C3 Integration

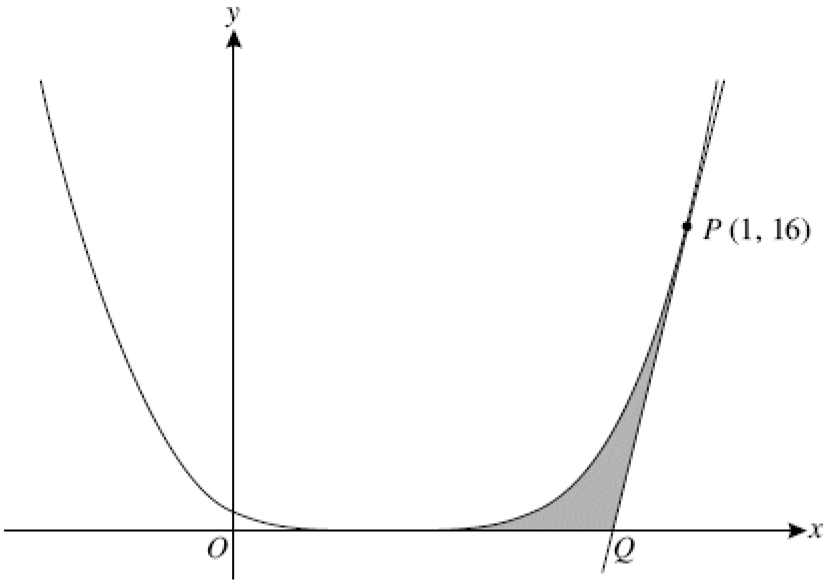
1. June 2010 qu. 4

The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x-coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is ln 81.

(i) Show that $k = 4$. [3]

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x-axis. [4]

2. June 2010 qu.7



The diagram shows the curve with equation $y = (3x - 1)^4$. The point P on the curve has coordinates (1, 16) and the tangent to the curve at P meets the x-axis at the point Q. The shaded region is bounded by PQ, the x-axis and that part of the curve for which $\frac{1}{3} \leq x \leq 1$. Find the exact area of this shaded region. [10]

3. Jan 2010 qu.1

Find $\int \frac{10}{(2x-7)^2}\,dx$. [3]

4. Jan 2010 qu.3

(i) Find, in simplified form, the exact value of $\int_{10}^{20} \frac{60}{x}\,\mathrm{d}x$. [2]

(ii) Use Simpson's rule with two strips to find an approximation to $\int_{10}^{20} \frac{60}{x}\,\mathrm{d}x$. [3]

(iii) Use your answers to parts (i) and (ii) to show that $\ln 2 \approx \frac{25}{36}$. [2]

5. Jan 2010 qu.6

Given that $\int_{0}^{\ln 4} (k\mathrm{e}^{3x} + (k-2)\mathrm{e}^{-\frac{1}{2}x})\,\mathrm{d}x = 185$, find the value of the constant k. [7]

6. June 2009 qu. 2

The diagram shows the curve with equation $y = (2x - 3)^2$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = 0$. Find the exact volume obtained when the shaded region is rotated completely about the x-axis. [5]

7. June 2009 qu. 4

It is given that $\int_{a}^{3a} (\mathrm{e}^{3x} + \mathrm{e}^{x})\mathrm{d}x = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{9}\ln(300 + 3\mathrm{e}^{a} - 2\mathrm{e}^{3a})$. [5]

(ii) Use an iterative process, based on the equation in part (i), to find the value of a correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process.

8. Jan 2009 qu.1

Find

(i) $\int 8\mathrm{e}^{-2x}\,\mathrm{d}x$,

(ii) $\int (4x + 5)^6\,\mathrm{d}x$. [5]

9. Jan 2009 qu.8

The diagram shows the curve with equation

$$y = \frac{6}{\sqrt{x}} - 3.$$

The point P has coordinates $(0, p)$.
The shaded region is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$.

The shaded region is rotated completely about the y-axis to form a solid of volume V.

(i) Show that $V = 16\pi\left(1 - \frac{27}{(p+3)^3}\right)$. [6]

(ii) It is given that P is moving along the y-axis in such a way that, at time t, the variables p and t are related by $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{3}p + 1$. Find the value of $\frac{\mathrm{d}V}{\mathrm{d}t}$ at the instant when $p = 9$. [4]

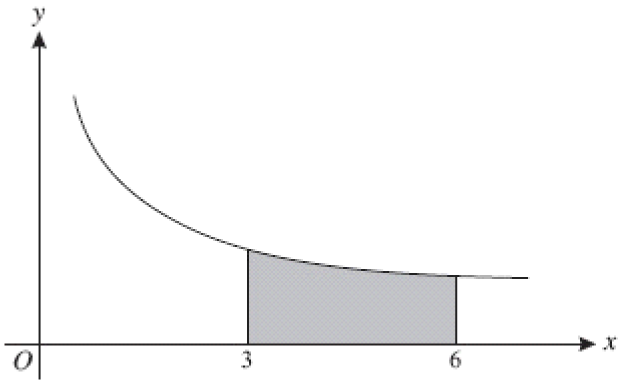
10. June 2008 qu. 6

The diagram shows the curves $y = e^{3x}$ and $y = (2x - 1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced. [9]

11. Jan 2008 qu.5

(a) Find $\int (3x + 7)^9 \, dx$. [3]

(b)



The diagram shows the curve $y = \frac{1}{2\sqrt{x}}$. The shaded region is bounded by the curve and the lines $x = 3$, $x = 6$ and $y = 0$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced, simplifying your answer. [5]

12. June 2007 qu. 4

The integral $\boldsymbol{I}$ is defined by $\boldsymbol{I} = \int_0^{13} (2x+1)^{\frac{1}{3}} dx$.

