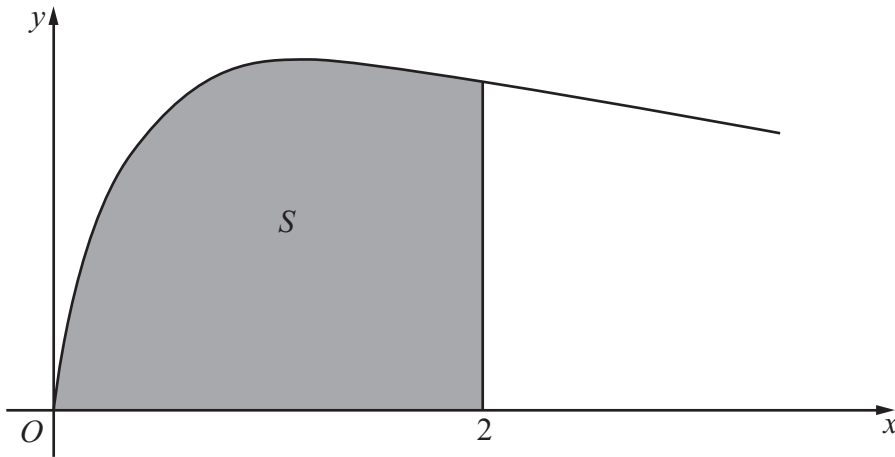


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0$$

The finite region S , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 2$

The region S is rotated 360° about the x -axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)



5.

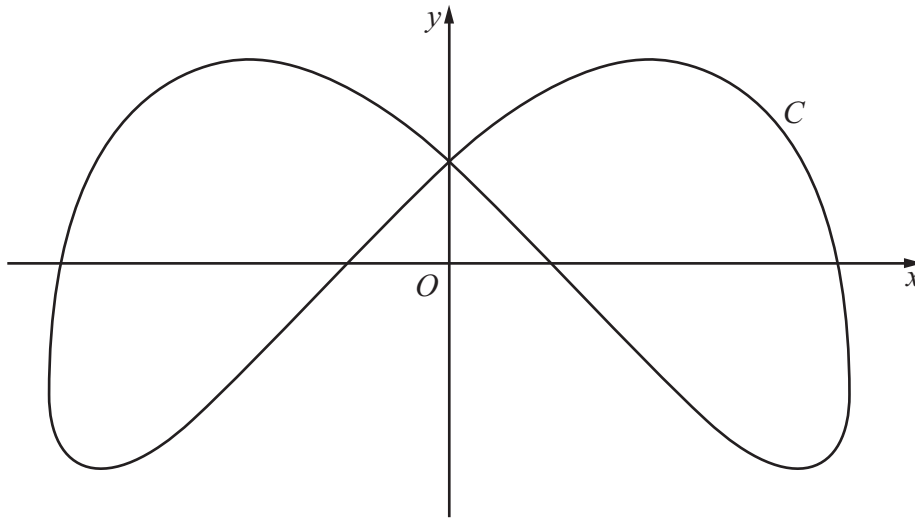


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \sin\left(t + \frac{\pi}{6}\right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . (3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$. (5)



6.

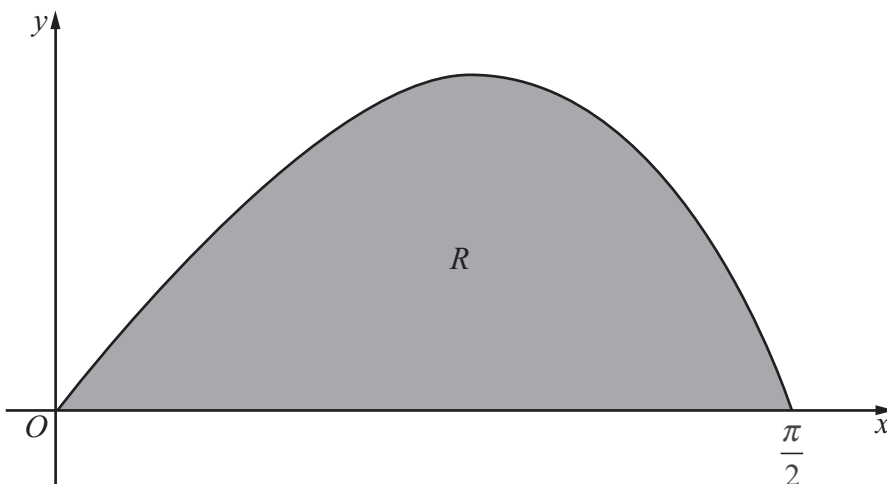


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2 \sin 2x}{(1 + \cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

