Pure Core 4 Past Paper Questions: Mark Scheme

Taken from MAP2, MAP3

Pure 2 June 2001

6 (a)	$\cos 2x = \cos^2 x - \sin^2 x$	B1		
	$= (\cos x - \sin x)(\cos x + \sin x)$	M1		Difference of two squares
	Hence result	A1	3	
(b)	$R = \sqrt{2}, \quad a = 45^\circ$	B1B1		
	$\sqrt{2}\sin(x+45) = \frac{1}{2}$	M1		
	$x = 114^\circ$	A1√		
	$x = 336^\circ$	A1√	5	AWRT these are OK –1 for any extra solutions
	Total		**8**	

Pure 2 January 2002

4 (a)	$\frac{2\tan x}{1+\tan^2 x} = \frac{2\sin x}{\cos x \sec^2 x}$	B1		Use of $\sec^2 x$ identity
	$= \frac{2\sin x \cos^2 x}{\cos x}$	M1 A1		reduction to sin/cos only
	$= 2\sin x \cos x$ $= \sin 2x$	A1	4	AG
(b)				Alternative method $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$
	$\sin 30^\circ = \frac{1}{2}$	B1		
	$= \frac{2t}{1+t^2}$	M1		$\tan(60-45)$ M1 $\tan(45-30)$ $= \frac{\tan 60 - \tan 45}{1+\tan 60 \tan 45}$ B1 $= \frac{\tan 45 - \tan 30}{1+\tan 45 \tan 30}$ $\tan 60 = \sqrt{3}$ $\quad \tan 30 = \frac{1}{\sqrt{3}}$
	$t^2 - 4t + 1 = 0$ $t = \frac{4 \pm \sqrt{16-4}}{2}$	M1		$= \frac{\sqrt{3}-1}{1+\sqrt{3}}$ A1 $= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$
	$= 2 \pm \sqrt{3}$	A1		Correct attempt to rationalise M1
	$a = 2, \; b = -1$	A1	5	$2-\sqrt{3}$ A1 $2-\sqrt{3}$
	Total		**9**	

Q	Solution	Marks	Total	Comments
5 (a)	$\cos\alpha = -\frac{5}{13}$	M1A1	2	$\cos\alpha = \frac{5}{13}$ M0A0 unless from $s^2 + c^2 = 1$ in which case M1A0
(b)	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	M1		use of
	$= \left(\frac{12}{13}\times\frac{4}{5}\right) - \left(\frac{5}{13}\times\frac{3}{5}\right)$	A1F		f.t. $\cos\alpha$ from (a) Non-exact value gets M1A0A0 or possibly M1A1A0
	$= \frac{33}{65}$	A1F	3	
	Total		**5**	

Pure 2 June 2002

8 (a)	Use of an appropriate identity Simplify/cancel to AG	B1 B2	3	
(b)	$\cos^2\theta = 2\sin 2\theta$ $= 4\sin\theta\cos\theta$	B1		
	$\cos\theta(\cos\theta - 4\sin\theta) = 0$	M1		Simplify and factorise Condone division by $\cos\theta$
	$\cos\theta = 0$ $\theta = 90,\ 270$	A1A1		
	$\tan\theta = \frac{1}{4}$ $\theta = 14^\circ, 194^\circ$	A1A1	6	
	Total		**9**	

Pure 2 January 2003

4 (a)(i)	$L = 2\sin\theta + 4\cos\theta$	B1	1	Accept unsimplified
(ii)	$R\sin(\theta+\alpha) = R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$ $R\cos\alpha = 2,\ R\sin\alpha = 4$	M1 A1F		**Alternative** $2\sin\theta + 4\cos\theta$
	$R = \sqrt{20}$, $\alpha = 1.107$ (AWRT 1.11)	A1FA1F	4	$= \sqrt{20}\left(\frac{2}{\sqrt{20}}\sin\theta + \frac{4}{\sqrt{20}}\cos\theta\right)$ M1A1F $= \sqrt{20}\,(\cos\alpha\sin\theta + \sin\alpha\cos\theta)$ A1F $= \sqrt{20}\ \sin(\theta+\alpha),\ \alpha = 1\cdot107$ A1F For ft, must be in form $a\sin\theta + b\cos\theta, \alpha$ in radians
(b)(i)	$L_{\max} = \sqrt{20}\ (4.47)$	B1F	1	
(ii)	Maximum when $\theta + \alpha = \frac{\pi}{2}$	M1		Or $\theta + \alpha = 90^\circ$
	$\theta \approx 0\cdot46$	A1F	2	CAO
	Total		**8**	

Pure 2 June 2003

3 (a)	$\tan(45^\circ + \theta) = \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ \tan\theta}$	M1		Use of correct formula for $\tan(A+B)$
	$= \frac{1+\tan\theta}{1-\tan\theta}$	A1	2	Replace $\tan 45^\circ = 1$
(b)	Put $\theta = 60^\circ$: $\tan 105^\circ = \frac{1+\sqrt{3}}{1-\sqrt{3}}$	M1		use of $\theta = 60^\circ$ i.e. $\tan 105 = \frac{1+\tan 60}{1-\tan 60}$
		A1		use of $\tan 60 = \sqrt{3}$ in correct formula for $\tan(A+B)$ or equiv $\frac{3}{\sqrt{3}}$
	$= \frac{(1+\sqrt{3})^2}{(1+\sqrt{3})(1-\sqrt{3})}$	M1		attempt at rationalisation
	$= \frac{1+2\sqrt{3}+3}{-2}$			
	$= -2-\sqrt{3}$	A1F	4	ft if of required form
	Total		**6**	

