Functions

$y = \text{range}$

$x = \text{domain}$

$fg(x)$ means do $g(x)$first, then substitute in $f(x)$.

To find the inverse

$f^{-1}(x) =$

1) Put the function equal to y.
2) Re-arrange to find x in terms of y.
3) Re-write replacing y with x.

N.B. To have an inverse, a function must be One-One.

Trigonometry

$\frac{1}{\sin x} = \text{cosec}\, x$ $\frac{1}{\cos x} = \sec x$ $\frac{1}{\tan x} = \cot x$

Identities

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

Principal Values (PV)

$\sin^{-1} is -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\cos^{-1} is\, 0 \leq x \leq 0$

$\tan^{-1} is -\frac{\pi}{2} < x < \frac{\pi}{2}$

Differentiation (All of it)

Generally: $\frac{d}{dx} kx^n = knx^{n-1}$

Switching the respect: $\frac{d}{dx} = \frac{\frac{1}{dy}}{dx}$

The Chain Rule: $\frac{d}{dx}(ax + v) = \left(\frac{dy}{dx}(ax + v) \times n\right)(ax + v)^{n-1}$

This is expressed as "Multiply the brackets by the power, multiplied with the differential of the brackets and raise to the power, subtract one."

The Exponential Function: $\frac{d}{dx}e^{kx} = ke^{kx}$

The Natural Log Function: $\frac{d}{dx}\ln kx = \frac{1}{x}$

The Product Rule: $\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$

This is expressed as "the first function multiplied by the derivative of the second, added to the derivative of the first function, multiplied by the second function."

The Quotient Rule: $\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dy} - u\frac{dv}{dy}}{v^2}$

This is expressed as "the bottom function multiplied by the derivative of the top, subtract the top function multiplied by the derivative of the bottom – divided by the bottom squared."

Trig Functions:

$$\frac{d}{dx}\sin f(x) = f'(x)\cos(f(x))$$
$$\frac{d}{dx}\cos f(x) = -f'(x)\sin(f(x))$$
$$\frac{d}{dx}\tan f(x) = f'^{(x)}\sec^2 f(x)$$

Integration (All of it)

Generally: $\int kx^n\,dx = \frac{kx^{n+1}}{n+1}$

Area under a graph: **Between the x-axis:** $\int_a^b y^2 dx$

Between the y-axis: $\int_a^b x^2 dy$

Volumes of Rev: **Between the x-axis:** $\int_a^b \pi y^2 dx$

Between the x-axis: $\int_a^b \pi x^2 dy$

The Exponential Function: $\int e^{kx}dx = \frac{1}{k}e^x + c$

The Natural Log Function: $\int \frac{1}{x}\,dx = \ln|x| + c$

Trig Functions:

$$\int \sin kx\,dx = -\frac{1}{k}\cos kx + c$$
$$\int \cos kx\,dx = \frac{1}{k}\sin kx + c$$
$$\int \sec^2 kx\,dx = \frac{1}{k}\tan kx + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

By Recognition: $\int g'\big(f(x)\big)f'(x)dx = g\big(f(x)\big) + c$

By Parts: $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx + c$

Numerical Methods

Decimal Search: If two values have different signs then the function has at least one root in the interval. From here, decrease the size of the interval.

Iteration: Re-arrange the formula in the form of $x = f(x)$.

Numerical Integration

Mid-Ordinate Rule: $\int_b^a y\,dx = h\left(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \cdots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}}\right)$

Simpson's Rule: $\int_b^a y\,dx = \frac{1}{3}h\{(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})\}$