The table below shows corresponding values of x and y for $y = \frac{2 \sin 2x}{(1 + \cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 4 decimal places. (3)
- (c) Using the substitution $u = 1 + \cos x$, or otherwise, show that
- $$\int \frac{2 \sin 2x}{(1 + \cos x)} dx = 4 \ln(1 + \cos x) - 4 \cos x + k$$
- where k is a constant. (5)
- (d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)



7. Relative to a fixed origin O , the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B .

- (a) Find the vector \overrightarrow{AB} . **(2)**
- (b) Find a vector equation for the line l . **(2)**
- (c) Show that the size of the angle BAD is 109° , to the nearest degree. **(4)**

The points A , B and D , together with a point C , are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

- (d) Find the position vector of C . **(2)**
- (e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures. **(3)**
- (f) Find the shortest distance from the point D to the line l , giving your answer to 3 significant figures. **(2)**



8. (a) Express $\frac{1}{P(5-P)}$ in partial fractions. (3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5 - P), \quad t \geq 0$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

- (b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where a , b and c are integers. (8)

- (c) Hence show that the population cannot exceed 5000 (1)



2.

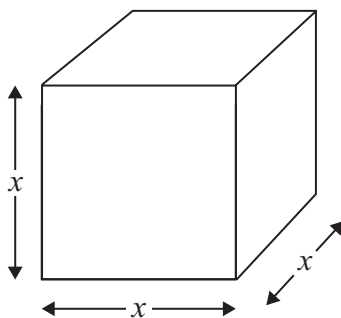


Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated. At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm³.

(a) Show that $\frac{dV}{dx} = 3x^2$ (1)

Given that the volume, V cm³, increases at a constant rate of 0.048 cm³s⁻¹,

(b) find $\frac{dx}{dt}$, when $x = 8$ (2)

(c) find the rate of increase of the total surface area of the cube, in cm²s⁻¹, when $x = 8$ (3)



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3.
$$f(x) = \frac{6}{\sqrt{(9 - 4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient in its simplest form. (6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x , up to and including the term in x^3 , of

(b) $g(x) = \frac{6}{\sqrt{(9 + 4x)}}, \quad |x| < \frac{9}{4}$ (1)

(c) $h(x) = \frac{6}{\sqrt{(9 - 8x)}}, \quad |x| < \frac{9}{8}$ (2)



4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y \cos^2 x} \quad (5)$$



6.

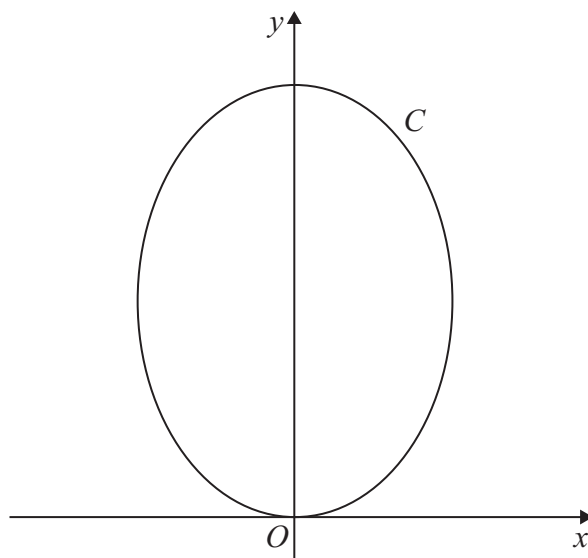


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4 \cos^2 t, \quad 0 \leq t \leq \pi$$

(a) Show that $\frac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined. (5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.
 Give your answer in the form $y = ax + b$, where a and b are constants. (4)

(c) Find a cartesian equation of C . (3)



7.

