

Solutionbank C3

Edexcel AS and A Level Modular Mathematics

2 Review Exercise

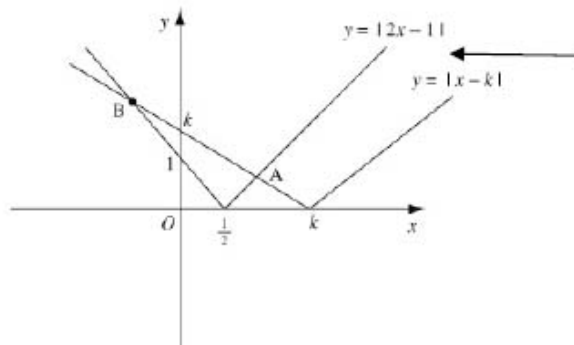
Exercise A, Question 1

Question:

- a On the same set of axes sketch the graphs of $y = |2x - 1|$ and $y = |x - k|$, $k > 1$.
- b Find, in terms of k , the values of x for which $|2x - 1| = |x - k|$.

Solution:

a



For $y = |2x - 1|$ draw $y = 2x - 1$ and reflect part below x -axis in the x -axis. For $y = |x - k|$, intersects x -axis at $(k, 0)$ where $k > 1$ and gradient is 1, whereas the other line has gradient 2.

b For point A

$$\begin{aligned} 2x - 1 &= -(x - k) \\ \Rightarrow 3x &= 1 + k \\ x &= \frac{1 + k}{3} \end{aligned}$$

The line $|x - k|$ has been reflected so equation is $y = -(x - k)$.

For point B

$$\begin{aligned} -(2x - 1) &= -(x - k) \\ \Rightarrow -2x + 1 &= -x + k \\ x &= 1 - k \end{aligned}$$

Both lines have been reflected.

As $k > 0$, this value is negative, which agrees with diagram.

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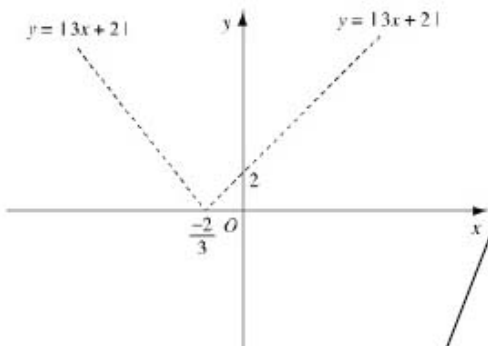
2 Review Exercise
Exercise A, Question 2

Question:

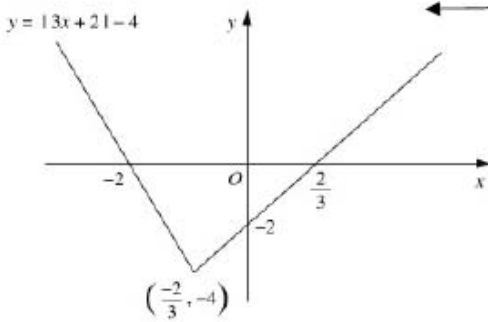
- a Sketch the graph of $y = |3x + 2| - 4$, showing the coordinates of the points of intersection of the graph with the axes.
- b Find the values of x for which $|3x + 2| = 4 + x$.

Solution:

a



First draw $y = |3x + 2|$ and then translate by 4 units in negative y -direction.



Meets x -axis where

i

$$y = 3x + 2 - 4 = 0$$

$$\Rightarrow 3x = 2$$

$$x = \frac{2}{3}$$

ii

$$y = -3x - 2 - 4 = 0$$

$$\Rightarrow 3x = -6$$

$$x = -2$$

b

$$|3x + 2| = 4 + x$$

$$|3x + 2| - 4 = x$$

The values of x are the x -coordinates of the intersections of $y = x$ and $y = |3x + 2| - 4$.

So

i

$$3x + 2 - 4 = x$$

$$\Rightarrow 2x = 2$$

$$x = 1$$

Alternatively, you could solve $(3x + 2)^2 = (4 + x)^2$.

ii

$$-3x - 2 - 4 = x$$

$$\Rightarrow 4x = -6$$

$$x = -1\frac{1}{2}$$

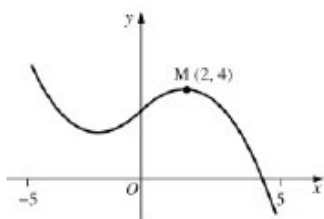
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2 Review Exercise

Exercise A, Question 3

Question:



The figure shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.

The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

a $y = f(x) + 3$

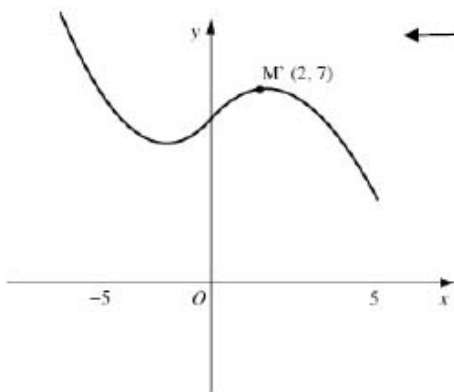
b $y = |f(x)|$

c $y = f(|x|)$.

Show on each graph the coordinates of any maximum turning points.

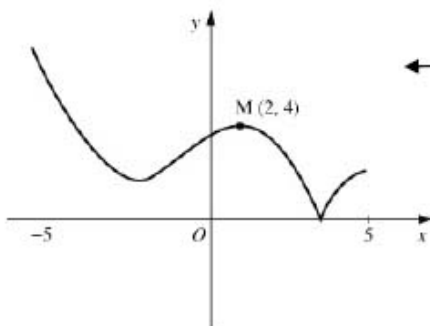
Solution:

a



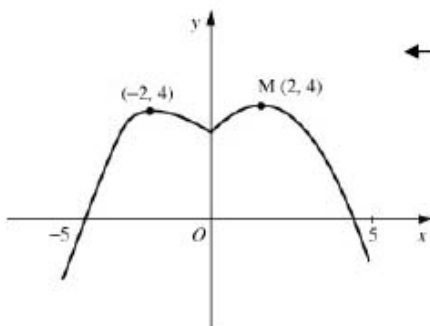
← Translation of +3 in the y direction.

b



← For $y \geq 0$, curve is $y = f(x)$.
For $y < 0$, reflect in x -axis.

c



← For $x < 0$
 $f|x| = f(-x)$
so draw $y = f(x)$
for $x \geq 0$, and then reflect this in
 $x = 0$.

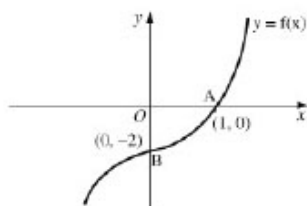
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2 Review Exercise

Exercise A, Question 4

Question:



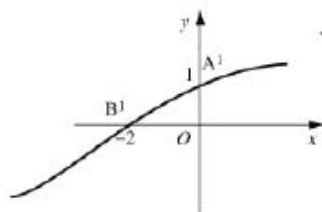
The diagram shows a sketch of the graph of the increasing function f . The curve crosses the x -axis at the point $A(1, 0)$ and the y -axis at the point $B(0, -2)$. On separate diagrams, sketch the graph of:

- a $y = f^{-1}(x)$
- b $y = f(|x|)$
- c $y = f(2x) + 1$
- d $y = 3f(x-1)$.

