Edexcel Maths C3

Topic Questions from Papers

Numerical Methods

Leave blank

2. (a) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(b) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)

Leave blank

5. $$f(x) = 2x^3 - x - 4.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)

Leave blank

5. **Figure 2**

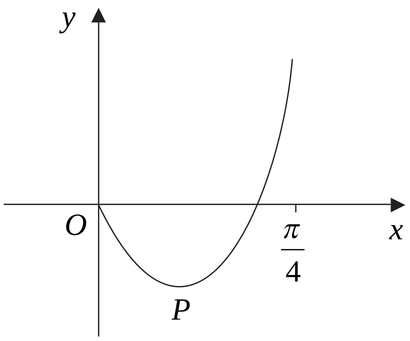

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leqslant x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

N 2 3 5 8 1 A 0 1 0 2 4

Leave blank

Question 5 continued

Leave blank

7. $$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.
(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.
(3)

(c) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.
(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.
(3)

(e) Hence sketch the graph of $y = |f(x)|$.
(1)

Leave blank

Question 7 continued

Q7

(Total 13 marks)

Leave blank

4. $$f(x) = -x^3 + 3x^2 - 1.$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(2)

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(c) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

Leave blank

3. $$\mathrm{f}(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $\mathrm{f}(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \; x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places.

(2)

Leave blank

7.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that f(x) = 0 has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation f(x) = 0 can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

Leave blank

Question 7 continued

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

7. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

Leave blank

Question 7 continued

Leave blank

1.

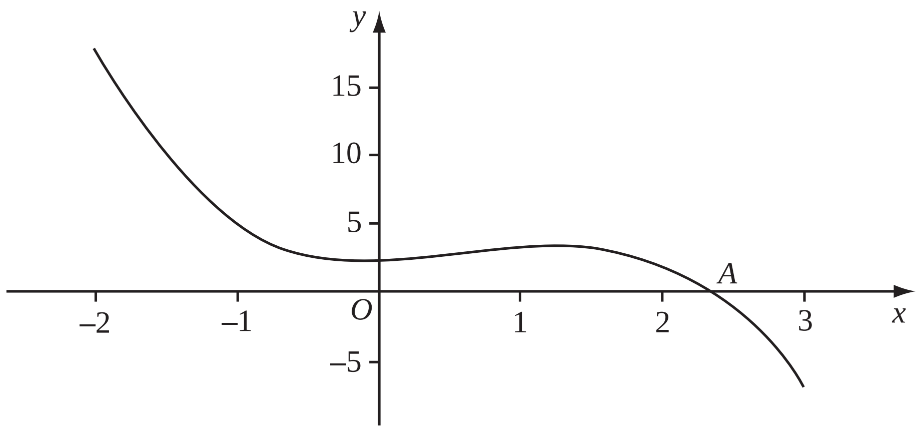

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

Leave blank

2.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

(a) Show that f(x) = 0 can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

(2)

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

Leave blank

4. (i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$.

(5)

Leave blank

3. $$\mathrm{f}(x) = 4\operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(a) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval $[1.2, 1.3]$.

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of $\mathrm{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

Leave blank

Question 3 continued

H 3 5 3 8 5 A 0 7 2 8

Turn over

Leave blank

4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.
(2)

(b) Solve $f(x) = 15 + x$.
(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find fg(2).
(2)

(d) Find the range of g.
(3)

Leave blank

2. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$

(2)

The equation $f(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

Leave blank

2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

Leave blank

2. $$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

Leave blank

7.

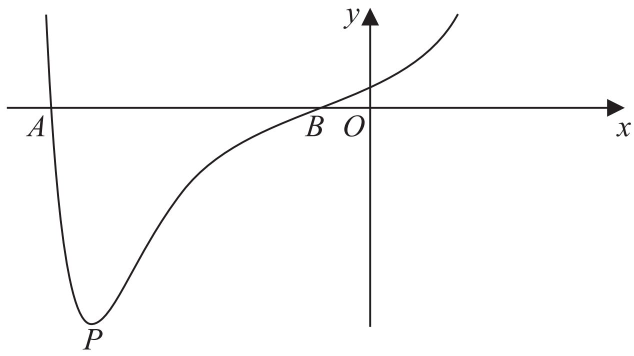

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)\mathrm{e}^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

The curve has a minimum turning point at the point P as shown in Figure 2.

(c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$

(3)

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \qquad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

Leave blank

Question 7 continued

Leave blank

4. $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

Leave blank

Question 4 continued

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$