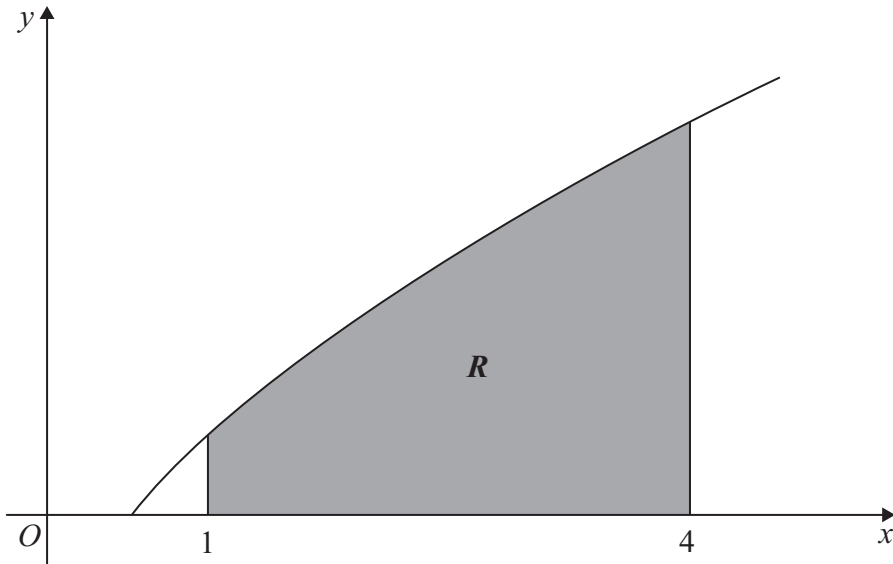


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 4$

- (a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R , giving your answer to 2 decimal places. (4)

- (b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$. (4)

- (c) Hence find the exact area of R , giving your answer in the form $a \ln 2 + b$, where a and b are exact constants. (3)



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Question 7 continued

Lined area for writing the answer to Question 7.



8. Relative to a fixed origin O , the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B .

- (a) Find the vector \vec{AB} . (2)

- (b) Find a vector equation for the line l . (2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l . Given that the vector \vec{CP} is perpendicular to l ,

- (c) find the position vector of the point P . (6)



1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3}$$

find the binomial expansion of $f(x)$, in ascending powers of x , up to and including the term in x^3 .

Give each coefficient as a simplified fraction.

(5)



4.

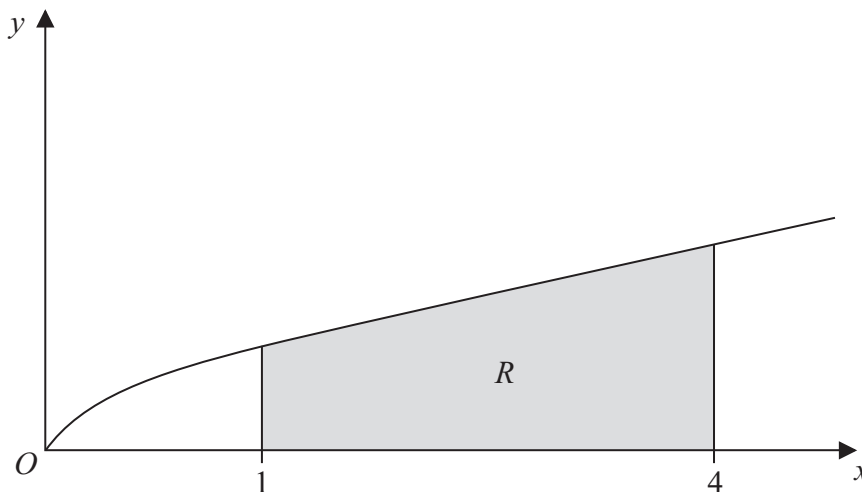


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R , giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R .

(8)



5.

