C3 Numerical Methods

1. June 2010 qu. 6

(i) Show by calculation that the equation $\tan^2 x - x - 2 = 0$, where x is measured in radians, has a root between 1.0 and 1.1. [3]

(ii) Use the iteration formula $x_{n+1} = \tan^{-1}\sqrt{2 + x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. [4]

(iii) Deduce a root of the equation $\sec^2 2x - 2x - 3 = 0$. [3]

2. Jan 2010 qu.3

(i) Find, in simplified form, the exact value of $\int_{10}^{20} \frac{60}{x}\,dx$. [2]

(ii) Use Simpson's rule with two strips to find an approximation to $\int_{10}^{20} \frac{60}{x}\,dx$. [3]

(iii) Use your answers to parts (i) and (ii) to show that $\ln 2 \approx \frac{25}{36}$. [2]

3. Jan 2010 qu. 8

(i) The curve $y = \sqrt{x}$ can be transformed to the curve $y = \sqrt{2x+3}$ by means of a stretch parallel to the y-axis followed by a translation. State the scale factor of the stretch and give details of the translation. [3]

(ii) It is given that N is a positive integer. By sketching on a single diagram the graphs of $y = \sqrt{2x+3}$ and $y = \frac{N}{x^3}$, show that the equation $\sqrt{2x+3} = \frac{N}{x^3}$ has exactly one real root. [3]

(iii) A sequence $x_1, x_2, x_3, \ldots$ has the property that $x_{n+1} = N^{\frac{1}{3}}(2x_n + 3)^{-\frac{1}{6}}$.
For certain values of x_1 and N, it is given that the sequence converges to the root of the equation $\sqrt{2x+3} = \frac{N}{x^3}$.

(a) Find the value of the integer N for which the sequence converges to the value 1.9037 (correct to 4 decimal places). [2]

(b) Find the value of the integer N for which, correct to 4 decimal places, $x_3 = 2.6022$ and $x_4 = 2.6282$. [3]

4. FP2 Jan 2010 qu 1 part i)

It is given that $f(x) = x^2 - \sin x$.

(i) The iteration $x_{n+1} = \sqrt{\sin x_n}$, with $x_1 = 0.875$, is to be used to find a real root, α, of the equation $f(x) = 0$. Find x_2, x_3 and x_4, giving the answers correct to 6 decimal places. [2]

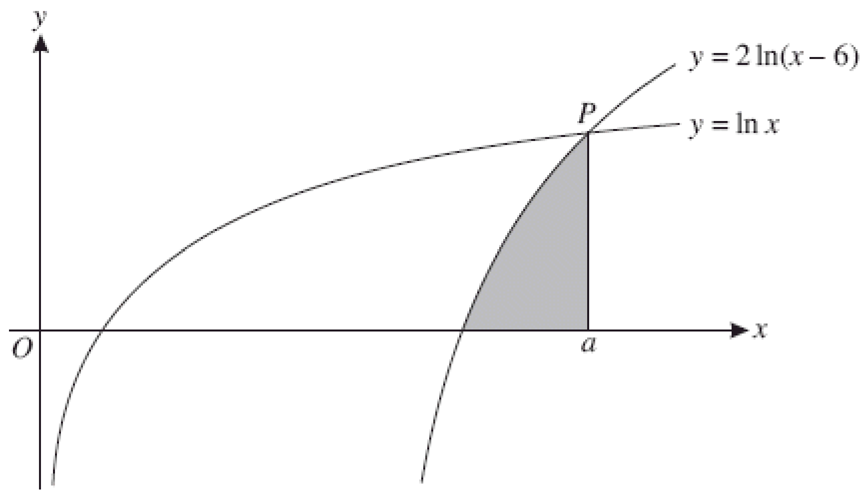
5. June 2009 qu. 4

It is given that $\int_a^{3a} (e^{3x} + e^x)\,dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$. [5]

(ii) Use an iterative process, based on the equation in part (i), to find the value of a correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process.

6. June 2009 qu. 8



The diagram shows the curves $y = \ln x$ and $y = 2\ln(x - 6)$. The curves meet at the point P which has x-coordinate a. The shaded region is bounded by the curve $y = 2\ln(x - 6)$ and the lines $x = a$ and $y = 0$.

