Core 3 Trigonometry Questions

1 (a) Find $\dfrac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

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4 It is given that $2\operatorname{cosec}^2 x = 5 - 5\cot x$.

(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

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3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3\sec x + 9$ can be written as

$$\sec^2 x - 3\sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3\sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

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7 (a) Given that $z = \dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{dz}{dx} = \sec^2 x$. *(3 marks)*

(b) Sketch the curve with equation $y = \sec x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. *(2 marks)*

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2}\sin y$. *(1 mark)*

(ii) Given that $x = \frac{1}{2}\sin y$, find $\dfrac{dx}{dy}$ in terms of y. *(1 mark)*

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - 4x^2}}$$ *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

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5 (a) (i) Show that the equation

$$2\cot^2 x + 5\operatorname{cosec} x = 10$$

can be written in the form $2\operatorname{cosec}^2 x + 5\operatorname{cosec} x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$.
(3 marks)

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6 (a) Find $\dfrac{dy}{dx}$ when:

(ii) $y = x^2 \tan x$. *(2 marks)*

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8 (a) Write down $\displaystyle\int \sec^2 x \, dx$. *(1 mark)*

(b) Given that $y = \dfrac{\cos x}{\sin x}$, use the quotient rule to show that $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

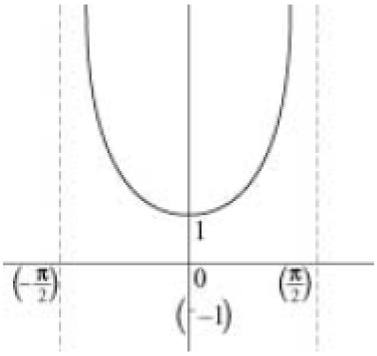
(d) Hence find $\displaystyle\int_{0.5}^{1} (\tan x + \cot x)^2 \, dx$, giving your answer to two significant figures.
(4 marks)

Core 3 Trigonometry Answers

1(a)	$\frac{dy}{dx} = 3\sec^2 3x$	M1		for sec 3x SC/$3\sec^2 x$ B1
		A1	2	
	Alternative			
	Use of product/Quotient rule (M1)			Good attempt
	$\frac{3\cos^2 3x + 3\sin^2 3x}{\cos^2 3x}$ (A1)			Correct

4(a)	$2\mathrm{cosec}^2 x = 5(1-\cot x)$			
	$2 + 2\cot^2 x = 5 - 5\cot x$	M1		use of $\mathrm{cosec}^2 x = 1 + \cot^2 x$
	$2\cot^2 x + 5\cot x - 3 = 0$	A1	2	AG
(b)	$(2\cot x - 1)(\cot x + 3) = 0$	M1		or $2 + 5t - 3t^2 = 0$ Or in $\tan x$ $(2-t)(1+3t) = 0$
	$\cot x = \frac{1}{2}, -3$			
	$\tan x = 2, -\frac{1}{3}$	A1	2	AG
(c)	$x = 1.1, -2.0$ } AWRT	B1		Any 2 correct — In degrees: B0
	$x = -0.3, 2.8$ }	B1		Any 3 correct — B1
		B1	3	4 correct — B2

3(a)	$\sec x = 5$			
	$\cos x = 0.2$	M1		
	$x = 1.37, 4.91$ AWRT	A1A1	3	
(b)	$\tan^2 x = 3\sec x + 9$			
	$\sec^2 x - 1 = 3\sec x + 9$	M1		for using $\sec^2 x = 1 + \tan^2 x$ OE
	$\sec^2 x - 3\sec x - 10 = 0$	A1	2	AG
(c)	$(\sec x - 5)(\sec x + 2) = 0$	M1		or use of formula (attempt)
	$\sec x = 5, -2$	A1		
	$\cos x = 0.2, -0.5$			
	$x = 1.37, 4.91$	B1F		any 2 correct or ft their 2 answers in (a)
	$2.09, 4.19$	A1	4	all 4 correct, no extras