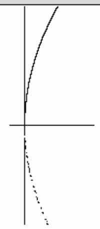
Pure 2 January 2004

3 (a)	$\beta = \tan^{-1}(2.4) = 1.176°$	B1	1	
(b)	$10\sin\theta + 24\cos\theta \equiv R\sin(\theta+\alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R\sin\alpha = 24$ $R\cos\alpha = 10$	M1		Any correct attempt at finding R or α
	$\tan\alpha = 2.4$ $\quad \therefore \alpha = 1.176°$	A1		Correct α (AWRT 1.18)
	$R^2 = 24^2 + 10^2 = 676$ $\quad R = 26$	A1		Correct R
	$\Rightarrow 26\sin(\theta + 1.176)$		3	
(c)(i)	Maximum value = 26	B1√	1	On their answer to part (b) $(\pm 26$ gets B0$)$ (based on a valid method used in (b))
(ii)	$\sin(\theta + 1.176) = 1$	M1		
	$\therefore \theta + 1.176 = \frac{\pi}{2}$ $\theta = 0.395°$	A1√	2	On their value of α (6.68, 13.0, ………..)
	Total		**7**	

Pure 2 June 2004

2(a)	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$……(i) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$…….(ii)			
	add the two equations (i) & (ii) together $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$	M1 A1	2	AG
(b)(i)	$2\sin 8x\cos 2x = \sin(8x + 2x) + \sin(8x - 2x)$	M1		
	$= \sin 10x + \sin 6x$	A1	2	
(ii)	$\int 6\sin 8x\cos 2x\,dx$			
	$= 3\int(\sin 10x + \sin 6x)\,dx$	M1ft		Use their (i)
	$= 3\left(\frac{-\cos 10x}{10} - \frac{\cos 6x}{6}\right) + c$	M1ft		Integration attempted
	$= -\frac{3}{10}\cos 10x - \frac{1}{2}\cos 6x + c$	A1ft	3	Any correct form
	Total		**7**	

Pure 3 June 2001

Q	Solution	Marks	Total	Comments
1 (a)		B1	1	
(b)	$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{8}{8t}$	M1 A1	2	Use of chain rule
(c)	$t=0 \Rightarrow x=1, \quad y=4$	B1		Use of $(1, 4)$
	Gradient $=\frac{y-4}{x-1}=\frac{1}{0.5}$	M1		
	$y=2x+2$	A1	3	OE: e.g. $(y-4)=2(x-1)$
	Total		**6**	

Q	Solution	Marks	Total	Comments
3 (a)	$\frac{dP}{dt}$ is rate of increase of population	B1		
	This is proportional to P $\left(\Rightarrow \frac{dP}{dt}=kP\right)$	B1	2	
(b)(i)	$\frac{dP}{P}=k\,dt$	M1		
	$(\ln P=kt+c)$			
	$P=\left(e^{kt+c}\right) \quad \left[=Ae^{kt}\right]$	A1		
	$t=0, \quad A=1000$	A1		
	$t=30, \quad k=\frac{1}{30}\ln 2$	A1	4	
(ii)	$1000\,e^{\frac{1}{30}\ln 2t}=5000e^{-0.05t}$	M1 m1		Equate populations Take logarithms
	$\frac{1}{30}\ln 2t+0.05t=\ln 5$	A1		OE any correct expression
	$t=22$	A1	4	
	Total		**10**	

Q	Solution	Marks	Total	Comments
6 (a)	$\begin{bmatrix}3\\-1\\-2\end{bmatrix}\bullet\begin{bmatrix}3\\4\\5\end{bmatrix}(=-5)$ attempted	M1		
	$\cos\theta=\frac{-5}{\sqrt{14}\sqrt{50}}$	m1 B1		$\sqrt{14}$ or $\sqrt{50}$ seen
	$\theta=100.9^\circ \quad \Rightarrow 79.1^\circ$ line and normal	A1		
	$\Rightarrow 10.9^\circ$ line and plane	B1√	5	$90^\circ-\angle$ between line and normal
(b)			5	
	$\mathbf{AB}=\begin{bmatrix}2\\3\\3\end{bmatrix}$	B1		
	Line l_2 is $\mathbf{r}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+t\begin{bmatrix}2\\3\\3\end{bmatrix}$	B1√		OE: ft on $\begin{bmatrix}2\\3\\3\end{bmatrix}$
	$2+3s=3+2t$ $4s=-2+3t$	M1		Set up and attempt to solve any two simultaneous equations
	$s=7, \quad t=10$	A1√		ft on equations
	$4+3t=-1+5s=34$	A1		Check: 3rd equation
	Total		**10**	

7 (a)	$f(x)=\frac{A}{1+2x}+\frac{B}{4-x}$ $=\frac{2}{1+2x}+\frac{1}{4-x}$	B1 M1 A1	3	Any appropriate method
(b)(i)	$\frac{1}{4-x}=\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$ $=\frac{1}{4}\left[1+(-1)\left(-\frac{x}{4}\right)+\frac{(-1)(-2)}{2}\left(-\frac{x}{4}\right)^2\right]$	B1 M1 A1	3	$4\left(1-\frac{x}{4}\right)$ AG
(ii)	$\frac{1}{1+2x}=(1+2x)^{-1}$ $=\left[1+(-1)(2x)+\frac{(-1)(-2)}{2}(2x)^2\right]$ $=1-2x+4x^2$	M1 A1	2	
(iii)	$f(x)=2\left(1-2x+4x^2\right)+\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}$ $=\frac{9}{4}-\frac{63}{16}x+\frac{513}{64}x^2$	M1 A1	2	Accept $2.25-3.94x+8.02x^2$
(iv)	$-4<x<4,\quad -\frac{1}{2}<x<\frac{1}{2}$ valid for $-\frac{1}{2}<x<\frac{1}{2}$	B1 B1	2	B2 for $-\frac{1}{2}<x<\frac{1}{2}$ stated
(c)(i)	$\int f(x)\,dx=\ln\lvert 1+2x\rvert-\ln\lvert 4-x\rvert$	M1 A1	2	$k\ln\lvert 1+2x\rvert$ $l\ln\lvert 4-x\rvert$
(ii)	$\int_0^{0.25} f(x)=[0.4055-1.3218]-[1.3863]$ $=0.470$ $\left[\frac{9}{4}x-\frac{63}{16}\cdot\frac{x^2}{2}+\frac{513}{64}\cdot\frac{x^3}{3}\right]_0^{0.25}$ $=0.481$ Error $=0.011$	B1√ M1 A1 A1√	4	ft on $k\ln\lvert 1+2x\rvert+l\ln\lvert 4-x\rvert$ ft on difference between integrals
	Total		**18**	