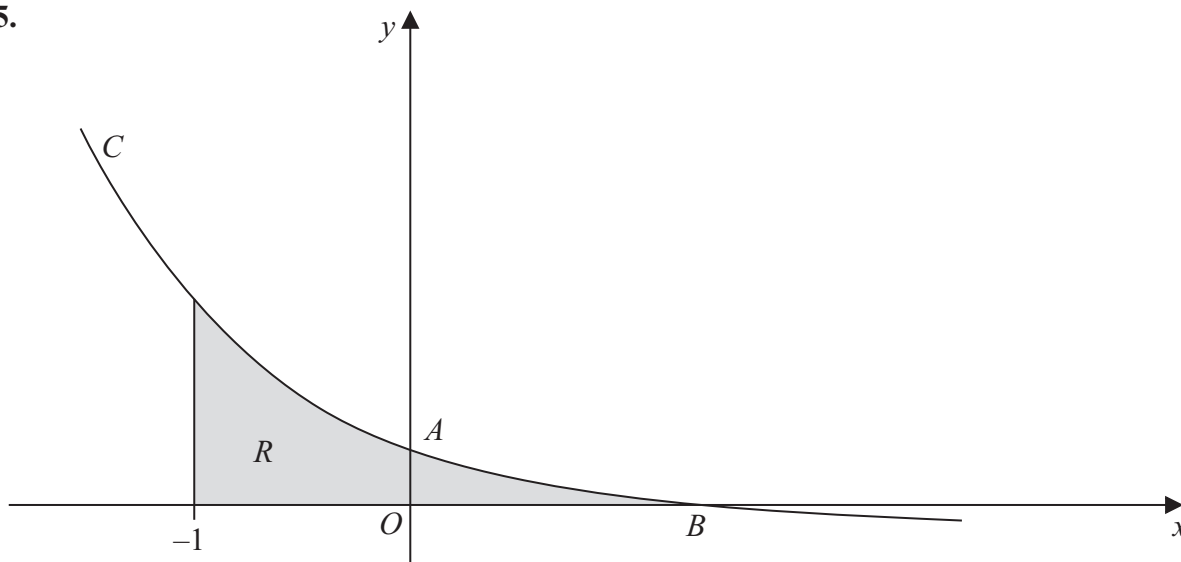


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

- (a) Show that A has coordinates $(0, 3)$. (2)
- (b) Find the x coordinate of the point B . (2)
- (c) Find an equation of the normal to C at the point A . (5)

The region R , as shown shaded in Figure 2, is bounded by the curve C , the line $x = -1$ and the x -axis.

- (d) Use integration to find the exact area of R . (6)



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6.

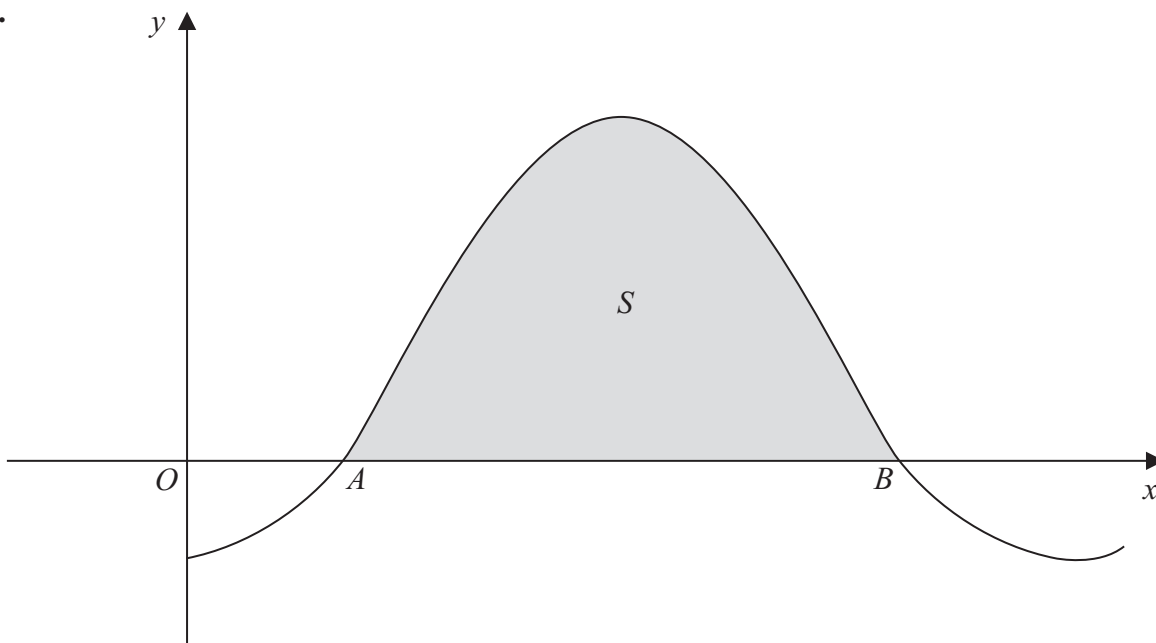


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2 \cos x$, where x is measured in radians. The curve crosses the x -axis at the point A and at the point B .

- (a) Find, in terms of π , the x coordinate of the point A and the x coordinate of the point B . (3)

The finite region S enclosed by the curve and the x -axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x -axis.