In each case, show the images of the points A and B .

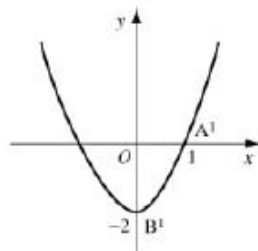
Solution:

a



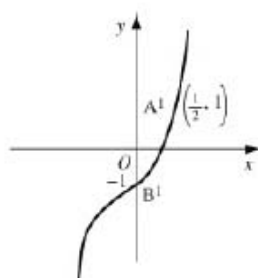
← Reflect $y = f(x)$ in $y = x$.
 $(1, 0) \rightarrow (0, 1)$
 $(0, -2) \rightarrow (-2, 0)$

b



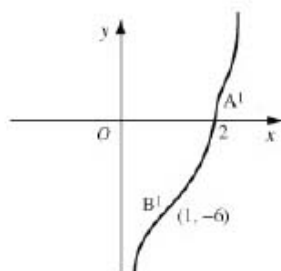
← Draw $y = f(x), x \geq 0$
 Reflect this in y -axis.

c



← Sketch $y = f(x)$ in x -axis with scale factor $\frac{1}{2}$, and translate in the y direction by 1 unit.

d



← Translate by +1 in the x direction and stretch in the y direction with scale factor 3.

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2 Review Exercise

Exercise A, Question 5

Question:

For the positive constant k , where $k > 1$ the functions f and g are defined by

$$f : x \rightarrow \ln(x+k), x > -k,$$

$$g : x \rightarrow |2x-k|, x \in \mathbb{R}$$

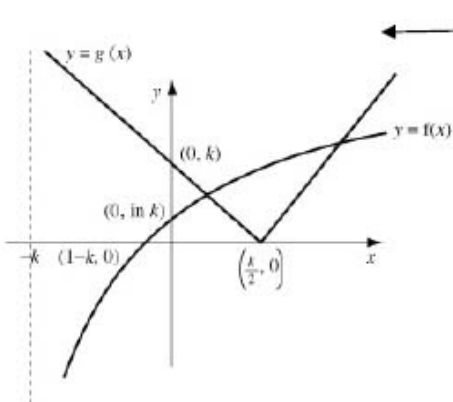
- Sketch, on the same set of axes, the graphs of f and g . Give the coordinates of points where the graphs meet the axes.
- Write down the range of f .
- Find, in terms of k , $fg\left(\frac{k}{4}\right)$.

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

- Find the value of k . *E*

Solution:

a



Graph is for $k > 1$ $y = f(x)$ has asymptote $x = -k$.
 It meets the x -axis where $\ln(x+k) = 0$
 $\Rightarrow x+k = 1$
 $\Rightarrow x = 1-k$
 It meets the y -axis where $x = 0$, i.e. $y = \ln k$.
 $y = g(x)$ is v-shaped passing through $(\frac{k}{2}, 0)$ and $(0, k)$.

b

Range of f is $f(x) \in \mathbb{R}$

f is an increasing function.

c

$$\begin{aligned} fg\left(\frac{k}{4}\right) &= f\left(-\frac{k}{2}\right) \\ &= f\left(\frac{k}{2}\right) \\ &= \ln\left(\frac{3k}{2}\right) \end{aligned}$$

d

$$y = \ln(x+k)$$

$$\frac{dy}{dx} = \frac{1}{x+k}$$

$$\text{So when } x = 3 \quad \frac{1}{3+k} = \frac{2}{9}$$

Gradient of $9y = 2x + 1$ is $\frac{2}{9}$.

$$\Rightarrow 7 = 6 + 2k$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

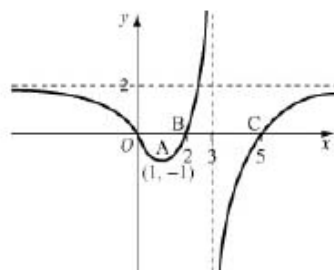
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2 Review Exercise

Exercise A, Question 6

Question:



The diagram shows a sketch of the graph of $y = f(x)$.

The curve has a minimum at the point $A(1, -1)$, passes through x -axis at the origin, and the points $B(2, 0)$ and $C(5, 0)$; the asymptotes have equations $x = 3$ and $y = 2$.

a Sketch, on separate axes, the graph of

i $y = |f(x)|$

ii $y = -f(x+1)$

iii $y = f(-2x)$

in each case, showing the images of the points A, B and C.

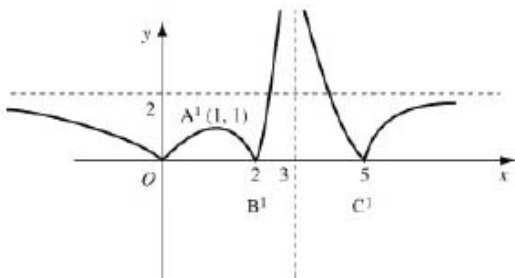
b State the number of solutions to the equation

i $3|f(x)| = 2$

ii $2|f(x)| = 3$.

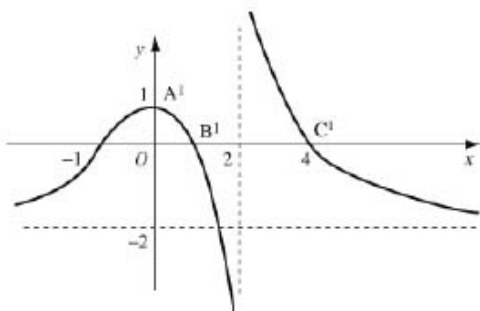
Solution:

a i



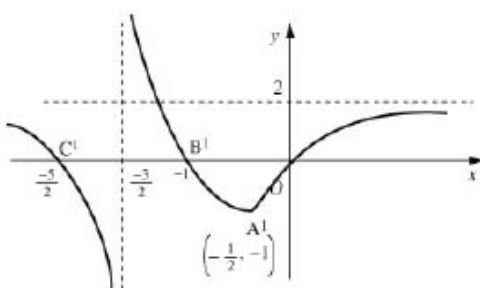
All parts of curve $y = f(x)$ below x -axis are reflected in x -axis.
 $A \rightarrow (1, 1)$
 B and C do not move.

ii



Translate by -1 in the x direction and reflect in the x - axis.

iii



Stretch in the x direction with scale factor $-\frac{1}{2}$ (or stretch in the x direction with scale factor $\frac{1}{2}$ and reflect in the y -axis).

b i

$3|f(x)| = 2 \Rightarrow |f(x)| = \frac{2}{3}$
 number of solutions is 6

ii $2|f(x)| = 3 \Rightarrow |f(x)| = \frac{3}{2}$
 number of solutions is 4

ie $2|f(x)| = 3 \Rightarrow |f(x)| = \frac{3}{2}$

Consider graph a i.
i How many times does the line $y = \frac{2}{3}$ cross the curve?
 Line is below A' .
ii Draw the line $y = \frac{3}{2}$.

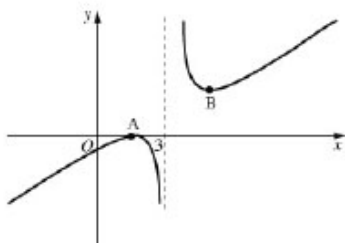
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2 Review Exercise

Exercise A, Question 7

Question:



The diagram shows part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{(x-1)^2}{(x-3)}.$$

The points A and B are the stationary points of C .