Pure 3 January 2002

2	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{-2}{t^2}\times\frac{1}{2}$ $t=2\quad \frac{dy}{dx}=\frac{-1}{4}$ gradient of normal = 4 $y=4x+c$ $t=2,\ x=7,\ y=2$ $y=4x-27$ **Alternative** Eliminate t $y=\frac{4}{x-3},\ xy=4+3y,\ x=\frac{4}{y}+3$ $\frac{dy}{dx}=\frac{-4}{(x-3)^2}\times\frac{dy}{dx}+y=3\frac{dy}{dx}\frac{dx}{dy}=\frac{-4}{y^2}$ t = 2 $\quad x=7\quad y=1\quad \frac{dy}{dx}=\frac{-1}{4}$ gradient of normal = 4 $y=4x+c$ y = $4x-27$	M1 A1 B1F B1F M1 A1 *(M1)* *(A1)* *(B1ft)* *(B1ft)* *(M1)* *(A1)*	6	Use chain rule Substitute $t=2$ in $\frac{dy}{dx}$ Follow on gradient Use (7,1) and gradient Attempt to differentiate correct expression Follow on $\frac{dy}{dx}$ Follow on gradient
	Total		**6**	

Q	Solution	Marks	Total	Comments
4 (a)(i)	$P = 15000$	B1	1	
(ii)	$11000 = 15000\,e^{-2k}$	M1		
	$-2k = \ln\left(\frac{11}{15}\right)$	m1		
	$k = 0.155$	A1	3	
(b)	$18000\,e^{-0.175t} = 15000\,e^{-kt}$	M1		
	$1.2 = e^{0.02t}$	A1		OE
	$\ln 1.2 = 0.02t$	M1		
	$t = 9.1 \quad \text{year} = 2009$	A1	4	AWRT 9.1; accept 9.5
		(B2)		Special case – use of trial values of t t=9 B2(Max 2/4)
	Total		**8**	

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\frac{dh}{dt} = \pm k\sqrt{h}$	M1 A1	2	$\frac{dh}{dt} = \ldots \quad \frac{dh}{dt}\alpha\sqrt{h}\ldots$
(ii)	$\int \frac{dh}{\sqrt{h}} = \int \pm k\,dt$	M1		
	$2h^{1/2} = \pm kt + C$ At $t = 0,\ h = 1 \quad C = 2$	A1		
	$2\sqrt{h} = 2 - kt$	A1	3	AG
(iii)	At $t = 2,\ h = 1/2$			
	$k = \frac{2 - 2\sqrt{1/2}}{2} = 0.293$	M1 A1	2	Use AG and solve for k
(b)	At $h = 0,\ t = \frac{C}{k} = \frac{2}{0.293}$	M1		Use h=0 and solve for t
	$= 6.8$ hours $=$ 6 hrs 50 mins	A1	2	Accept 410 minutes
	Total		**9**	

Pure 3 June 2002

Q	Solution	Marks	Total	Comments
1 (a)	$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = 2 \times \frac{-1}{2t}$	M1 A1	2	
(b)	$t = 3 \qquad$ gradient normal $= 3$	B1ft		Ft on gradient tangent $\left(\frac{-1}{\text{gradient tangent}}\right)$
	$t = 3 \quad x = -8 \quad y = 6$	B1		
	$y = 3x + c$	M1		
	$y = 3x + 30$	A1	4	
	Total		**6**	

2 **(a)**	$\dfrac{4-x}{(1-x)(2+x)}=\dfrac{A}{1-x}+\dfrac{B}{2+x}$			
	$4-x=A(2+x)+B(1-x)$	M1		
	$x=1\quad A=1\quad x=-2\quad B=2$	M1A1	3	Attempt to find A and B
(b)(i)	$\dfrac{1}{2+x}=\dfrac{1}{2}\left(1+\dfrac{x}{2}\right)^{-1}$	B1		
	$=\dfrac{1}{2}\left(1+-1\times\dfrac{x}{2}+\dfrac{-1\times-2}{2}\left(\dfrac{x}{2}\right)^2\right)$	M1		Use of binomial series $n=-1$ use $\dfrac{x}{2}$
		A1	3	AG convincingly obtained
(ii)	$\dfrac{1}{1-x}=(1-x)^{-1}$			
	$=1+-1\times(-x)+\dfrac{-1\times-2}{2}(-x)^2$	M1		
	$=1+x+x^2$	A1	2	
	Alternative to part (b) by Maclaurin			
	$f(x)=\quad(2+x)^{-1}\quad f(0)=\dfrac{1}{2}$	*(M1)*		Differentiate twice
	$f'(x)=\quad-(2+x)^{-2}\quad f'(0)=-\dfrac{1}{4}$	*(A1)*		
	$f''(x)=\quad 2(2+x)^{-3}\quad f''(0)=\dfrac{2}{8}$			
	$f(x)=\quad\dfrac{1}{2}-\dfrac{1}{4}x+\dfrac{1}{2}\times\dfrac{2}{8}x^2$	*(A1)*		AG obtained using $x=0$, in Maclaurin's series
	OR			
	$(2+x)^{-1}\quad=2^{-1}+(-1)\times2^{-2}x+\dfrac{-1\times-2\times2^{-3}}{2!}x^2$	*(M1A1)* *(A1)*		use negative powers of 2 all correct

Q	Solution	Marks	Total	Comments
(c)	$\dfrac{4-x}{(1-x)(2+x)}=2+\dfrac{x}{2}+\dfrac{5}{4}x^2$	M1A1	2	Ignore extra terms
	Alternative to part (c)			
	$(4-x)\left(\dfrac{1}{2}-\dfrac{x}{4}+\dfrac{x^2}{8}\right)(1+x+x^2)=a+bx$	*(M1)*		
	Correct expansion	*(A1)*		
	Total		**10**	