- (b) Find, by integration, the exact value of the volume of the solid generated. (6)



7. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection. **(5)**

(b) Find the acute angle between l_1 and l_2 , giving your answer in degrees to 1 decimal place. **(3)**

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1 ,

(c) find the exact coordinates of P . **(6)**



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8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{d\theta}{dt} = \frac{(3 - \theta)}{125}$$

- (a) By solving the differential equation, show that,

$$\theta = Ae^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

- (b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)



Centre No.						Paper Reference					Surname	Initial(s)		
Candidate No.						6	6	6	6	/	0	1	R	Signature

Paper Reference(s)

6666/01R

**Edexcel GCE
Core Mathematics C4
Advanced**

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
 Answer ALL the questions.
 You must write your answer for each question in the space following the question.
 When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
 Full marks may be obtained for answers to ALL questions.
 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 8 questions in this question paper. The total mark for this paper is 75.
 There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
 You should show sufficient working to make your methods clear to the Examiner.
 Answers without working may not gain full credit.

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5.

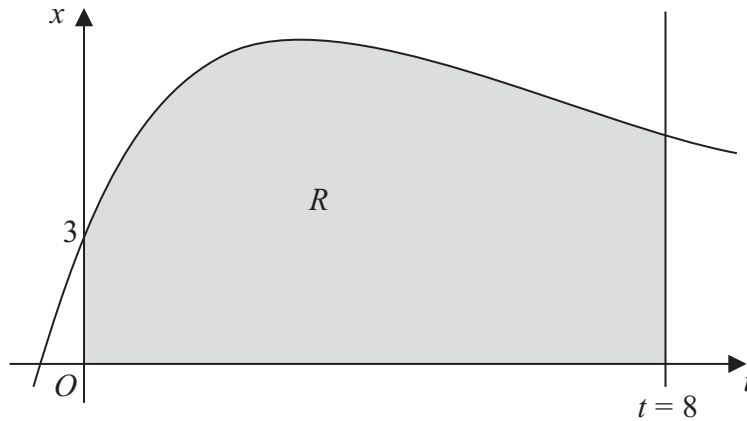


Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x -axis, the t -axis and the line $t = 8$.

- (a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

- (b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R , giving your answer to 2 decimal places.

(3)

- (c) Use calculus to find the exact value for the area of R .

(6)

- (d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

(1)



6. Relative to a fixed origin O , the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a , b and c are constants and λ is a parameter.

Given that the point A lies on the line l ,

- (a) find the value of a .

(3)

Given also that the vector \vec{AB} is perpendicular to l ,

- (b) find the values of b and c ,

(5)

- (c) find the distance AB .

(2)

The image of the point B after reflection in the line l is the point B' .

- (d) Find the position vector of the point B' .

(2)



7.

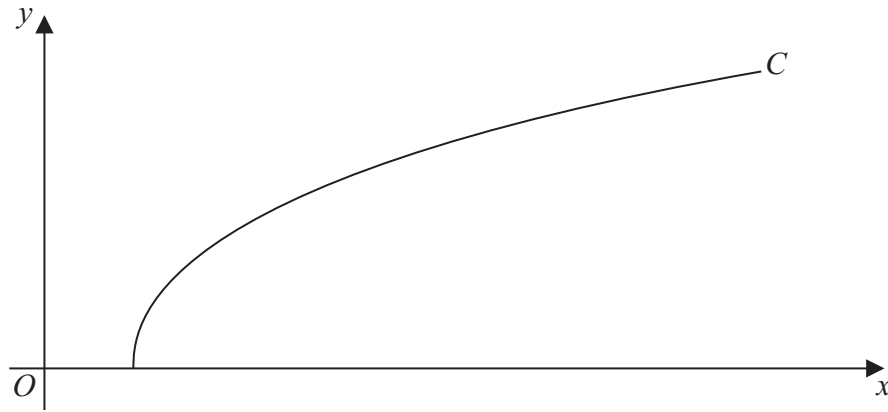


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27 \sec^3 t, \quad y = 3 \tan t, \quad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \frac{\pi}{6}$ (4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \quad a \leq x \leq b$$

stating the values of a and b . (3)

