C3 EXPONENTIALS AND LOGARITHMS — Answers - Worksheet A

1 **a** 20.1 **b** 0.135 **c** 13.6 **d** –0.598 **e** 1.97 **f** 0.434

2 **a** $= 4$ **b** $= e^{\ln 3} = 3$ **c** $= 2e^{\ln \frac{1}{6}} = \frac{1}{3}$ **d** $= 7$ **e** $= \ln e^{-1} = -1$ **f** $= -0.5$

3 **a** $x = 4$ **b** $x = 17$ **c** $x^2 = 25$, $x > 0$ $\therefore x = 5$ **d** $\frac{1}{x} = \frac{1}{3}$, $x = 3$

4 **a** $x = e^{15}$

b $\ln t = 6$
$t = e^6$

c $x - 4 = e^7$
$x = e^7 + 4$

d $\ln 5y = 8$
$5y = e^8$
$y = \frac{1}{5}e^8$

e $\frac{1}{2}x + 3 = e^{2.5}$
$\frac{1}{2}x = e^{2.5} - 3$
$x = 2e^{2.5} - 6$

f $4 - 3x = e^{11}$
$3x = 4 - e^{11}$
$x = \frac{1}{3}(4 - e^{11})$

5 **a** $x = \ln 0.7$

b $e^y = 2$
$y = \ln 2$

c $5x = \ln 3$
$x = \frac{1}{5}\ln 3$

d $4t + 1 = \ln 12$
$t = \frac{1}{4}(\ln 12 - 1)$

e $e^{2x-3} = 14$
$2x - 3 = \ln 14$
$x = \frac{1}{2}(\ln 14 + 3)$

f $e^{4-5x} = \frac{7}{2}$
$4 - 5x = \ln \frac{7}{2}$
$x = \frac{1}{5}(4 - \ln \frac{7}{2})$

6 **a** $e^x = 12$
$x = \ln 12 = 2.48$

b $15x - 7 = e^4$
$x = \frac{1}{15}(e^4 + 7) = 4.11$

c $e^{\frac{1}{2}y+3} = \frac{11}{4}$
$\frac{1}{2}y + 3 = \ln \frac{11}{4}$
$y = 2(\ln \frac{11}{4} - 3) = -3.98$

d $\ln(5 - 2x) = \frac{7}{3}$
$5 - 2x = e^{\frac{7}{3}}$
$x = \frac{1}{2}(5 - e^{\frac{7}{3}}) = -2.66$

e $10 - 3y = e^e$
$y = \frac{1}{3}(10 - e^e) = -1.72$

f $2\ln x + 3\ln x = 19$
$\ln x = \frac{19}{5}$
$x = e^{\frac{19}{5}} = 44.70$

g $e^{\frac{9}{4}x} = 3$
$\frac{9}{4}x = \ln 3$
$x = \frac{4}{9}\ln 3 = 0.49$

h $e^{3t-1} = 4$
$3t - 1 = \ln 4$
$t = \frac{1}{3}(\ln 4 + 1) = 0.80$

i $\ln \frac{2x-5}{x} = \frac{1}{4}$
$2x - 5 = e^{\frac{1}{4}}x$
$(2 - e^{\frac{1}{4}})x = 5$
$x = \frac{5}{2 - e^{\frac{1}{4}}} = 6.98$

7 $2e^{2x} - 11e^x + 12 = 0$
$(2e^x - 3)(e^x - 4) = 0$
$e^x = \frac{3}{2}, 4$
$x = \ln \frac{3}{2}, \ln 4$

8 **a** $= \frac{(3x-4)(x-2)}{(x-2)(x-3)} = \frac{3x-4}{x-3}$

b $\ln \frac{3x^2-10x+8}{x^2-5x+6} = \ln 2x$

$\frac{3x^2-10x+8}{x^2-5x+6} = 2x$

$\frac{3x-4}{x-3} = 2x$

$3x - 4 = 2x(x - 3)$

$2x^2 - 9x + 4 = 0$

$(2x - 1)(x - 4) = 0$

$x = \frac{1}{2}, 4$

9 $e^{5y} - x = 0 \quad \Rightarrow \quad 5y = \ln x$

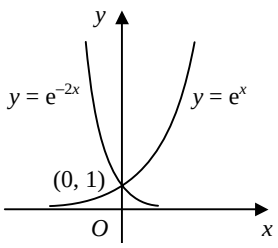
$\ln x^4 = 7 - y \quad \Rightarrow \quad 4 \ln x = 7 - y$