3 (a)	$x = 2 \quad y = \pm\dfrac{5\sqrt{5}}{3} = \pm 3.73$	M1A1	2	allow ± 3.7, or any correct numerical form
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x^2}{9}+\dfrac{y^2}{25}\right) = \dfrac{\mathrm{d}}{\mathrm{d}x}(1)$	M1		attempt implicit differentiation LHS only, with use of chain rule.
	$\dfrac{2x}{9}+\dfrac{2y}{25}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1		OE correct differentiation
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm\dfrac{2}{9}\ 2\ \dfrac{25}{2}\ \dfrac{3}{5}\quad \dfrac{1}{\sqrt{5}} = \pm 1.5$	M1A1	4	substitute $x = 2$, and values for y. Accept ±1.49…
	Alternative to part (b)			
	$y = 5\sqrt{1-\dfrac{x^2}{9}}$	*(M1)*		differentiate a function of form $y = a\sqrt{c+bx^2}$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5\times\dfrac{1}{2}\times -\dfrac{2}{9}x\left(1-\dfrac{x^2}{9}\right)^{-\frac{1}{2}}$	*(m1)*		use chain rule
	$x = 2;\ y = \pm 3.73$	*(A1)* *(A1)*		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm 1.5$
	Total		**6**	
4 (a)	$\dfrac{1}{2}m_0 = m_0\,\mathrm{e}^{-28k}$	M1		Allow any value for m_0
	$\dfrac{1}{2} = \mathrm{e}^{-28k}$	A1		
	$\ln\dfrac{1}{2} = -28k$	M1		Take lns of an exponential expression
	$k = 0.024755(256)$	A1	4	AG convincingly obtained NB sign $\dfrac{-\ln\frac{1}{2}}{28}$ SC – substitute $k = 0.024755$ into e^{-28k} explain why this shows mass is halved max 2/4
(b)	$1 = m\,\mathrm{e}^{-100k}$ $m = 11.9$ g	M1 A1	2	accept 11.89
	Alternative to part (b) $t = -100 \quad m = 1\mathrm{e}^{100k}$ $= 11.9$	*(M1)* *(A1)*		$\dfrac{100}{28} = 3.57$ number of half lives $2^{3.57}$
	Total		**6**	

Q	Solution	Marks	Total	Comments
5 (a)	$\int y^2\,\mathrm{d}y = \int 1\,\mathrm{d}x$	M1		Separate; attempt to integrate both sides
	$\dfrac{1}{3}y^3 = x + c$	A1A1		or $\dfrac{1}{3}y^3 + c = x$ A1A0 2 out of $\dfrac{1}{3}y^3,\ x,\ c$
	$y = \sqrt[3]{3x+K}$	A1	4	Accept $3c$ for K.
(b)	$-1^3 = 3\times 1 + K$	M1		Use of (1,-1) in an expression with a constant.
	$y^3 = 3x - 4 \qquad y = \sqrt[3]{3x-4}$	A1	2	Correct expression connecting y and x. Allow $K = -4$
	Total		**6**	

8 (a)	$3+4t=8-s$	M1		Set up and attempt to solve
	$-2+4t=-1+3s$			
	$t=1 \quad s=1$	m1A1		
	$1+3\times1=2+1\times2=4$	A1		Check third equation
	$(x,y,z)=(7,\ 2,\ 4)$	B1ft	5	ft on consistent use of s or t
(b)(i)	$4\times1+4\times11+3\times-16=0$	M1		Use scalar product with a direction
	$-1\times1+3\times11+2\times-16=0$	A1	2	Both equal zero

Pure 3 January 2003

1 (a)	$(1+x)^{\frac{1}{2}}=1+\frac{1}{2}x+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)x^2}{2}$	M1		
	$=1+\frac{1}{2}x-\frac{1}{8}x^2$	A1	2	
(b)(i)	$\sqrt{4+2x}=2\sqrt{1+\frac{x}{2}}$	B1		(b)(i)Specialcases
	$=2\left(1+\frac{1}{2}\left(\frac{x}{2}\right)-\frac{1}{8}\left(\frac{x}{2}\right)^2\right)$	M1		Allow A1F for $2+\frac{x}{2}+\frac{x^2}{16}$ follow $1+\frac{1}{2}x+\frac{1}{8}x^2$
	$=2+\frac{x}{2}-\frac{x^2}{16}$	A1	3	or $4\left(1+\frac{x}{4}-\frac{x^2}{32}\right)=4+x-\frac{x^2}{8}$
	Alternative using $(a+x)^n=a^n+na^{n-1}x+\frac{n(n-1)a^{n-2}}{2}x^2\ldots$			
	$(4+2x)^{\frac{1}{2}}=4^{\frac{1}{2}}+\frac{1}{2}4^{-\frac{1}{2}}2x+$	*(M1*		M1 – use of $n=\frac{1}{2};a=4\ \ x\to 2x$
	$\frac{\frac{1}{2}\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}}{2}(2x)^2$	*A1)*		A1 – correct
	$=2+\frac{1}{2}.\frac{1}{2}.2x-\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{8}4x^2$			
	$=2+\frac{1}{2}x-\frac{1}{16}x^2$	*(A1)*		A1 – correct simplification
(ii)	$-2<x<2$	B1	1	
	Total		**6**	