The line $x = 3$ is a vertical asymptote to C .

- Using calculus, find the coordinates of A and B .
- Sketch the curve C^* , with equation $y = f(-x) + 2$, showing the coordinates of the images of A and B .
- State the equation of the vertical asymptote to C^* .

Solution:

a

$$\frac{dy}{dx} = \frac{(x-3)2(x-1) - (x-1)^2(1)}{(x-3)^2}$$

Use the quotient rule.

$$= \frac{(x-1)[2x-6 - (x-1)]}{(x-3)^2}$$

$$= \frac{(x-1)(x-5)}{(x-3)^2}$$

$f(x) = 0 \Rightarrow x = 1$, so, as a check, you know that the coordinates of A are (1, 0).

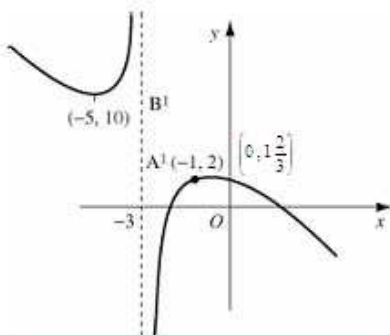
$$\frac{dy}{dx} = 0 \Rightarrow x = 1 \text{ or } x = 5$$

Coordinates of A: (1, 0)

B: (5, 8)

$$y = \frac{(5-1)^2}{(5-3)} = \frac{16}{2} = 8$$

b



Reflect in the y -axis and translate by 2 units in the y direction.

c $x = -3$

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2 Review Exercise

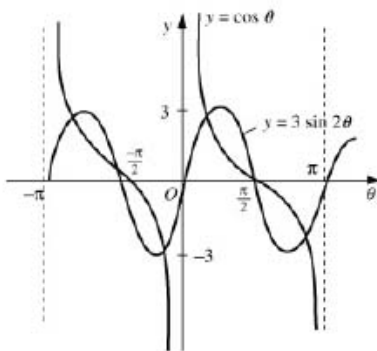
Exercise A, Question 8

Question:

- a On the same set of axes, in the interval $-\pi < \theta < \pi$, sketch the graphs of
- $y = \cot \theta$,
 - $y = 3 \sin 2\theta$
- b Solve, in the interval $-\pi < \theta < \pi$, the equation $\cot \theta = 3 \sin 2\theta$ giving your answers, in radians, to 3 significant figures where appropriate.

Solution:

a



b

$$\begin{aligned} \cot \theta &= 3 \sin 2\theta \\ \Rightarrow \frac{\cos \theta}{\sin \theta} &= 6 \sin \theta \cos \theta \end{aligned}$$

Do not cancel $\cos \theta$.

$$\Rightarrow \cos \theta (1 - 6 \sin^2 \theta) = 0$$

$$\Rightarrow \cos \theta = 0 \text{ or } \sin \theta = \pm \sqrt{\frac{1}{6}}$$

Do not forget \pm for $\sin \theta$.

$$\cos \theta = 0 \Rightarrow \theta = \pm \frac{\pi}{2} = 1.57$$

$$\sin \theta = \pm \sqrt{\frac{1}{6}}$$

$$\theta =$$

$$0.421, 2.72, -0.421, -2.72$$

Give values in all 4 quadrants.

Remember to use the radian mode on your calculator.

$$\alpha, \pi - \alpha, -\alpha, -\pi + \alpha$$

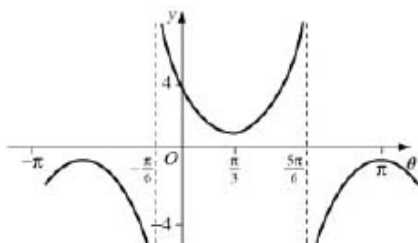
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2 Review Exercise

Exercise A, Question 9

Question:



The diagram shows, in the interval $-\pi \leq \theta \leq \pi$, the graph of $y = k \sec(\theta - \alpha)$.

The curve crosses the y -axis at the point $(0, 4)$ and the θ -coordinate of its minimum point is $\pi/3$.

- State, as a multiple of π , the value of α .
- Find the value of k .
- Find the exact values of θ at the points where the graph crosses the line $y = -2\sqrt{2}$.
- Show that the gradient at the point on the curve with θ -coordinate $\frac{7\pi}{12}$ is $2\sqrt{2}$.

Solution:

a $\frac{\pi}{3}$

$y = k \sec \theta$ has minimum on y -axis.

This curve has been translated by $\frac{\pi}{3}$
in the x direction.

b As $(0, 4)$ lies on curve

$$\begin{aligned} 4 &= k \sec\left(-\frac{\pi}{3}\right) \\ \Rightarrow 4 &= 2k \\ \Rightarrow k &= 2 \end{aligned}$$

$$\begin{aligned} \cos\left(-\frac{\pi}{3}\right) &= \frac{1}{2} \\ \Rightarrow \sec\left(-\frac{\pi}{3}\right) &= 2 \end{aligned}$$

c Solve

$$\begin{aligned} 2 \sec\left(\theta - \frac{\pi}{3}\right) &= -2\sqrt{2} \\ \Rightarrow \sec\left(\theta - \frac{\pi}{3}\right) &= -\sqrt{2} \\ \Rightarrow \cos\left(\theta - \frac{\pi}{3}\right) &= -\frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \Rightarrow \theta - \frac{\pi}{3} &= -\frac{5\pi}{4}, -\frac{3\pi}{4} \\ \theta &= \frac{\pi}{3} - \frac{5\pi}{4}, \frac{\pi}{3} - \frac{3\pi}{4} \\ &= -\frac{11\pi}{12}, -\frac{5\pi}{12} \end{aligned}$$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ gives values in the

2nd and 3rd quadrants.

$$-\frac{4\pi}{3} \leq \theta - \frac{\pi}{3} \leq \frac{2\pi}{3}$$

[$y = -2\sqrt{2}$ meets the graph
where θ is negative.]

d

$$\begin{aligned} \frac{dy}{d\theta} &= 2 \sec\left(\theta - \frac{\pi}{3}\right) \tan\left(\theta - \frac{\pi}{3}\right) \\ \text{At } \theta &= \frac{7\pi}{12}, \frac{dy}{dx} = 2 \sec \frac{\pi}{4} \tan \frac{\pi}{4} = 2\sqrt{2}(1) = 2\sqrt{2} \end{aligned}$$

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2 Review Exercise

Exercise A, Question 10

Question:

- a Given that $\sin^2 \theta + \cos^2 \theta = 1$, show that $1 + \tan^2 \theta \equiv \sec^2 \theta$.
- b Solve, for $0 \leq \theta < 360^\circ$, the equation $2 \tan^2 \theta + \sec \theta = 1$, giving your answers to 1 decimal place. *E*

Solution:

a

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &\equiv 1 \\ \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} &\equiv \frac{1}{\cos^2 \theta} \\ \Rightarrow \tan^2 \theta + 1 &\equiv \sec^2 \theta \end{aligned}$$

b

$$2 \tan^2 \theta + \sec \theta = 1$$

$$\Rightarrow 2(\sec^2 \theta - 1) + \sec \theta = 1$$

$$\Rightarrow 2 \sec^2 \theta + \sec \theta - 3 = 0$$

$$\Rightarrow (2 \sec \theta + 3)(\sec \theta - 1) = 0$$

$$\Rightarrow \sec \theta = -\frac{3}{2} \quad \text{or} \quad \sec \theta = +1$$

$$\Rightarrow \cos \theta = -\frac{2}{3} \quad \text{or} \quad \cos \theta = +1$$

$$\cos \theta = -\frac{2}{3} \Rightarrow \theta = 131.8^\circ, 228.2^\circ$$

$$\cos \theta = +1 \Rightarrow \theta = 0^\circ$$

Use result in a to form a quadratic in $\sec \theta$.