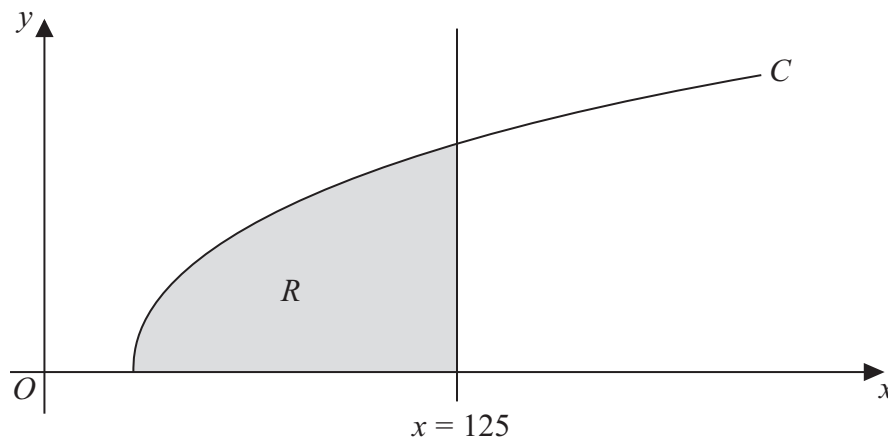


Figure 3

The finite region R which is bounded by the curve C , the x -axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution. (5)



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Question 7 continued

Lined area for writing the answer to Question 7 continued.



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- 8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{dx}{dt} = k(M - x), \text{ where } M \text{ is a constant.}$$

- (a) Explain, in the context of the problem, what $\frac{dx}{dt}$ and M represent. (2)

Given that initially the mass of waste products is zero,

- (b) solve the differential equation, expressing x in terms of k , M and t . (6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

- (c) find the value of x when $t = \ln 9$, expressing x in terms of M , in its simplest form. (4)



Centre No.						Paper Reference						Surname	Initial(s)
Candidate No.					6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced**

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
2	
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Total	

Materials required for examination Mathematical Formulae (Pink)	Items included with question papers Nil
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Advice to Candidates

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You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



2. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1 \tag{6}$$

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)



3.

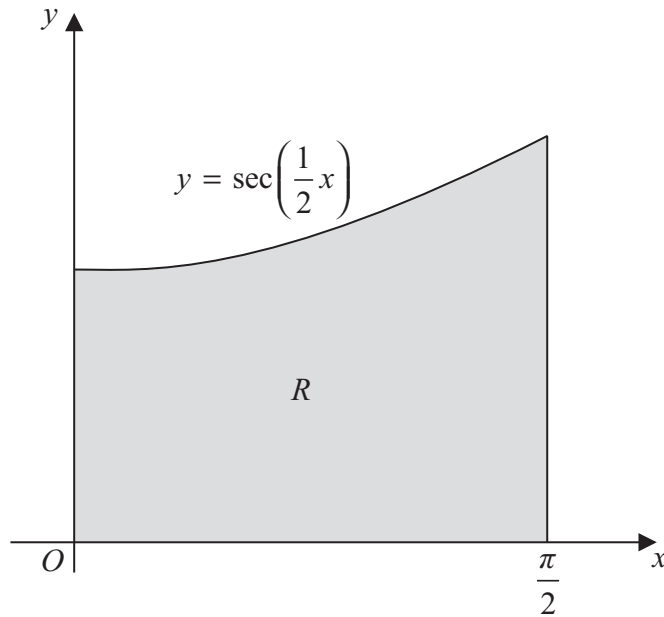


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

- (a) Complete the table above giving the missing value of y to 6 decimal places. (1)
- (b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

- (c) Use calculus to find the exact volume of the solid formed. (4)



4. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$ **(4)**

(b) Find a cartesian equation for C in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant k . **(3)**

(c) Write down the range of $f(x)$. **(2)**



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5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)} dx = \int \frac{2}{u(2u - 1)} du \quad (3)$$

- (b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)



Leave blank

6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \quad \theta \leq 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t} \tag{8}$$

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off. (3)



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8. With respect to a fixed origin O , the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O , where p is a constant.

Given that vector \vec{PA} is perpendicular to l ,

(a) find the value of p . (4)

Given also that B is a point on l such that $\angle BPA = 45^\circ$,

(b) find the coordinates of the two possible positions of B . (5)



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Question 8 continued

Lined writing area for the answer to Question 8.

Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END



Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$f(x)$	$\int f(x) \, dx$
$\sec^2 kx$	$\frac{1}{k} \tan kx$
$\tan x$	$\ln \sec x $
$\cot x$	$\ln \sin x $
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x , \quad \ln \tan(\frac{1}{2}x) $
$\sec x$	$\ln \sec x + \tan x , \quad \ln \tan(\frac{1}{2}x + \frac{1}{4}\pi) $

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$