2 (a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $= -\sin t \times \frac{1}{3\cos t}$	M1 A1		Chain rule and derivatives attempted.
	$t = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = -\frac{1}{3}$	B1	3	For B1, substitution of $t = \frac{\pi}{4}$ into expression for $\frac{dy}{dx}$ seen. Allow $-\frac{\sin\frac{\pi}{4}}{3\cos\frac{\pi}{4}} = -\frac{1}{3}$ or $-\frac{0.707}{2.121} = -\frac{1}{3}$ AG
	Alternative $\frac{x^2}{9} + y^2 = 1 \qquad y = \sqrt{1 - \frac{x^2}{9}}$			
	$\frac{dy}{dx} = \frac{1}{2}\left(1 - \frac{x^2}{9}\right)^{-\frac{1}{2}}\left(-\frac{2x}{9}\right)$	*(M1)* *(A1)*		
	$t = \frac{\pi}{4} \qquad x = \frac{3}{\sqrt{2}}$ $\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1}{2}}} \cdot \left(\frac{-6}{9\sqrt{2}}\right) = -\frac{3}{9} = -\frac{1}{3}$	*(A1)*		
(b)	$t = \frac{\pi}{4} \qquad x = \frac{3}{\sqrt{2}} \qquad y = \frac{1}{\sqrt{2}}$	M1 A1		allow 2.12, 0.71
	$y - \frac{1}{\sqrt{2}} = -\frac{1}{3}\left(x - \frac{3}{\sqrt{2}}\right)$	M1		
	$y = -\frac{1}{3}x + \sqrt{2}$	A1	4	$\sqrt{2}$ OE numerical form Accept 1.4
	Alternative $x = 3\sin\frac{\pi}{4} \quad y = \cos\frac{\pi}{4}$	*(M1)*		
	$y - \cos\frac{\pi}{4} = -\frac{1}{3}\left(x - 3\sin\frac{\pi}{4}\right)$	*(M1)*		
	$y = -\frac{1}{3}x + \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$	*(A1)*		
	$y = -\frac{1}{3}x + \sqrt{2}$	*(A1)*		
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)(i)	$\dfrac{x^2}{x^2-16}=\dfrac{x^2-16+16}{x^2-16}$	B1	1	OE eg by division ; AG Use of a particular value of $x, x=0,1,2...$ showing LHS=RHS is B0 (see equivalents)
	Accepted equivalents			
	$\dfrac{x^2}{x^2-16}=1+\dfrac{16}{x^2-16}$ $\Rightarrow x^2=x^2-16+16$ $=x^2$			
	$\dfrac{x^2}{x^2-16}=A+\dfrac{B}{x^2-16}$ $\Rightarrow x^2=A\left(x^2-16\right)+B$ $x=4 \quad \Rightarrow B=16$ $\Rightarrow A=1$			
	$\dfrac{x^2}{x^2-16}=1+\dfrac{A}{x^2-16}$ $\Rightarrow x^2=x^2-16+A$ $\Rightarrow A=16$			
(ii)	$\dfrac{16}{x^2-16}=\dfrac{A}{x-4}+\dfrac{B}{x+4}$ $16=A(x+4)+B(x-4)$			
	$x=4 \Rightarrow A=2$	M1		Any equivalent method
	$x=-4 \Rightarrow B=-2$	A1	2	
(b)	$\int_5^8\left(1+\dfrac{2}{x-4}-\dfrac{2}{x+4}\right)\mathrm{d}x$			
	$=\left[x+2\ln\lvert x-4\rvert-2\ln\lvert x+4\rvert\right]_5^8$	M1 A1		$x+k\ln(x-4)+l\ln(x+4)$
	$=\left(8+2\ln\lvert 4\rvert-2\ln\lvert 12\rvert\right)-\left(5+2\ln\lvert 1\rvert-2\ln\lvert 9\rvert\right)$	m1		Allow both M1, m1 if $\int 1\mathrm{d}x=x$ is omitted.
	$=3+2\ln 3$	A1	4	ft on both A marks on values of A, B. Accept $3+\ln 9$
	Total		7	

Q	Solution	Marks	Total	Comments
7 (a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{7}{14\,000x}$	B1	1	AG $\dfrac{7}{14\,000x}$ seen; $\left(--7=7 \text{ not required}\right)$
(b)	$\int_2^3 2000x=\int_0^t 1\mathrm{d}t$	M1		Attempt separation and integration
	$\left[2000\dfrac{x^2}{2}\right]_2^3=[t]_0^t$	A1		or $1000x^2=t+c$
	$2000\left[\dfrac{9}{2}-\dfrac{4}{2}\right]=t$	m1		$x=2, t=0 \Rightarrow c=4000$
	$t=5000$ (sec)	A1		$x=3 \Rightarrow t=5000$
	$t=1.39$ hrs $\Rightarrow 1.23$ pm	A1F	5	Accept 1 hour 23min (ignore seconds) ft on $0<t<20\,000$
	Total		6	

Pure 3 June 2003

2 (a)	$\frac{dx}{dt}=3,\ \frac{dy}{dt}=\frac{-1}{t^2}$	M1		Use chain rule
	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-1}{3t^2}$	A1	2	
(b)	$t=1,\ \frac{dy}{dx}=\frac{-1}{3}$	B1F		
	Gradient of normal = 3	B1F		Follow on gradient
	$\left.\begin{array}{l} y=3x+c \\ t=1,\ x=2,\ y=1 \end{array}\right\}$	M1		Use (2, 1) and gradient
	$y=3x-5$	A1F	4	ft on gradient
				Accept $y-1=3(x-2)$
	Total		**6**	

4 (a)	$A=1000$	B1	1	
(b)	$c^{60}=\frac{12000}{A}$	M1		
	$60\log c=\log 12$ or $c=\sqrt[60]{12}$	m1		Or ln used. $c=\sqrt[60]{\frac{12000}{A}}$, their A
	$c=1.04228....$	A1	3	AG convincingly obtained
				SC: $1000\times 1.0423^{60}=12011$
				(12.011 or AWRT seen or 12.011×1000 SC1)
(c)(i)	$\log N=\log Ac^t$	M1		Attempt to take logs (log or ln seen)
	$\log N=\log A+t\log c$	m1		Correct use of log laws
	$t=\frac{\log N-\log A}{\log c}$	A1F	3	OE: $t=\frac{\log N-3}{0.01798}$ etc. ft their A
(ii)	$t=167$ minutes	B1	1	condone more figures if penalised in 3(b)
	Total		**8**	