360° not in interval.

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2 Review Exercise

Exercise A, Question 11

Question:

- a Prove that $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$.
- b Find all the values of x , in the interval $0 \leq x \leq 360^\circ$, for which $\sec^4 2x = \tan 2x(3 + \tan^3 2x)$.
Give your answers correct to 1 decimal place, where appropriate.

Solution:

a

$$\begin{aligned} \sec^4 \theta - \tan^4 \theta &= (\sec^2 \theta + \tan^2 \theta)(\sec^2 \theta - \tan^2 \theta) \\ &= (1 + \tan^2 \theta + \tan^2 \theta)(1) \\ &= 1 + 2 \tan^2 \theta \end{aligned}$$

Use $a^2 - b^2 = (a + b)(a - b)$.

Use $\sec^2 \theta = 1 + \tan^2 \theta$.

b

$$\begin{aligned} \sec^4 2x &= 3 \tan 2x + \tan^4 2x \\ \Rightarrow \sec^4 2x - \tan^4 2x &= 3 \tan 2x \\ \Rightarrow 1 + 2 \tan^2 2x &= 3 \tan 2x \\ \Rightarrow 2 \tan^2 2x - 3 \tan 2x + 1 &= 0 \\ \Rightarrow (2 \tan 2x - 1)(\tan 2x - 1) &= 0 \\ \tan 2x = \frac{1}{2} \text{ or } \tan 2x = 1 & \\ \tan 2x = 1 \Rightarrow 2x &= 45^\circ, 225^\circ, 405^\circ, 585^\circ \\ x &= 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ \\ \tan 2x = \frac{1}{2} \Rightarrow 2x &= 26.6^\circ, 206.6^\circ, 386.6^\circ, 566.6^\circ \\ x &= 13.3^\circ, 103.3^\circ, 193.3^\circ, 283.3^\circ \end{aligned}$$

Use result in a with $\theta = 2x$.

$0 \leq x \leq 360^\circ$
 $\Rightarrow 0 \leq 2x \leq 720^\circ$

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Exercise A, Question 12

Question:

a Prove that

$$\cot \theta - \tan \theta = 2 \cot 2\theta, \theta \neq \frac{n\pi}{2}.$$

b Solve, for $-\pi < \theta < \pi$, the equation

$$\cot \theta - \tan \theta = 5,$$

giving your answers to 3 significant figures.

Solution:

a

$$\begin{aligned} \text{LHS} &= \cot \theta - \tan \theta \\ &= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos 2\theta}{\frac{1}{2} \sin 2\theta} \\ &= 2 \cot 2\theta \end{aligned}$$

b Solve

$$2 \cot 2\theta = 5$$

$$\Rightarrow \cot 2\theta = \frac{5}{2}$$

$$\tan 2\theta = 0.4$$

$$\Rightarrow 2\theta = -5.903, -2.761, 0.3805, \dots, 3.522$$

$$\theta = -2.95, -1.38, 0.190, 1.76 \text{ (3 s.f.)}$$

$\begin{aligned} -\pi &< \theta < \pi \\ -2\pi &< 2\theta < 2\pi \end{aligned}$

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2 Review Exercise

Exercise A, Question 13

Question:

- a Solve, in the interval $0 \leq \theta \leq 2\pi$, $\sec \theta + 2 = \cos \theta + \tan \theta(3 + \sin \theta)$, giving your answers to 3 significant figures.
- b Solve, in the interval $0 \leq x \leq 360^\circ$, $\cot^2 x = \operatorname{cosec} x(2 - \operatorname{cosec} x)$, giving your answers to 1 decimal place.

Solution:

a

$$\begin{aligned} \sec \theta + 2 &= \cos \theta + \tan \theta(3 + \sin \theta) \\ \Rightarrow 1 + 2 \cos \theta &= \cos^2 \theta + 3 \sin \theta + \sin^2 \theta \\ \Rightarrow 1 + 2 \cos \theta &= 1 + 3 \sin \theta \\ \Rightarrow 3 \sin \theta &= 2 \cos \theta \\ \Rightarrow \tan \theta &= \frac{2}{3} \\ \Rightarrow \theta &= 0.588, 3.73 \text{ (3 s.f.)} \end{aligned}$$

Multiply by $\cos \theta$.
Use $\sin^2 \theta + \cos^2 \theta = 1$.

b

$$\begin{aligned} \cot^2 x &= \operatorname{cosec} x(2 - \operatorname{cosec} x) \\ &= 2 \operatorname{cosec} x - \operatorname{cosec}^2 x \\ \Rightarrow \operatorname{cosec}^2 x - 1 &= 2 \operatorname{cosec} x - \operatorname{cosec}^2 x \\ \Rightarrow 2 \operatorname{cosec}^2 x - 2 \operatorname{cosec} x - 1 &= 0 \\ \Rightarrow \operatorname{cosec} x &= \frac{2 \pm \sqrt{4 + 8}}{4} \\ &= \frac{1 \pm \sqrt{3}}{2} \end{aligned}$$

Use $1 + \cot^2 x = \operatorname{cosec}^2 x$ to form a quadratic equation in $\operatorname{cosec} x$.

$$\sqrt{12} = 2\sqrt{3}$$

As $\operatorname{cosec} x \geq 1$ or $\operatorname{cosec} x \leq -1$

$$\operatorname{cosec} x = \frac{1 + \sqrt{3}}{2} = 1.366\dots$$

$$\sin x = 0.732\dots$$

$$x = 47.1^\circ, 132.9^\circ$$

$$-1 \leq \frac{1 - \sqrt{3}}{2} \leq 1 \text{ so invalid.}$$

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2 Review Exercise
Exercise A, Question 14

Question:

Given that

$$y = \arcsin x, -1 \leq x \leq 1 \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2},$$

- a express $\arccos x$ in terms of y .
- b Hence find, in terms of π the value of $\arcsin x + \arccos x$.

Given that

$$y = \arccos x, -1 \leq x \leq 1 \text{ and } 0 \leq y \leq \pi,$$

- c sketch, on the same set of axes, the graphs of $y = \arcsin x$ and $y = \arccos x$, making it clear which is which.
- d Explain how your sketches can be used to evaluate $\arcsin x + \arccos x$.