(i) Give details of the pair of transformations which transforms the curve $y = \ln x$ to the curve $y = 2\ln(x - 6)$. [3]

(ii) Solve an equation to find the value of a. [4]

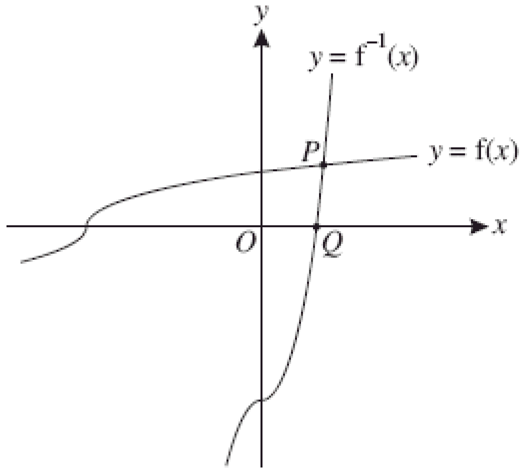
(iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region. [3]

7. Jan 2009 qu. 2

(i) Use Simpson's rule with four strips to find an approximation to $\int_4^{12} \ln x \, dx$, giving your answer correct to 2 decimal places. [4]

(ii) Deduce an approximation to $\int_4^{12} \ln(x^{10}) \, dx$. [1]

8. Jan 2009 qu. 6



The function f is defined for all real values of x by

$$f(x) = \sqrt[3]{\frac{1}{2}x + 2}.$$

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at the point P, and the graph of $y = f^{-1}(x)$ meets the x-axis at Q (see diagram).

(i) Find an expression for $f^{-1}(x)$ and determine the x-coordinate of the point Q. [3]

(ii) State how the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related geometrically, and hence show that the x-coordinate of the point P is the root of the equation $x = \sqrt[3]{\frac{1}{2}x + 2}$. [2]

(iii) Use an iterative process, based on the equation $x = \sqrt[3]{\frac{1}{2}x + 2}$, to find the x-coordinate of P, giving your answer correct to 2 decimal places. [4]

9. FP2 Jan 2009 qu. 2 part i)

It is given that α is the only real root of the equation $x^5 + 2x - 28 = 0$ and that $1.8 < \alpha < 2$.

(i) The iteration $x_{n+1} = \sqrt[5]{28 - 2x_n}$, with $x_1 = 1.9$, is to be used to find α. Find the values of x_2, x_3 and x_4, giving the answers correct to 7 decimal places. [3]

10. June 2008 qu. 4

The gradient of the curve $y = (2x^2 + 9)^{\frac{5}{2}}$ at the point P is 100.

(i) Show that the x-coordinate of P satisfies the equation $x = 10(2x^2 + 9)^{-\frac{3}{2}}$. [3]

(ii) Show by calculation that the x-coordinate of P lies between 0.3 and 0.4. [3]

(iii) Use an iterative formula, based on the equation in part (i), to find the x-coordinate of P correct to 4 decimal places. You should show the result of each iteration. [3]

11. Jan 2008 qu. 2

The sequence defined by $x_1 = 3,\quad x_{n+1} = \sqrt[3]{31 - \frac{5}{2}x_n}$ converges to the number α.

(i) Find the value of α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) Find an equation of the form $ax^3 + bx + c = 0$, where a, b and c are integers, which has α as a root. [3]

12. June 2007 qu. 6

(i) Given that $\int_0^a (6e^{2x} + x)\,dx = 42$, show that $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$. [5]

(ii) Use an iterative formula, based on the equation in part (i), to find the value of a correct to 3 decimal places. Use a starting value of 1 and show the result of each iteration. [4]

13. Jan 2007 qu. 3

(a) It is given that a and b are positive constants. By sketching graphs of

$$y = x^5 \text{ and } y = a - bx$$

on the same diagram, show that the equation $x^5 + bx - a = 0$ has exactly one real root. [3]

(b) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the real root of the equation $x^5 + 2x - 53 = 0$. Show the result of each iteration, and give the root correct to 3 decimal places. [4]

14. Jan 2007 qu. 8

The diagram shows the curve with equation $y = x^8 e^{-x^2}$. The curve has maximum points at P and Q. The shaded region A is bounded by the curve, the line $y = 0$ and the line through Q parallel to the y-axis. The shaded region B is bounded by the curve and the line PQ.