Pure 3 January 2004

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=6.\frac{1}{6t}$	M1 A1	2	
(ii)	$t=\frac{1}{2}$ gradient $=2$	B1√	1	ft only on $\frac{dy}{dx}=f(t)$
(b)(i)	$t=\frac{y}{6}$ $x=3\left(\frac{y}{6}\right)^2=\left[\frac{y^2}{12}\right]$	M1A1	2	Accept $\frac{3y^2}{36}$ Use of tangent $y=2x+\frac{3}{2}$ or $x=\frac{y}{2}-\frac{3}{4}$ instead of curve: no marks
(ii)	$\frac{dx}{dy}=\frac{2y}{12}$ $t=\frac{1}{2}$ $y=3$ $\frac{dx}{dy}=\frac{6}{12}$ $\frac{dy}{dx}=\frac{12}{6}=2$	M1 B1 M1A1	4	Alternative: $\frac{dx}{dy}=\frac{2y}{12}$ M1 $\frac{y}{6}=t$ B1 $t=\frac{1}{2};\frac{dx}{dy}=\frac{1}{2}$ M1 $\frac{dy}{dx}=2$ A1
	Total		**9**	

3 (a)(i)	$t=0$ $P=50$	B1	1	
(ii)	$e^{\frac{-t}{4}}\to 0$ $P\to 100$	B1	1	
(b)	$75=100-50e^{\frac{-t}{4}}$ $\frac{1}{2}=e^{\frac{-t}{4}}$	M1A1		Allow $\frac{25}{50}$ for $\frac{1}{2}$
	$\ln\frac{1}{2}=\frac{-t}{4}$ $t=2.8$	M1A1	4	SC trial and improvement 2.8 4/4, 2.77 3/4 else 0
	Total		**6**	

Q	Solution	Marks	Total	Comments
4 (a)	$8 + 3x = A(2 - x) + B(1 + 3x)$	M1		Any equivalent method
	$x = 2 \quad 14 = 7B \quad B = 2$	M1		
	$x = \frac{-1}{3} \quad 7 = \frac{7}{3}A \quad A = 3$	A1	3	
(b)	$\frac{1}{1+3x} = (1+3x)^{-1}$			Alternative by Maclaurin
				$f' = \frac{\pm 3}{(1+3x)^2}$; $f'' = \frac{\pm 18 \text{ or } 6}{(1+3x)^3}$ M1
				and $f(0)$ $f'(0)$ $f''(0)$ seen
	$=1 + -1(3x) + \frac{-1.-2}{2}(3x)^2$	M1		Allow $3x^2$
	$=1 - 3x + 9x^2$	A1	2	
(c)	$\frac{1}{2-x} = \frac{1}{2(1-\frac{x}{2})}$	B1		Alternative:
				$(2-x)^{-1} = 2^{-1} + (-1)2^{-2}(-x)$ $+\frac{(-1.-2)}{2!}2^{-3}(-x)^2$
	$= \left(1 + -1\left(\frac{-x}{2}\right) + \frac{-1.-2}{2}\left(\frac{-x^2}{2}\right)\right)$	M1		M1 – use negative powers of 2 A1 – coefficients correct A1 – all correct, with use of $-x$ seen
	$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}$	A1	3	Answer given, convincingly obtained Alternative: $(1-\frac{x}{2})^{-1}$ by Maclaurin $f' = \frac{\pm 1}{(2-x)^2}$ $\quad f'' = \frac{\pm 2}{(2-x)^3}$ M1 $f(0)$ $f'(0)$ $f''(0)$ seen M1 AG convincingly obtained A1
(d)	$\frac{8+3x}{(1+3x)(2-x)}$			
	$= 3(1 - 3x + 9x^2) + 2\left(\frac{1}{2} + \frac{x}{4} + \frac{x^2}{8}\right)$	M1M1		M1 – use series M1 – use PFs and multiply out
	$4 - \frac{17}{2}x + \frac{109}{4}x^2$	A1	3	Alternative: $(8+3x)(1-3x+9x^2)\left(\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}\right)$ M1 Multiply out M1
(e)	Valid for $\lvert x\rvert < \frac{1}{3}$	B2	2	B1 for $x < \frac{1}{3}$ B1 for $\lvert x\rvert < \frac{1}{3}$ and $\lvert x\rvert < 2$ or $\lvert x\rvert < 1$
	Total		**13**	

6 (a)	$\int \frac{dv}{10-5v} = \int dt$	M1		Attempt to separate and integrate
	$-\frac{1}{5}\ln(10-5v) = t + c$	M1 A1A1		$\pm k\ln(10-5v)$ c required
	$t = 0 \quad v = 0 \quad c = -\frac{1}{5}\ln 10$	B1√		Find c or use limits
	$t = \frac{1}{5}\ln\left(\frac{10}{10-5v}\right) = \frac{1}{5}\ln\left(\frac{2}{2-v}\right)$	A1	6	AG convincingly obtained
(b)	$e^{5t} = \frac{2}{2-v}$	M1		Alternative: $0.5 = \frac{1}{5}(\ln 2 - \ln(2-v))$ M1
	$t = 0.5 \quad 2 - v = 2e^{-2.5}$	m1		$e^{\ln 2 - 2.5} = e^{\ln(2-v)}$ M1
	$v = 1.8358 \quad v = 1.8$ m s^{-1}	A1	3	$v = 1.8$ A1
	Total		**9**	

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\overrightarrow{AB}=\begin{bmatrix}2\\4\\-4\end{bmatrix}$			No marks for $\overrightarrow{AB}$ alone
	$\lvert\overrightarrow{AB}\rvert=\sqrt{2^2+4^2+4^2}=6$	M1A1	2	
(ii)	M is (4, 1, 0)	B1	1	Accept $\begin{bmatrix}4\\1\\0\end{bmatrix}$
(b)	$\overrightarrow{CM}\bullet\overrightarrow{AB}=\begin{bmatrix}-4\\3\\1\end{bmatrix}\bullet\begin{bmatrix}2\\4\\-4\end{bmatrix}$	M1A1	2	M1 – sensible attempt at $\overrightarrow{CM}\bullet\overrightarrow{AB}$ Allow $\overrightarrow{MC}$ for $\overrightarrow{CM}$
	$=-8+12-4=0$			$\mp 8\mp 12\mp 4=0$ must be seen