Solution:

a $y = \arcsin x$

$$\Rightarrow \sin y = x$$

$$x = \cos\left(\frac{\pi}{2} - y\right)$$

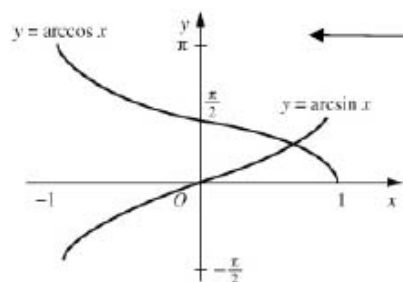
$$\Rightarrow \frac{\pi}{2} - y = \arccos x$$

Using $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$.

b

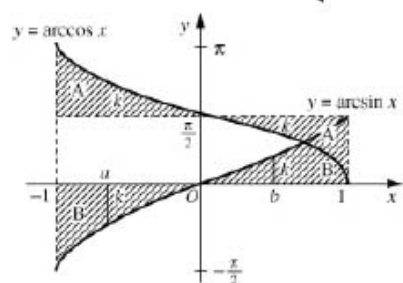
$$\begin{aligned} \arcsin x + \arccos x &= y + \frac{\pi}{2} - y \\ &= \frac{\pi}{2} \end{aligned}$$

c



$y = \arccos x$ is a reflection in the line $y = x$ of $y = \cos x, 0 \leq x \leq \pi$.
 $y = \arcsin x$ is a reflection in the line $y = x$ of $y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

d



The shaded areas A and B are congruent due to the symmetries of the graphs.
 Consider $x = a$, where $-1 \leq a < 0$
 $\arccos x = \frac{\pi}{2} + h$, see diagram
 $\arcsin x = -h$
 $\Rightarrow \arcsin x + \arccos x = \frac{\pi}{2}$
 Consider $x = b$, where $0 \leq b \leq 1$
 $\arccos x = \frac{\pi}{2} - k$
 $\arcsin x = k$
 $\Rightarrow \arcsin x + \arccos x = \frac{\pi}{2}$

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise Exercise A, Question 15

Question:

- a By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$, show that
 $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$.
- b Given that $\cos \theta = \frac{\sqrt{2}}{3}$, find the exact value of $\sec 3\theta$.

Solution:

a

$$\begin{aligned} \cos(2\theta + \theta) &= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta \\ &= (2\cos^2 \theta - 1)\cos \theta - 2\sin \theta \cos \theta \sin \theta \\ &= 2\cos^3 \theta - \cos \theta - 2\cos \theta(1 - \cos^2 \theta) \\ &= 2\cos^3 \theta - \cos \theta - 2\cos \theta + 2\cos^3 \theta \\ &= 4\cos^3 \theta - 3\cos \theta \end{aligned}$$

Use
 $\cos(A + B) = \cos A \cos B - \sin A \sin B$
 with $A = 2\theta, B = \theta$.

b

Use $\sin^2 \theta = 1 - \cos^2 \theta$.

$$\begin{aligned} \cos 3\theta &= 4\left(\frac{\sqrt{2}}{3}\right)^3 - 3\left(\frac{\sqrt{2}}{3}\right) \\ &= \frac{8\sqrt{2}}{27} - \frac{\cancel{3}\sqrt{2}}{\cancel{3}} \\ &= \frac{-19\sqrt{2}}{27} \end{aligned}$$

$(\sqrt{2})^3 = 2\sqrt{2}$

$$\Rightarrow \sec 3\theta = \frac{-27}{19\sqrt{2}} = \frac{-27\sqrt{2}}{38}$$

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise

Exercise A, Question 16

Question:

Given that $\sin(x+30^\circ) = 2\sin(x-60^\circ)$,

- a show that $\tan x = 8+5\sqrt{3}$.
 b Hence express $\tan(x+60^\circ)$ in the form $a+b\sqrt{3}$.

Solution:

a $\sin(x+30^\circ) = 2\sin(x-60^\circ)$

$$\Rightarrow \sin x \cos 30^\circ + \cos x \sin 30^\circ = 2\sin x \cos 60^\circ - 2\cos x \sin 60^\circ$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \sin x - \sqrt{3}\cos x$$

$$\Rightarrow \sqrt{3}\sin x + \cos x = 2\sin x - 2\sqrt{3}\cos x$$

$$\Rightarrow (2\sqrt{3}+1)\cos x = (2-\sqrt{3})\sin x$$

$$\Rightarrow \tan x = \frac{2\sqrt{3}+1}{2-\sqrt{3}}$$

$$= \frac{(2\sqrt{3}+1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{4\sqrt{3} + \sqrt{3} + 2 + 6}{1}$$

$$= 8+5\sqrt{3}$$

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

Use $\frac{\sin x}{\cos x} = \tan x$.

Rationalise the denominator.

b

$$\tan(x+60^\circ) = \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}$$

$$= \frac{8+6\sqrt{3}}{1-8\sqrt{3}-15}$$

$$= \frac{8+6\sqrt{3}}{-8\sqrt{3}-14} = \frac{-4-3\sqrt{3}}{4\sqrt{3}+7}$$

$$= \frac{-(4+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$= \frac{-[(28-36)+(21\sqrt{3}-16\sqrt{3})]}{1}$$

$$= 8-5\sqrt{3}$$

$\tan 60^\circ = \sqrt{3}$

Use a.

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2 Review Exercise

Exercise A, Question 17

Question:

a Given that $\cos A = \frac{3}{4}$ where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$.

b i Show that

$$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) = \cos 2x$$

Given that

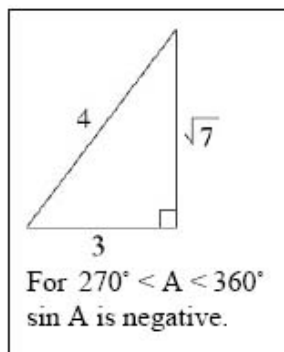
$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

ii show that $\frac{dy}{dx} = \sin 2x$ **E**

Solution:

a

$$\begin{aligned}\cos A &= \frac{3}{4}, \quad 270^\circ < A < 360^\circ \\ \Rightarrow \sin A &= \frac{-\sqrt{7}}{4} \\ \text{so } \sin 2A &= 2 \sin A \cos A \\ &= -2 \frac{\sqrt{7}}{4} \times \frac{3}{4} \\ &= \frac{-3\sqrt{7}}{8}\end{aligned}$$



b i

$$\begin{aligned}\cos\left(2x + \frac{\pi}{3}\right) &= \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} \\ &= \frac{1}{2} \cos 2x - \frac{\sqrt{3}}{2} \sin 2x \\ \cos\left(2x - \frac{\pi}{3}\right) &= \frac{1}{2} \cos 2x + \frac{\sqrt{3}}{2} \sin 2x \\ \text{so } \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) &= \cos 2x \quad \leftarrow \text{Add two results above.}\end{aligned}$$

ii

$$\begin{aligned}y &= 3 \sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \\ &= 3 \sin^2 x + \cos 2x \\ \frac{dy}{dx} &= 3(2 \sin x \cos x) - 2 \sin 2x \\ &= \sin 2x\end{aligned}$$

Use b i.

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2 Review Exercise

Exercise A, Question 18

Question:

Solve, in the interval $-180^\circ \leq x < 180^\circ$, the equations

- a $\cos 2x + \sin x = 1$
 b $\sin x(\cos x + \operatorname{cosec} x) = 2\cos^2 x$, giving your answers to 1 decimal place.