7(a)	$z=\frac{\sin x}{\cos x}$			
	$\frac{dz}{dx}=\frac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}$	M1 A1		use of quotient rule $\left(\frac{\pm\cos^2 x\pm\sin^2 x}{\cos^2 x}\right)$
	$=\frac{1}{\cos^2 x}$ $=\sec^2 x$	A1	3	AG (be convinced)
(b)		M1		correct shape including asymptotic behaviour and symmetrical about $x=0$ and $y>0$
		A1	2	use of 1

(b)(i)	$y=\sin^{-1}2x$ $\sin y=2x$ and $\frac{1}{2}\sin y=x$	B1	1	AG (be convinced)
(ii)	$\frac{dx}{dy}=\frac{1}{2}\cos y$	B1	1	
(c)	$\frac{dy}{dx}=\frac{2}{\cos y}$	M1A1		M1 for $\frac{k}{\cos y}$
	$\sin y=2x$ and $\sin^2+\cos^2=1$	M1		use of to get $\cos y$ or $\cos^2 y$
	$\cos y=\sqrt{1-4x^2}$ $\frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$	A1	4	AG; condone omission of proof of sign

2	Stretch (I) SF $\frac{1}{3}$ (II)	M1		For I + (II or III)
	Parallel to $x-$axis (III)	A1		All correct
	Translate	E1		Allow translation
	$\begin{pmatrix}0\\1\end{pmatrix}$	B1	4	Correct vector or description

5(a)(i)	$2(\text{cosec}^2 x - 1) + 5\,\text{cosec}\,x = 10$	M1		
	$2\,\text{cosec}^2 x - 2 + 5\,\text{cosec}\,x - 10 = 0$			
	$2\,\text{cosec}^2 x + 5\,\text{cosec}\,x - 12 = 0$	A1	2	**AG**
(ii)	$(2\,\text{cosec}\,x - 3)(\text{cosec}\,x + 4) = 0$	M1		Attempt to solve
	$\text{cosec}\,x = \frac{3}{2}$ or -4	A1		Condone answers with no method shown
	$\sin x = \frac{2}{3}$ or $-\frac{1}{4}$	A1	3	**AG**
(b)	$(\theta - 0.1) = 0.73, 2.41, -0.25, -2.89$ AWRT	B1		2 correct values, may be implied later $(41.8, 138.2, -165.5, -14.5)$
	$\theta = 0.83, 2.51, -0.15, -2.79$ AWRT	B1		2 correct answers
		B1	3	+ 2 correct answers and no extra within range

(ii)	$y = x^2 \tan x$	M1		Product rule
	$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$	A1	2	

8(a)	$\tan x \ (+c)$	B1	1	
(b)	$f(x) = \frac{\cos x}{\sin x}$			
	$f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$	M1 A1		quotient rule $\frac{\pm \sin^2 x \pm \cos^2 x}{\sin^2 x}$
	$= \frac{-1}{\sin^2 x}$	A1		use of $\sin^2 x + \cos^2 x = 1$
	$= -\text{cosec}^2 x$	A1	4	AG CSO **Special cases** $f(x) = \frac{\cot x}{1}$ $f'(x) = \frac{1 \times -\text{cosec}^2 x - \cot x \times 0}{1^2}$ M1 $= -\text{cosec}^2 x$ A1 (max 2/4) **Or** $f(x) = \frac{1}{\tan x}$ $f'(x) = \frac{\tan x \times 0 - 1 \times \sec^2 x}{\tan^2 x}$ M1 A1 $= \frac{-\sec^2 x}{\tan^2 x}$ $= \frac{-1}{\sin^2 x} = -\text{cosec}^2$ A1 (max 3/4)

(c)	LHS $= \tan^2 x + \cot^2 x + 2\tan x \cot x$	M1		expanding
	$= \tan^2 x + 1 + \cot^2 x + 1$	M1		correct use of trig identities
	$= \sec^2 x + \text{cosec}^2 x$	A1	3	CSO
	$=$ RHS			
(d)	$\int (\tan x + \cot x)^2 \, dx = \int \sec^2 x + \text{cosec}^2 x \, dx$	M1		use of identity
	$= [\tan x - \cot x]_{0.5}^{1}$	M1 A1		$\pm \tan x \pm \cot x$ OE
	$= 0.9153 - -1.2842$			
	$= 2.2$	A1	4	AWRT