Question			Answer	Marks	Guidance	
1	(i)		$\frac{dy}{dx} = \sin 2x + 2x\cos 2x$ $dy/dx = 0$ when $\sin 2x + 2x\cos 2x = 0$ $\Rightarrow \quad \frac{\sin 2x + 2x\cos 2x}{\cos 2x} = 0$ $\Rightarrow \quad \tan 2x + 2x = 0$ *	M1 A1 M1 A1 **[4]**	$d/dx(\sin 2x) = 2\cos 2x$ soi cao, mark final answer equating their derivative to zero, **provided it has two terms** must show evidence of division by $\cos 2x$	can be inferred from $dy/dx = 2x\cos 2x$ e.g. $dy/dx = \tan 2x + 2x$ is A0
	(ii)		At P, $x\sin 2x = 0$ $\Rightarrow \sin 2x = 0,\ 2x = (0),\ \pi \Rightarrow x = \pi/2$ At P, $dy/dx = \sin \pi + 2(\pi/2)\cos \pi = -\pi$ Eqn of tangent: $y - 0 = -\pi(x - \pi/2)$ $\Rightarrow \quad y = -\pi x + \pi^2/2$ $\Rightarrow \quad 2\pi x + 2y = \pi^2$ * When $x = 0$, $y = \pi^2/2$, so Q is $(0, \pi^2/2)$	M1 A1 B1 ft M1 A1 M1A1 **[7]**	 $x = \pi/2$ ft their $\pi/2$ and their derivative substituting 0, their $\pi/2$ and their $-\pi$ into $y - y_1 = m(x - x_1)$ **NB AG** can isw inexact answers from $\pi^2/2$	Finding $x = \pi/2$ using the given line equation is M0 or their $-\pi$ into $y = mx+c$, and then evaluating c: $y = (-\pi)x + c$, $0 = (-\pi)(\pi/2) + c$ M1 $\Rightarrow c = \pi^2/2$ $\Rightarrow y = -\pi x + \pi^2/2 \Rightarrow 2\pi x + 2y = \pi^2$ *A1
	(iii)		Area = triangle OPQ – area under curve Triangle OPQ = ½ × $\pi/2$ × $\pi^2/2$ [= $\pi^3/8$] Parts: $u = x$, $dv/dx = \sin 2x$ $du/dx = 1$, $v = -½\cos 2x$ $\int_0^{\pi/2} x\sin 2x\,dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos 2x\,dx$ $= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$ $= -\frac{1}{4}\pi\cos\pi + \frac{1}{4}\sin\pi - (-0\cos 0 + \frac{1}{4}\sin 0) = \frac{1}{4}\pi[-0]$ So shaded area = $\pi^3/8 - \pi/4 = \pi(\pi^2 - 2)/8$*	M1 B1cao M1 A1ft A1 A1cao A1 **[7]**	soi (or area under PQ – area under curve allow art 3.9 condone $v = k\cos 2x$ soi ft their $v = -½\cos 2x$, ignore limits [− ½ $x\cos 2x$ + ¼ $\sin 2x$] o.e., must be correct at this stage, ignore limits (so dep previous A1) **NB AG** must be from fully correct work	area under line may be expressed in integral form or using integral: $\int_0^{\pi/2}(\frac{1}{2}\pi^2 - \pi x)\,dx = \left[\frac{1}{2}\pi^2 x - \frac{1}{2}\pi x^2\right]_0^{\pi/2} = \frac{\pi^3}{4} - \frac{\pi^3}{8} [= \frac{\pi^3}{8}]$ v can be inferred from their 'uv'