Solution:

a

$$\begin{aligned}\cos 2x + \sin x &= 1 \\ \Rightarrow (1 - \sin^2 x) + \sin x &= 1 \\ \Rightarrow 2\sin^2 x - \sin x &= 0 \\ \sin x(2\sin x - 1) &= 0 \\ \Rightarrow \sin x = 0 \quad \text{or} \quad \sin x &= \frac{1}{2}\end{aligned}$$

$$\Rightarrow x = -180^\circ, 0^\circ, 30^\circ, 150^\circ$$

b

$$\begin{aligned}\sin x \cos x + \sin x \cdot \frac{1}{\sin x} &= 2\cos^2 x \\ \Rightarrow \sin x \cos x &= 2\cos^2 x - 1 \\ \Rightarrow \frac{1}{2}\sin 2x &= \cos 2x \\ \Rightarrow \tan 2x &= 2 \\ \Rightarrow 2x &= -116.57^\circ, 63.43^\circ \\ x &= -58.3^\circ, 31.7^\circ \text{ (1 d.p.)}\end{aligned}$$

← Choose the appropriate form of $\cos 2x$ to give a quadratic in $\sin x$. Do NOT cancel $\sin x$, always factorise.

$$\begin{aligned}-180^\circ \leq x &< 180^\circ \\ \Rightarrow -360^\circ \leq 2x &< 360^\circ\end{aligned}$$

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2 Review Exercise

Exercise A, Question 19

Question:

- a Prove that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n^\circ.$$
- b Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$.
- c Solve, for $0^\circ < \theta < 360^\circ$, the equation

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 3,$$

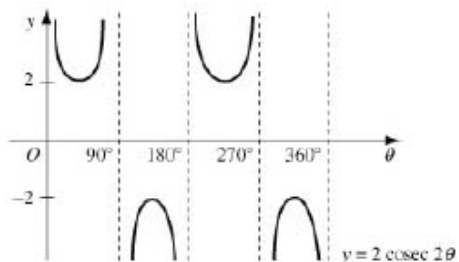
 giving your answers to 1 decimal place. *E*

Solution:

a

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\frac{1}{2} \sin 2\theta} \\ &= 2 \operatorname{cosec} 2\theta \end{aligned}$$

b



Note that maxima and minima are at -2 and $+2$ respectively.

c Solve

$$2 \operatorname{cosec} 2\theta = 3$$

$$\text{i.e. } \sin 2\theta = \frac{2}{3}$$

$$\Rightarrow 2\theta = 41.8^\circ, 138.2^\circ, 401.8^\circ, 498.2^\circ$$

$$\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ \text{ (1 d.p.)}$$

$$\begin{aligned} 0^\circ < \theta &< 360^\circ \\ \Rightarrow 0^\circ < 2\theta &< 720^\circ \end{aligned}$$

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2 Review Exercise
Exercise A, Question 20

Question:

- a Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
- b Hence find the greatest value of $(3\sin x + 2\cos x)^4$.
- c Solve, for $0 < x < 2\pi$, the equation $3\sin x + 2\cos x = 1$, giving your answers to 3 decimal places. *E*

Solution:

a Set

$$3 \sin x + 2 \cos x \equiv R \sin(x + \alpha)$$

$$\equiv R \sin x \cos \alpha + R \cos x \sin \alpha$$

$$\Rightarrow R \cos \alpha = 3$$

$$R \sin \alpha = 2$$

$$\Rightarrow \tan \alpha = \frac{2}{3}$$

$$\Rightarrow \alpha = 0.588\dots$$

$$R = \sqrt{13}$$

← Comparing $\sin x$.
Comparing $\cos x$.

← Divide.

← α should be in radians as
 $0 < \alpha < \frac{\pi}{2}$.

For R :

either square and add

$$R^2(\sin^2 \alpha + \cos^2 \alpha) = 3^2 + 2^2,$$

$$R > 0$$

or use $R \cos \alpha = 3$ or $R \sin \alpha = 2$ with
 α found above.

$$\Rightarrow 3 \sin x + 2 \cos x = \sqrt{13} \sin(x + 0.588\dots)$$

b The maximum value of $\sqrt{13} \sin(x + 0.588\dots)$ is $\sqrt{13}$ and occurs when
 $\sin(x + 0.588\dots) = 1$ so maximum value of

$$\{\sqrt{13} \sin(x + 0.588\dots)\}^4 = (\sqrt{13})^4$$

$$= 169$$

c Solve

$$\sqrt{13} \sin(x + 0.588\dots) = 1$$

$$\Rightarrow \sin(x + 0.588\dots) = \frac{1}{\sqrt{13}}$$

$$\Rightarrow x + 0.588\dots = \pi - 0.2810\dots \quad 2\pi + 0.2810\dots$$

$$= 2.8606, 6.5642\dots$$

$$\Rightarrow x = 2.273, 5.976 \text{ (3 d.p.)}$$

$$0 < x < 2\pi$$

$$\Rightarrow 0.588 < x + 0.588 < 6.87$$

$$\sin^{-1} \frac{1}{\sqrt{13}} = \sin^{-1} 0.277$$

$$= 0.2810\dots$$

is outside above interval for $x + 0.588$.

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2 Review Exercise
Exercise A, Question 21

Question:

The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$.

The x -coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form

$y = ax + b$, where a and b are constants. E

Solution:

$$y = \ln\left(\frac{1}{3}x\right)$$

$$x = 3, y = \ln 1 = 0, \text{ so } P \equiv (3, 0)$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\text{At } P, \text{ gradient of tangent} = \frac{1}{3}$$

$$\text{so gradient of normal} = -3$$

Equation of normal is

$$y - 0 = -3(x - 3)$$

$$y = -3x + 9$$

Remember that
 $\log ax = \log a + \log x$ in any
base

$$\text{so } \frac{d}{dx}(\ln ax) = \frac{d}{dx}(\ln x)$$

Use $m_1 m_2 = -1$ for
perpendicular lines.

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2 Review Exercise

Exercise A, Question 22

Question:

a Differentiate with respect to x

i $3\sin^2 x + \sec 2x$,

ii $\{x + \ln(2x)\}^3$,

Given that $y = \frac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq -1$,

b show that $\frac{dy}{dx} = -\frac{8}{(x-1)^3}$

E

Solution:

a i

$$y = 3\sin^2 x + \sec 2x$$

$$\frac{dy}{dx} = 6\sin x \cos x + 2\sec 2x \tan 2x$$

$$= 3\sin 2x + 2\sec 2x \tan 2x$$

$$\begin{aligned} \frac{d}{dx}(\sin x)^2 &= 2\sin x \cos x \\ \frac{d}{dx}\sec ax &= a\sec ax \tan ax \end{aligned}$$

ii

$$y = \{x + \ln(2x)\}^3$$

$$\frac{dy}{dx} = 3\{x + \ln(2x)\}^2 \left\{1 + \frac{1}{x}\right\}$$

$$\begin{aligned} y &= u^3 \text{ where } u = f(x) \\ \frac{dy}{dx} &= 3u^2 \frac{du}{dx} \\ \text{Note } \frac{d}{dx}(\ln ax) &= \frac{1}{x} \end{aligned}$$

b

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \quad x \neq -1$$

$$\frac{dy}{dx} = \frac{(x-1)^2(10)(x-1) - (5x^2 - 10x + 9)(2)(x-1)}{(x-1)^4}$$

$$= \frac{10(x^2 - 2x + 1) - 2(5x^2 - 10x + 9)}{(x-1)^3}$$

$$= \frac{10x^2 - 20x + 10 - 10x^2 + 20x - 18}{(x-1)^3}$$

$$= \frac{-8}{(x-1)^3}$$

$$\begin{aligned} \text{Use quotient rule} \\ u &= 5x^2 - 10x + 9 \\ \frac{du}{dx} &= 10x - 10 \\ v &= (x-1)^2 \\ \frac{dv}{dx} &= 2(x-1) \end{aligned}$$

Be careful to use brackets and signs appropriately.

