**1.** $$\mathrm{f}(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \; x \neq -2.$$

(a) Show that $\mathrm{f}(x) = \dfrac{x^2 + x + 1}{(x+2)^2}, x \neq -2.$

**(4)**

(b) Show that $x^2 + x + 1 > 0$ for all values of $x$.

**(3)**

(c) Show that f $(x) > 0$ for all values of $x$, $x \neq -2$.

**(1)**

**(Total 8 marks)**

**2.** The function f is even and has domain $\mathbb{R}$. For $x \geq 0$, $\mathrm{f}(x) = x^2 - 4ax$, where $a$ is a positive constant.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x)$, showing the coordinates of all the points at which the curve meets the axes.

**(3)**

(b) Find, in terms of $a$, the value of $\mathrm{f}(2a)$ and the value of $\mathrm{f}(-2a)$.

**(2)**

Given that $a = 3$,

(c) use algebra to find the values of $x$ for which $\mathrm{f}(x) = 45$.

**(4)**

**(Total 9 marks)**

**1.** (a) $f(x) = \dfrac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ M1A1, A1

$= \dfrac{x^2+4x+4-3x-6+3}{(x+2)^2} = \dfrac{x^2+x+1}{(x+2)^2}*$ cso A1 4

(b) $x^2 + x + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of $x$. M1A1, A1 3

Alternative

$\frac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$ M1A1

A parabola with positive coefficient of $x^2$ has a minimum

$\Rightarrow x^2 + x + 1 > 0$

Accept equivalent arguments A1 3

(c) $f(x) = \dfrac{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$

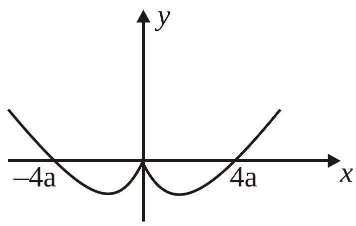
Numerator is positive from (b)

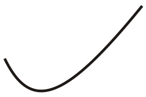
$x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive)

Hence $f(x) > 0$ B1 1

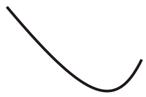
**[8]**

**2.** (a)



$x > 0$ B1



$x < 0$ B1 ft

$(4a, 0)$ & $(-4a, 0)$ and shape at $(0, 0)$ B1 3

(b) $f(2a) = (2a)^2 - 4a(2a) = 4a^2 - 8a^2 = \underline{-4a^2}$ B1

$f(-2a)$ [= $f(2a)$ ($\because$ *even function*) ] $= \underline{-4a^2}$ B1 ft 2

*B1 ft their f(2a)*

| | | | |
|---|---|---|---|
| (c) | $a = 3$ and $f(x) = 45 \Rightarrow 45 = x^2 - 12x \quad (x > 0)$ | M1 | |
| | $0 = x^2 - 12x - 45$ | | |
| | $0 = (x - 15)(x + 3)$ | M1 | |
| | $x = 15$ (or $-3$) | A1 | |
| | $\therefore$ Solutions are $\underline{x = \pm 15}$ <u>only</u> | A1 | 4 |

*M1 Attempt 3TQ in x*

*M1 Attempt to solve*

*A1 At least x = 15 can ignore x = –3*

*A1 To get final A1 must make clear <u>only</u> answers are ±15.*

**[9]**

**1.** Part (a) was very well done, the great majority of candidates gaining full marks. Part (b), however, proved very demanding and there were many who had no idea what is required to show a general algebraic result. It was common to see candidates, both here and in part (c), who substituted into the expression a number of isolated values of $x$, noted that they were all positive, and concluded the general result. Those who did complete the square correctly, obtaining $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$, did not always explain the relevance of this to the required result. Many tried calculus but, to complete the proof this way, it was necessary to show that $\left(-\frac{1}{2}, \frac{3}{4}\right)$,was a minimum and this was rarely seen. Those who tried to solve $x^2 + x + 1 = 0$ or just calculated the discriminant often correctly concluded that the graph did not cross the axis but, to complete this proof, it was necessary to use the fact that $x^2 + x + 1$ has a positive coefficient if $x^2$ and this was not often seen. A suitable diagram was accepted here as a sufficient supporting argument. A correct argument for part (c) was often given by those who were unable to tackle part (b) successfully and this was allowed the mark.

**2.** This question caused problems for all but a handful of very good candidates. The vast majority did not deal with an even function. There were some good sketches of $y = x^2 - 4ax$ but rarely was a completely correct sketch seen. In part (b) most obtained the value for f(2$a$), though some failed to simplify it, but f(– 2$a$) was rarely correct, 12$a^2$ being a common error.

The correct quadratic was usually solved successfully in part (c) and sometimes the –3 root was rejected but only a handful gave ±15 as their final answer.