(i) Show by differentiation that the x-coordinate of Q is 2. **[5]**

(ii) Use Simpson's rule with 4 strips to find an approximation to the area of region A. Give your answer correct to 3 decimal places. **[4]**

(iii) Deduce an approximation to the area of region B. **[2]**

15. June 2006 qu. 3

The equation $2x^3 + 4x - 35 = 0$ has one real root.

(i) Show by calculation that this real root lies between 2 and 3. **[3]**

(ii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{17.5 - 2x_n},$$

with a suitable starting value, to find the real root of the equation $2x^3 + 4x - 35 = 0$ correct to 2 decimal places. You should show the result of each iteration. [3]

16. Jan 2006 qu. 7

The diagram shows the curve with equation $y = \cos^{-1}x$.

(i) Sketch the curve with equation $y = 3\cos^{-1}(x - 1)$, showing the coordinates of the points where the curve meets the axes. **[3]**

(ii) By drawing an appropriate straight line on your sketch in part (i), show that the equation $3\cos^{-1}(x - 1) = x$ has exactly one root. **[1]**

(iii) Show by calculation that the root of the equation $3\cos^{-1}(x - 1) = x$ lies between 1.8 and 1.9. **[2]**

(iv) The sequence defined by $x_1 = 2,\ x_{n+1} = 1 + \cos\left(\frac{1}{3}x_n\right)$

converges to a number α. Find the value of α correct to 2 decimal places and explain why α is the root of the equation $3\cos^{-1}(x - 1) = x$. **[5]**

17. Jan 2006 qu. 8

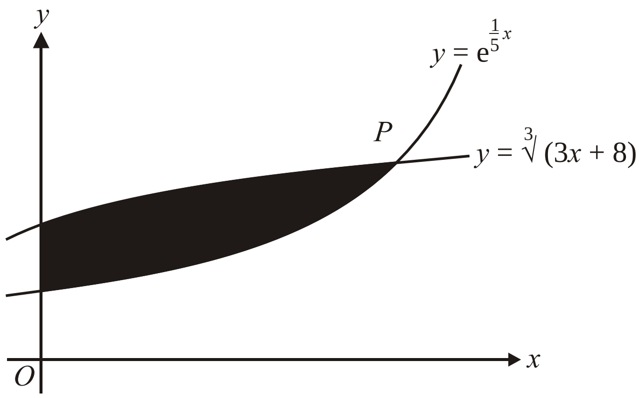
The diagram shows part of the curve $y = \ln(5 - x^2)$ which meets the x-axis at the point P with coordinates (2, 0). The tangent to the curve at P meets the y-axis at the point Q. The region A is bounded by the curve and the lines $x = 0$ and $y = 0$. The region B is bounded by the curve and the lines PQ and $x = 0$.

(i) Find the equation of the tangent to the curve at P. [5]

(ii) Use Simpson's Rule with four strips to find an approximation to the area of the region A, giving your answer correct to 3 significant figures. [4]

(iii) Deduce an approximation to the area of the region B. [2]

18. June 2005 qu. 8



The diagram shows part of each of the curves $y = e^{\frac{1}{5}x}$ and $y = \sqrt[3]{(3x+8)}$. The curves meet, as shown in the diagram, at the point P. The region R, shaded in the diagram, is bounded by the two curves and by the y-axis.

(i) Show by calculation that the x-coordinate of P lies between 5.2 and 5.3. [3]

(ii) Show that the x-coordinate of P satisfies the equation $x = \frac{5}{3}\ln(3x + 8)$. [2]

(iii) Use an iterative formula, based on the equation in part (ii), to find the x-coordinate of P correct to 2 decimal places. [3]

(iv) Use integration, and your answer to part (iii), to find an approximate value of the area of the region R. [5]

19. June 2005 qu. 4

(a) The diagram shows the curve $y = \frac{2}{\sqrt{x}}$.

The region R, shaded in the diagram, is bounded by the curve and by the lines $x = 1$, $x = 5$ and $y = 0$.
The region R is rotated completely about the x-axis.
Find the exact volume of the solid formed.

[4]

(b) Use Simpson's rule, with 4 strips, to find an approximate value for

$$\int_1^5 \sqrt{(x^2 + 1)}\, dx,$$

giving your answer correct to 3 decimal places. **[4]**