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2 Review Exercise

Exercise A, Question 23

Question:

Given that $y = \ln(1 + e^x)$,

a show that when $x = -\ln 3$, $\frac{dy}{dx} = \frac{1}{4}$

b find the exact value of x for which $e^x \frac{dy}{dx} = 6$.

Solution:

$$y = \ln(1 + e^x)$$

a $\frac{dy}{dx} = \frac{e^x}{1 + e^x}$

when $x = -\ln 3$

$$\begin{aligned} e^x &= e^{-\ln 3} = e^{\ln 3^{-1}} \\ &= \frac{1}{3} \end{aligned}$$

$$\text{so } \frac{dy}{dx} = \frac{\frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4}$$

b

$$e^x \frac{dy}{dx} = 6$$

$$\Rightarrow (1 + e^x) \frac{e^x}{1 + e^x} = 6$$

$$\Rightarrow e^x = 6$$

$$\Rightarrow x = \ln 6$$

If $y = \ln u$ where $u = f(x)$
 $\frac{dy}{dx} = \frac{1}{u} \frac{du}{dx}$

Remember that $e^{\ln k} = k$

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2 Review Exercise

Exercise A, Question 24

Question:

a Differentiate with respect to x

i $x^2 e^{3x+2}$,

ii $\frac{\cos(2x^3)}{3x}$

b Given that $x = 4\sin(2y+6)$, find $\frac{dy}{dx}$ in terms of x . **E**

Solution:

a i

$$y = x^2 e^{3x+2}$$

$$\frac{dy}{dx} = x^2 \cdot 3e^{3x+2} + 2xe^{3x+2}$$

$$= (3x+2)xe^{3x+2}$$

Use the product rule with

$$u = x^2 \Rightarrow \frac{du}{dx} = 2x$$

$$v = e^{3x+2} \Rightarrow \frac{dv}{dx} = 3e^{3x+2}$$

ii

$$y = \frac{\cos(2x^3)}{3x}$$

$$\frac{dy}{dx} = \frac{3x[-6x^2 \sin(2x^3)] - 3 \cos(2x^3)}{9x^2}$$

$$= \frac{-[6x^3 \sin(2x^3) + \cos(2x^3)]}{3x^2}$$

Use the quotient rule with

$$u = \cos(2x^3)$$

$$\Rightarrow \frac{du}{dx} = -6x^2 \sin(2x^3)$$

$$v = 3x \Rightarrow \frac{dv}{dx} = 3$$

b

$$x = 4\sin(2y+6)$$

$$\Rightarrow \frac{dx}{dy} = 8\cos(2y+6)$$

$$\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$$

$$= \pm \frac{1}{8\sqrt{1-\sin^2(2y+6)}}$$

$$= \pm \frac{1}{8\sqrt{1-\frac{x^2}{16}}}$$

$$= \pm \frac{1}{2\sqrt{16-x^2}}$$

If

$$y = \sin(ax+b)$$

$$\frac{dy}{dx} = a \cos(ax+b)$$

$$\frac{dy}{dx} \text{ is in terms of } y.$$

Use $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

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Edexcel AS and A Level Modular Mathematics

2 Review Exercise

Exercise A, Question 25

Question:

Given that $x = y^2 e^{\sqrt{y}}$,

- a find, in terms of y , $\frac{dx}{dy}$
- b show that when $y = 4$, $\frac{dy}{dx} = \frac{e^{-2}}{12}$.

Solution:

$$x = y^2 e^{\sqrt{y}}$$

a

$$\begin{aligned} \frac{dx}{dy} &= y^2 \frac{1}{2\sqrt{y}} e^{\sqrt{y}} + 2ye^{\sqrt{y}} \\ &= \frac{1}{2} y^{\frac{3}{2}} e^{\sqrt{y}} + 2ye^{\sqrt{y}} \\ &= \frac{y}{2} e^{\sqrt{y}} (\sqrt{y} + 4) \end{aligned}$$

b When $y = 4$

$$\begin{aligned} \frac{dx}{dy} &= \frac{4}{2} e^2 (2 + 4) \\ &= 12e^2 \\ \text{so } \frac{dy}{dx} &= \frac{1}{12e^2} = \frac{e^{-2}}{12} \end{aligned}$$

Use the product rule with

$$u = y^2 \Rightarrow \frac{du}{dy} = 2y$$

$$v = e^{\sqrt{y}} \Rightarrow \frac{dv}{dy} = \frac{1}{2\sqrt{y}} e^{\sqrt{y}}$$

Remember

If

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f(x) e^{f(x)}$$

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2 Review Exercise

Exercise A, Question 26

Question:

- a Given that $y = \sqrt{1+x^2}$, show that $\frac{dy}{dx} = \frac{\sqrt{3}}{2}$ when $x = \sqrt{3}$.
- b Given that $y = \ln\{x + \sqrt{1+x^2}\}$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$.

Solution:

a

$$\begin{aligned}
 y &= \sqrt{1+x^2} = (1+x^2)^{\frac{1}{2}} \\
 \Rightarrow \frac{dy}{dx} &= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x \\
 &= \frac{2x}{2\sqrt{1+x^2}} \\
 &= \frac{x}{\sqrt{1+x^2}}
 \end{aligned}$$

When $x = \sqrt{3}$

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{\sqrt{3}}{\sqrt{1+3}} \\
 &= \frac{\sqrt{3}}{2}
 \end{aligned}$$

b

$$\begin{aligned}
 y &= \ln\{x + \sqrt{1+x^2}\} \\
 \frac{dy}{dx} &= \frac{1}{x + \sqrt{1+x^2}} \cdot \frac{d}{dx}\{x + \sqrt{1+x^2}\} \\
 &= \frac{1}{x + \sqrt{1+x^2}} \times \left[1 + \frac{x}{\sqrt{1+x^2}}\right] \\
 &= \frac{1}{\cancel{x + \sqrt{1+x^2}} \cdot \frac{(\sqrt{1+x^2} + x)}{\sqrt{1+x^2}}} \\
 &= \frac{1}{\sqrt{1+x^2}}
 \end{aligned}$$

Note that this cannot be simplified in particular,
 $\ln\{x + \sqrt{1+x^2}\} \neq \ln x + \ln \sqrt{1+x^2}$

If
 $y = \ln f(x)$
 $\frac{dy}{dx} = \frac{1}{f(x)} f'(x)$

$$\begin{aligned}
 \frac{d}{dx}(1+x^2)^{\frac{1}{2}} &= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x \\
 &= \frac{x}{\sqrt{1+x^2}}
 \end{aligned}$$

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2 Review Exercise

Exercise A, Question 27

Question:

Given that $f(x) = x^2e^{-x}$,

- find $f'(x)$, using the product rule for differentiation
 - show that $f''(x) = (x^2 - 4x + 2)e^{-x}$.
- A curve C has equation $y = f(x)$.
- Find the coordinates of the turning points of C .
 - Determine the nature of each turning point of the curve C .