sub. $20y = 7 - y$

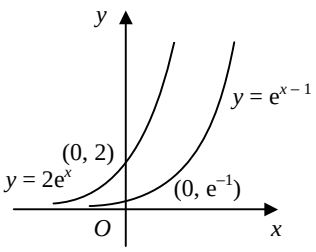
$y = \frac{1}{3}$

$\therefore\ x = e^{\frac{5}{3}} = 5.29,\ y = 0.33$

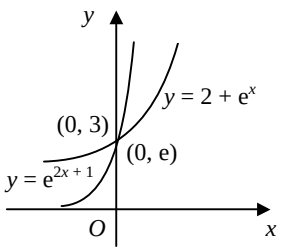
10 **a**



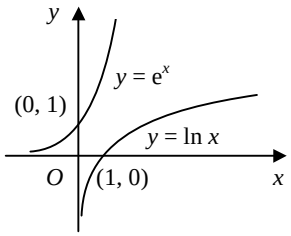
b



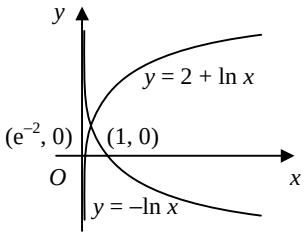
c



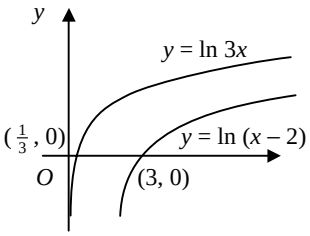
d



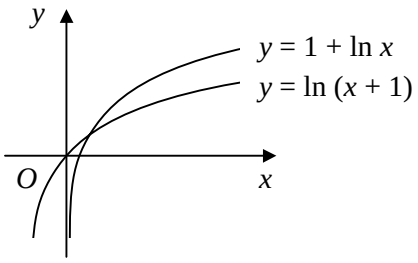
e



f



11 **a**



b $\ln (x + 1) = 1 + \ln x$

$\ln (x + 1) - \ln x = 1$

$\ln \frac{x+1}{x} = 1$

$\frac{x+1}{x} = e$

$x + 1 = ex$

$1 = x(e - 1)$

$x = \frac{1}{e-1}$

12 **a** 3

b $x = 0 \ \therefore y = 3 + e^{-1}$
$\therefore \ (0, 3 + e^{-1})$

c $3 + e^{2x-1} = 7$
$e^{2x-1} = 4$
$2x - 1 = \ln 4$
$x = \frac{1}{2}(1 + \ln 4)$
$x = \frac{1}{2} + \ln 2$

13 **a** $t = 10$, $N = 50e^{-2} = 6.77$ (3sf)

b $3 = 50e^{-0.2t}$
$t = -5 \ln \frac{3}{50} = 14.1$ (3sf)

14 **a** $160 = 240e^{180k}$
$k = \frac{1}{180} \ln \frac{2}{3} = -0.00225$ (3sf)

b $m = 240e^{-0.002253t}$
$120 = 240e^{-0.002253t}$
$t = \frac{-1}{0.002253} \ln \frac{1}{2} = 308$ years (3sf)

15 **a** $t = 15$, $N = 20e^{0.6} = 36.4$ (3sf)

b **i** $k = 20e^{0.04t}$

$$t = \frac{\ln(\frac{k}{20})}{0.04} = 25 \ln \frac{k}{20}$$

ii $2k = 20e^{0.04t}$

$$t = \frac{\ln(\frac{k}{10})}{0.04} = 25 \ln \frac{k}{10}$$

c time for N to increase from k to $2k$

$$= 25 \ln \frac{k}{10} - 25 \ln \frac{k}{20}$$

$$= 25 \ln \frac{(\frac{k}{10})}{(\frac{k}{20})}$$

$$= 25 \ln 2$$

$\therefore$ time for N to double is constant

16 **a** $300 = N_0e^{10k} \ \Rightarrow \ N_0 = \frac{300}{e^{10k}}$
$225 = N_0e^{20k}$

$$\therefore \ 225 = \frac{300}{e^{10k}} \times e^{20k}$$

$e^{10k} = \frac{3}{4}$
$k = \frac{1}{10} \ln \frac{3}{4} = -0.0288$ (3sf)
$N_0 = \frac{300}{\frac{3}{4}} = 400$

b $N = 400e^{-0.02877t}$
$150 = 400e^{-0.02877t}$
$t = \frac{-1}{0.02877} \ln \frac{3}{8} = 34.1$ (3sf)