Pure 3 June 2004

1(a)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-1}{2t^2}\frac{1}{2}$	M1		attempt $\frac{dy}{dt}$ & $\frac{dx}{dt}$; use $\frac{dy}{dt}\cdot\frac{dt}{dx}$ $\left(\frac{dy}{dt}\cdot\frac{dx}{dt}\text{ M0}\right)$
		A1	2	Use chain rule (ISW at this stage)
(b)	$t=1 \quad \frac{dy}{dx}=\frac{-1}{4}$	B1F		ft $t=1$ subst in their $\frac{dy}{dx}$
	gradient of normal = 4	B1F		Follow on gradient $\frac{-1}{\text{their } -\frac{1}{4}}$
	$y=4x+c \quad t=1 \quad x=1 \quad y=\frac{1}{2}$	M1		Use $(1, \frac{1}{2})$ and gradient $\frac{1}{2}=\text{their } 4+\text{c}$; $\frac{y-\frac{1}{2}}{x-1}=\text{their } 4$
	$y=4x-\frac{7}{2}$	A1F	4	OE: F on gradient; $y=(\text{their } m_N)\,x+c$
	<u>Special Cases</u> **Eliminate t in part (a)** $y=\frac{1}{x+1}$; $\frac{dy}{dx}=\frac{\pm 1}{(x+1)^2}$ M1 $=\frac{-1}{(2t)^2}$ A1 $m_T=-\frac{1}{4}$ B1F $\frac{1}{2}=-\frac{1}{4}\times 1+c$; $c=\frac{3}{4}$ M1 $y=-\frac{1}{4}x+\frac{3}{4}$ A1F(5/6) **<u>Common error</u>** $y=\frac{1}{2t}=2t^{-1}$; $\frac{dy}{dt}=2t^{-2}$; $\frac{dx}{dt}=2$ $\frac{dy}{dx}=\frac{2t^{-2}}{2}=-t^{-2}\ (\text{or } t^{-2})$ M1A0 $m_N=-1, +1$ B1F $m_T=+1, -1$ B0F (no ft for just changing sign) $x=1, y=\frac{1}{2}$, $\frac{1}{2}=-1+c$ or $\frac{1}{2}=1+c$ M1			**Tangent instead of normal** $m_T=\frac{1}{4}$ $\frac{1}{2}=\frac{1}{4}\times 1+c$; $c=\frac{1}{4}$ $y=\frac{1}{4}x+\frac{1}{4}$ (4/6) **NB** late substitution for t (could be retrospective) B1F B1F but if t's in final answer & no subst'n: either 0/4 or 1/4 if (1,1/2) and gradient used in linear equation

2(a)	$(1+x)^{\frac{1}{3}} = 1+\frac{1}{3}x+\frac{1}{3}\left(\frac{1}{3}-1\right)\frac{x^2}{2}$	M1		
	$= 1+\frac{1}{3}x-\frac{1}{9}x^2$	A1	2	
(b)	$(8+4x)^{\frac{1}{3}} = \left(8\left(1+\frac{1}{2}x\right)\right)^{\frac{1}{3}}$	B1		
	$= 2\left(1+\frac{1}{3}\frac{1}{2}x-\frac{1}{9}\left(\frac{1}{2}x\right)^2+\ldots\right)$	M1		M1 for expression inside bracket SC: $(8+4x)^{\frac{1}{3}}$ $= 8^{\frac{1}{3}}+\frac{1}{3}8^{-\frac{2}{3}}.4x+\frac{1}{3}\left(-\frac{2}{3}\right)8^{-\frac{5}{3}}\frac{(4x)^2}{2}$ [M1 for $8^{\frac{1}{3}}$ $8^{-\frac{2}{3}}$, $8^{-\frac{5}{3}}$ M1 for $4x$, $\frac{(4x)^2}{2}$] $= 2+\frac{1}{3}x-\frac{1}{18}x^2$
	$= 2+\frac{1}{3}x-\frac{1}{18}x^2+\ldots$	A1	3	Accept recurring decimals or equiv fractions
	Total		**5**	

3(a)	$30 = A(7-2x)+B(x+4)$	M1		PFs: any valid method
	$x=-4$ $30=15A$ $A=2$	M1		for substituting values of x to find A, B
	$x=\frac{7}{2}$ $30=\frac{15}{2}B$ $B=4$	A1	3	
(b)	$\int_0^3 \frac{2}{x+4}+\frac{4}{7-2x}\,dx$			
	$= \left[2\ln(x+4)-2\ln(7-2x)\right]_0^3$	M1A1F		M1 for $[c\ln(x+4)+d\ln(7-2x)]$ Ignore limits here
	$= 2\ln 7-2\ln 1-2\ln 4+2\ln 7$	m1A1F		m1 for $(c\ln 7+d\ln 1)-(c\ln 4+d\ln 7)$ m1 Use limits right way round. A1 All correct and with $\ln 1 = 0$. A1F for $c\ln 7 - d\ln 7 - c\ln 4$
		A1	5	or $-2\ln\frac{4}{49}$ or $-4\ln\frac{2}{7}$ or $-1\ln\frac{16}{2401}$ or $1\ln\frac{2401}{16}$
	Total		**8**	