2(i) $\int_0^1 \frac{x^3}{1+x}\,\mathrm{d}x$ $\quad$ *let* $u = 1+x,\ du = dx$			
when $x = 0$, $u = 1$, when $x = 1$, $u = 2$	B1	$a = 1$, $b = 2$	seen anywhere, e.g. in new limits
$= \int_1^2 \frac{(u-1)^3}{u}\,\mathrm{d}u$	B1	$(u-1)^3/u$	
$= \int_1^2 \frac{(u^3 - 3u^2 + 3u - 1)}{u}\,\mathrm{d}u$	M1	expanding (correctly)	
$= \int_1^2 (u^2 - 3u + 3 - \frac{1}{u})\,\mathrm{d}u$ *	A1dep	dep du = dx (o.e.) **AG**	e.g. du/dx = 1, condone missing dx's and du's, allow du = 1
$\int_0^1 \frac{x^3}{1+x}\,\mathrm{d}x = \left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u\right]_1^2$	B1	$\left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u\right]$	
$= (\frac{8}{3} - 6 + 6 - \ln 2) - (\frac{1}{3} - \frac{3}{2} + 3 - \ln 1)$	M1	substituting correct limits dep integrated	upper – lower; may be implied from 0.140…
$= \frac{5}{6} - \ln 2$	A1cao [7]	must be exact – must be 5/6	must have evaluated ln 1 = 0
(ii) $y = x^2 \ln(1 + x)$	M1	Product rule	
$\Rightarrow \frac{dy}{dx} = x^2 . \frac{1}{1+x} + 2x.\ln(1+x)$	B1 A1	d/dx (ln(1 + x)) = 1/(1 + x) cao (oe) mark final ans	or d/dx (ln u) = 1/u where u = 1 + x ln1+x is A0
$= \frac{x^2}{1+x} + 2x\ln(1+x)$			
When x = 0, dy/dx = 0 + 0.ln 1 = 0 ($\Rightarrow$ Origin is a stationary point)	M1 A1cao [5]	substituting x = 0 into **correct** deriv www	when x = 0, dy/dx = 0 with no evidence of substituting M1A0 but condone missing bracket in ln(1+x)
(iii) $A = \int_0^1 x^2 \ln(1+x)\,\mathrm{d}x$	B1	Correct integral and limits	condone no dx, limits (and integral) can be implied by subsequent work
let $u = \ln(1+x)$, $\mathrm{d}v/dx = x^2$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{1+x}$, $v = \frac{1}{3}x^3$	M1	parts correct	u, du/dx, dv/dx and v all correct (oe)
$\Rightarrow A = \left[\frac{1}{3}x^3 \ln(1+x)\right]_0^1 - \int_0^1 \frac{1}{3}\frac{x^3}{1+x}\,\mathrm{d}x$	A1		condone missing brackets
$= \frac{1}{3}\ln 2 - (\frac{5}{18} - \frac{1}{3}\ln 2)$	B1	$= \frac{1}{3}\ln 2 - ...$	
$= \frac{1}{3}\ln 2 - \frac{5}{18} + \frac{1}{3}\ln 2$	B1ft	… – 1/3 (result from part (i))	condone missing bracket, can re-work from scratch
$= \frac{2}{3}\ln 2 - \frac{5}{18}$	A1 [6]	cao	oe e.g. $= \frac{12\ln 2 - 5}{18}, \frac{1}{3}\ln 4 - \frac{5}{18}$, etc but must have evaluated ln 1 =0 Must combine the two ln terms

3(i)	$\frac{dy}{dx}=\frac{x^2.\frac{1}{x}-\ln x.2x}{x^4}$ $=\frac{x-2x\ln x}{x^4}$ $=\frac{1-2\ln x}{x^3}$	M1 B1 A1 A1 [4]	quotient rule with $u = \ln x$ and $v = x^2$ d/dx (ln x) = 1/x soi correct expression (o.e.) o.e. cao, mark final answer, but must have divided top and bottom by x	Consistent with their derivatives. udv ± vdu in the quotient rule is M0 Condone ln$x$.2x = ln 2x^2 for this A1 (provided ln x.2x is shown) e. $\frac{1}{x^3}-\frac{2\ln x}{x^3}$, $x^{-3}-2x^{-3}\ln x$
or	$\frac{dy}{dx}=-2x^{-3}\ln x+x^{-2}(\frac{1}{x})$ $=-2x^{-3}\ln x+x^{-3}$	M1 B1 A1 A1 [4]	product rule with $u = x^{-2}$ and $v = \ln x$ d/dx (ln x) = 1/x soi correct expression o.e. cao, mark final answer, must simplify the x^{-2}.(1/x) term.	or vice-versa
(ii)	$\int\frac{\ln x}{x^2}dx$ let u = ln x, du/dx = 1/x dv/dx = 1/x^2, $v = -x^{-1}$ $=-\frac{1}{x}\ln x+\int\frac{1}{x}.\frac{1}{x}dx$ $=-\frac{1}{x}\ln x+\int\frac{1}{x^2}dx$ $=-\frac{1}{x}\ln x-\frac{1}{x}+c$ $=-\frac{1}{x}(\ln x+1)+c$ *	M1 A1 A1 A1 [4]	Integration by parts with u = ln x, du/dx = 1/x, dv/dx = 1/x^2, $v = -x^{-1}$ must be correct, condone + c condone missing c NB **AG** must have c shown in final answer	Must be correct at this stage . Need to see 1/x^2

4(i) $\int_0^1 \frac{2x}{x^2+1}\,dx = \left[\ln(x^2+1)\right]_0^1$ $= \ln 2$	M2 A1 [3]	$[\ln(x^2 + 1)]$ cao (must be exact)
or let $u = x^2 + 1$, $du = 2x\,dx$ $\Rightarrow$ $\int_0^1 \frac{2x}{x^2+1}\,dx = \int_1^2 \frac{1}{u}\,du$ $= [\ln u]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u}\,du$ or $\left[\ln(1+x^2)\right]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1}\,dx = \int_0^1 \frac{2x+2-2}{x+1}\,dx = \int_0^1 (2 - \frac{2}{x+1})\,dx$ $= [2x - 2\ln(x+1)]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x + 1)$ 2, $-2/(x+1)$
or $\int_0^1 \frac{2x}{x+1}\,dx$ let $u = x + 1$, $\Rightarrow du = dx$ $= \int_1^2 \frac{2(u-1)}{u}\,du$ $= \int_1^2 (2 - \frac{2}{u})\,du$ $= [2u - 2\ln u]_1^2$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$) *and* correct limits used for u or x $2(u-1)/u$ dividing through by u $2u - 2\ln u$ allow ft on $(u-1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)