(i) Use integration to find the exact value of $\boldsymbol{I}$. [4]

(ii) Use Simpson's rule with two strips to find an approximate value for $\boldsymbol{I}$. Give your answer correct to 3 significant figures. [3]

13. June 2007 qu. 6

(i) Given that $\int_0^a (6e^{2x} + x)dx = 42$, show that $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$. [5]

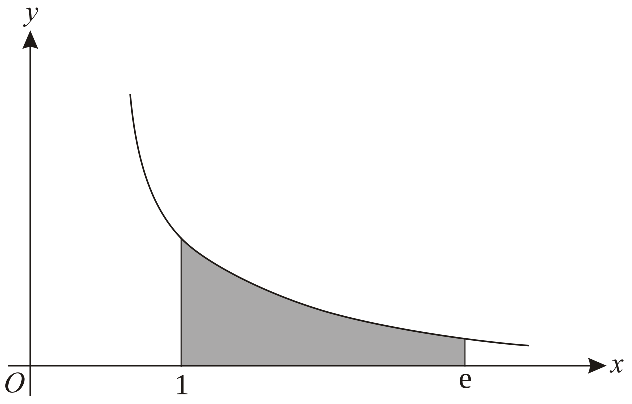
(ii) Use an iterative formula, based on the equation in part (i), to find the value of a correct to 3 decimal places. Use a starting value of 1 and show the result of each iteration. [4]

14. June 2007 qu. 8

(i) Given that $y = \dfrac{4\ln x - 3}{4\ln x + 3}$, show that $\dfrac{dy}{dx} = \dfrac{24}{x(4\ln x + 3)^2}$. [3]

(ii) Find the exact value of the gradient of the curve $y = \dfrac{4\ln x - 3}{4\ln x + 3}$ at the point where it crosses the x-axis. [4]

(iii)



The diagram shows part of the curve with equation $y = \dfrac{2}{x^{\frac{1}{2}}(4\ln x + 3)}$.

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = e$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. [4]

15. Jan 2007 qu.6

The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{3x + 2}}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$.

(i) Find the exact area of the shaded region. [4]

(ii) The shaded region is rotated completely about the x-axis. Find the exact volume of the solid formed, simplifying your answer. [5]

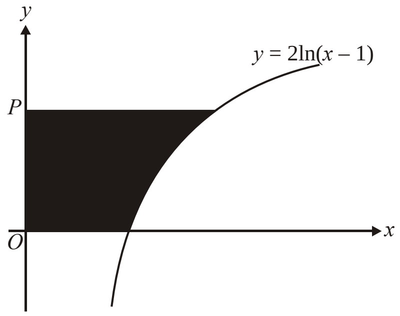
16. June 2006 qu. 7

(a) Find the exact value of $\displaystyle\int_1^2 \frac{2}{(4x - 1)^2}\,dx$. [4]

(b)

The diagram shows part of the curve $y = \dfrac{1}{x}$. The point P has coordinates $\left(a, \dfrac{1}{a}\right)$ and the point Q has coordinates $\left(2a, \dfrac{1}{2a}\right)$, where a is a positive constant. The point R is such that PR is parallel to the x-axis and QR is parallel to the y-axis. The region shaded in the diagram is bounded by the curve and by the lines PR and QR. Show that the area of this shaded region is $\ln(\frac{1}{2}e)$. [6]

17. June 2006 qu. 9



The diagram shows the curve with equation $y = 2\ln(x - 1)$. The point P has coordinates $(0, p)$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the ***y*-axis** to form a solid.

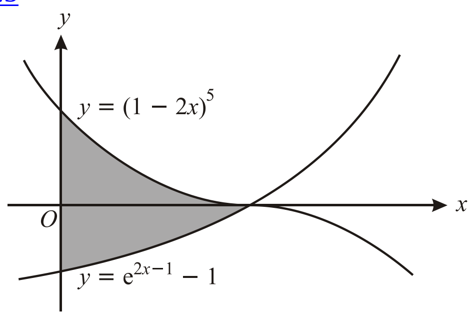
(i) Show that the volume, V cm^3, of the solid is given by $V = \pi(e^{P} + 4e^{\frac{1}{2}P} + p - 5)$. [8]

(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm min^{-1}. Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]

18. Jan 2006 qu.1

Show that $\int_2^8 \frac{3}{x}\,dx = \ln 64$. [4]

19. Jan 2006 qu.5



The diagram shows the curves $y = (1 - 2x)^5$ and $y = e^{2x-1} - 1$. The curves meet the point $(\frac{1}{2}, 0)$. Find the exact area of the region (shaded in the diagram) bounded by the y-axis and by part of each curve. [8]

20. June 2005 qu. 4

(a)

The diagram shows the curve $y = \frac{2}{\sqrt{x}}$. The region R, shaded in the diagram, is bounded by the curve and by the lines $x = 1$, $x = 5$ and $y = 0$. The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

(b) Use Simpson's rule, with 4 strips, to find an approximate value for $\int_1^5 \sqrt{(x^2 + 1)}\,dx$,

giving your answer correct to 3 decimal places. [4]