Q	Solution	Marks	Total	Comments
4(a)	$9(y+2)^2 = 5+4(x-1)^2$			
	$x=2 \quad 9(y+2)^2 = 5+4$	M1		Substitute $x = 2$ $9(y+2)^2 = 5+4 \times 3^2$ i.e. $(x+1)^2$
	$y+2=\pm 1 \qquad y=-1,-3$	m1A1	3	Find two y values. Coords not required $(y+2)^2 = \frac{41}{9}$, $y+2 = \pm\frac{\sqrt{41}}{3}$ M1A0
(b)	$\frac{d}{dx}\left(9(y+2)^2\right) = \frac{d}{dx}\left(5 + 4(x-1)^2\right)$	M1		Attempt implicit differentiation with use of chain rule: $\frac{dy}{dx}$ attached to y term, not x term
	$18(y+2)\frac{dy}{dx} = 0 + 8(x-1)$	A1A1		
	$(2,-1) \quad (2,-3)$	m1		Use $x = 2$ and candidate's y values
	$\frac{dy}{dx} = \frac{4}{9} \qquad \frac{dy}{dx} = -\frac{4}{9}$	A1	5	OE; CAO **<u>Alternative: explicit differentiation</u>** $y = \sqrt{\frac{5+4(x-1)^2}{9}} - 2$ $\frac{dy}{dx} = \frac{1}{2}\left(\frac{5+4(x-1)^2}{9}\right)^{-\frac{1}{2}} \frac{8}{9}(x-1)$ (M1A2 fully correct; M1A1 if 9 of $\frac{8}{9}$ missing $x=2$: $\frac{dy}{dx} = \pm\frac{1}{2}(1)\frac{8}{9} = \pm\frac{4}{9}$
	Total		**8**	

7(a)	$\int \frac{dx}{x} = \int (1-kt)dt$	M1		Attempt to separate and integrate. M0 if mixture of x's and t's
	$\ln x = t - \frac{1}{2}kt^2 + c$	A1A1		c required
	$x = e^{t-\frac{1}{2}kt^2+c}$	M1		**Alternatives**
	$x = 2000,\ t = 0 \Rightarrow A = 2000$	M1		(1) $c = \ln 2000$ M1
	$x = Ae^{t-\frac{1}{2}kt^2}$, where $A = e^c$	A1	6	$\ln \frac{x}{2000} = t - \frac{1}{2}kt^2$
	(if A suddenly appears without justification: A0)			$\frac{x}{2000} = e^{t-\frac{1}{2}kt^2}$ M1 $x = 2000\,e^{t-\frac{1}{2}kt^2}$ A1 (2) $c = \ln 2000$ M1 $x = e^{t-\frac{1}{2}kt^2} + \ln 2000$ M1 $= e^{t-\frac{1}{2}kt^2}\,e^{\ln 2000}$ $= 2000\ e^{t-\frac{1}{2}kt^2}$ A1 (3) $\int_0^t (1-kt)dt$ M1 $[\ln x]_{2000}^{x} = \left[t - \frac{1}{2}kt^2\right]_0^t$ — A1 for $\ln x$; A1 for $t - \frac{1}{2}kt^2$; A1 For both sets of limits $\ln x - \ln 2000 = t - \frac{1}{2}kt^2$ M1 $\ln\left(\frac{x}{2000}\right) = t - \frac{1}{2}kt^2$ A1 $x = 2000\,e^{t-\frac{1}{2}kt^2}$ AG AG convincingly obtained
(b)	Substituting $t = 12$ $x = 2000$	B1		No simplification required
	$12 - \frac{1}{2}k(12)^2 = \ln 1$	M1		For taking ln
	$k = \frac{1}{6}$	A1	3	OE
	Total		**9**	

8(a)	$\overrightarrow{AB} = \begin{bmatrix}2\\0\\2\end{bmatrix} - \begin{bmatrix}3\\-1\\2\end{bmatrix} = \begin{bmatrix}-1\\1\\0\end{bmatrix}$	M1		
	l_1 has equation $\mathbf{r} = \begin{bmatrix}3\\-1\\2\end{bmatrix} + \lambda\begin{bmatrix}-1\\1\\0\end{bmatrix}$.	A1	2	OE eg $\mathbf{r} = \begin{bmatrix}2\\0\\2\end{bmatrix} + \lambda\begin{bmatrix}1\\-1\\0\end{bmatrix}$
(b)	$3 - \lambda = 4 + \mu$ $-1 + \lambda = 1$ $2 = -1 - \mu$	M1		Set up at least 2 equations and attempt to solve.
	$\lambda = 2 \quad \mu = -3$	A1		
	Confirm in third equation	A1		
	Intersect at (1, 1, 2)	A1	4	Alternative: showing (1, 1, 2) lies on both lines A2
(c)	$\begin{bmatrix}4\\1\\-1\end{bmatrix} + \mu\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}9\\1\\-6\end{bmatrix}$.	M1		
	is satisfied by $\mu = 5$	A1	2	
(d)	$\overrightarrow{CD} \bullet \overrightarrow{AB} = 0$	B1		$\overrightarrow{CD} \cdot \begin{bmatrix}-1\\1\\0\end{bmatrix} = 0$ or $\overrightarrow{CD} \cdot \begin{bmatrix}1\\-1\\0\end{bmatrix} = 0$
	$\left(\begin{bmatrix}3\\-1\\2\end{bmatrix} + \lambda\begin{bmatrix}-1\\1\\0\end{bmatrix} - \begin{bmatrix}9\\1\\-6\end{bmatrix}\right) \bullet \begin{bmatrix}-1\\1\\0\end{bmatrix} = 0$	M1		not $\overrightarrow{CD} \cdot l_1$, unless corrected later
	$(-6-\lambda)(-1) + (-2+\lambda) = 0$	m1		
	$\lambda = -2 \quad D$ is $(5, -3, 2)$	A1	4	Answer may be in vector form **Alternative to part(d)** $\begin{bmatrix}x-9\\y-1\\z+6\end{bmatrix} \cdot \begin{bmatrix}-1\\1\\0\end{bmatrix} = 0$ B1 $\Rightarrow x - y = 8$ M1 $\begin{bmatrix}x\\y\\z\end{bmatrix} =$ their $\mathbf{r}$ from (a) M1 (5, –3, 2) A1
	Total		**12**	