Solution:

a

$$\begin{aligned} f(x) &= x^2e^{-x} \\ f'(x) &= x^2(-e^{-x}) + 2xe^{-x} \\ &= e^{-x}[-x^2 + 2x] \end{aligned}$$

Use the product rule.

b

$$\begin{aligned} f''(x) &= e^{-x}[-2x + 2] - e^{-x}[-x^2 + 2x] \\ &= e^{-x}[x^2 - 4x + 2] \end{aligned}$$

c Turning points when $f'(x) = 0$

$$\begin{aligned} \text{i.e. } x(2-x)e^{-x} &= 0 \\ \Rightarrow x(2-x) &= 0 \quad \text{As } e^{-x} \neq 0 \end{aligned}$$

So turning points when $x = 0, y = 0$

$$x = 2, y = 4e^{-2}$$

d When $x = 0, f''(0) = +2 > 0$

So $(0, 0)$ is a minimum point

When $x = 2, f''(2) = e^{-2}(4 - 8 + 2) < 0$

So $(2, 4e^{-2})$ is a maximum point

Solutionbank C3

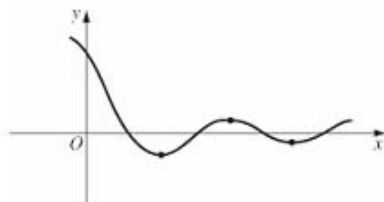
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2 Review Exercise

Exercise A, Question 28

Question:

- a Express $(\sin 2x + \sqrt{3} \cos 2x)$ in the form $R \sin(2x + k\pi)$, where $R > 0$ and $0 < k < \frac{1}{2}$.



The diagram shows part of the curve with equation

$$y = e^{-2\sqrt{2}x}(\sin 2x + \sqrt{3} \cos 2x).$$

- b Show that the x -coordinates of the turning points of the curve satisfy the equation

$$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

Solution:

$$\mathbf{a} \quad \sin 2x + \sqrt{3} \cos 2x = R \sin 2x \cos k\pi + R \cos 2x \sin k\pi$$

$$\Rightarrow R \cos k\pi = 1$$

$$R \sin k\pi = \sqrt{3}$$

$$\tan k\pi = \sqrt{3} \Rightarrow k = \frac{1}{3}$$

$$R = 2$$

b

$$y = 2e^{-2\sqrt{2}x} \sin\left(2x + \frac{\pi}{3}\right)$$

Use product rule.

$$\frac{dy}{dx} = 2e^{-2\sqrt{2}x} 2 \cos\left(2x + \frac{\pi}{3}\right) - 4\sqrt{2}e^{-2\sqrt{2}x} \sin\left(2x + \frac{\pi}{3}\right)$$

$$= 4e^{-2\sqrt{2}x} \left[\cos\left(2x + \frac{\pi}{3}\right) - \sqrt{2} \sin\left(2x + \frac{\pi}{3}\right) \right]$$

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \cos\left(2x + \frac{\pi}{3}\right) - \sqrt{2} \sin\left(2x + \frac{\pi}{3}\right) = 0$$

$$\Rightarrow \tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

Solutionbank C3

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2 Review Exercise
Exercise A, Question 29

Question:

The curve C has equation $y = x^2\sqrt{\cos x}$. The point P on C has x -coordinate $\frac{\pi}{3}$.

- a Show that the y -coordinate of P is $\frac{\sqrt{2}\pi^2}{18}$.
- b Show that the gradient of C at P is 0.809, to 3 significant figures.

In the interval $0 < x < \frac{\pi}{2}$, C has a maximum at the point A .

- c Show that the x -coordinate, k , of A satisfies the equation $x \tan x = 4$.

The iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{4}{x_n}\right), \quad x_0 = 1.25,$$

is used to find an approximation for k .

- d Find the value of k , correct to 4 decimal places.

Solution:

a

$$y = x^2 \sqrt{\cos x}$$

$$x = \frac{\pi}{3} \Rightarrow y = \frac{\pi^2}{9} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}\pi^2}{18}$$

$$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

b

$$\frac{dy}{dx} = 2x\sqrt{\cos x} + \frac{x^2(-\sin x)}{2\sqrt{\cos x}}$$

$$\text{When } x = \frac{\pi}{3}$$

Use product rule

$$u = x^2, \frac{du}{dx} = 2x$$

$$v = \sqrt{\cos x}, \frac{dv}{dx} = \frac{1}{2}(\cos x)^{-\frac{1}{2}}(-\sin x)$$

$$\frac{dy}{dx} = \left[\frac{2\pi}{3} \sqrt{\frac{1}{2}} + \frac{\pi^2}{18} \left(-\frac{\sqrt{3}}{2} \right) \right] \cdot \sqrt{2}$$

$$= 0.8094\dots$$

$$= 0.809 \text{ (3 s.f.)}$$

c Setting $\frac{dy}{dx} = 0$

$$\Rightarrow \frac{4x \cos x - x^2 \sin x}{2\sqrt{\cos x}} = 0$$

$$\Rightarrow 4 \cos x - x \sin x = 0$$

$$\Rightarrow 4 \cos x = x \sin x$$

$$\Rightarrow x \tan x = 4$$

Divide both sides by $\cos x$.

Ensure your calculator is in radian mode.

d $x_{n+1} = \tan^{-1}\left(\frac{4}{x_n}\right), x_0 = 1.25$

$$\Rightarrow x_1 = 1.26791\dots$$

$$x_2 = 1.26383\dots$$

$$x_3 = 1.26476\dots$$

$$x_4 = 1.26455\dots$$

$$x_5 = 1.26460\dots$$

$$x_6 = 1.26459\dots$$

$$x_7 = 1.26459\dots$$

$$x_8 = 1.26459\dots$$

$$\Rightarrow x = 1.2646 \text{ (4 d.p.)}$$

You can verify this by considering the sign of $f(x) = 4 \cos x - x \sin x$ in interval $[1.26455, 1.26465]$

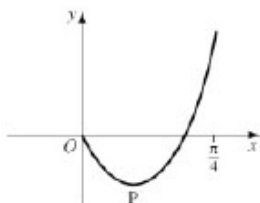
Solutionbank C3

Edexcel AS and A Level Modular Mathematics

2 Review Exercise

Exercise A, Question 30

Question:



The figure shows part of the curve with equation

$$y = (2x - 1) \tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x -coordinate of P is k .

a Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k .

b Calculate the values of x_1, x_2, x_3 and x_4 , giving your answers to 4 decimal places.

c Show that $k = 0.277$, correct to 3 significant figures. *E*

Solution:

a

$$y = (2x-1) \tan 2x$$

$$\frac{dy}{dx} = (2x-1)2 \sec^2 2x + 2 \tan 2x$$

$$\text{Setting } \frac{dy}{dx} = 0$$

$$\Rightarrow (2x-1) \sec^2 2x + \tan 2x = 0$$

$$\Rightarrow (2x-1) \frac{1}{\cos^2 2x} + \frac{\sin 2x}{\cos 2x} = 0$$

$$\Rightarrow (2x-1) + \sin 2x \cos 2x = 0$$

$$\Rightarrow 4x - 2 + 2 \sin 2x \cos 2x = 0$$

$$\Rightarrow 4x + \sin 4x - 2 = 0$$

$$\cos 2x \neq 0 \text{ in } 0 \leq x < \frac{\pi}{4}$$

$$\text{Use } \sin 2A = 2 \sin A \cos A \text{ with } A = 2x$$

$$\text{so } k \text{ satisfies } 4x + \sin 4x - 2 = 0$$

b

$$x_1 = 0.2670 \text{ (4 d.p.)}$$

$$x_2 = 0.2809 \text{ (4 d.p.)}$$

$$x_3 = 0.2746 \text{ (4 d.p.)}$$

$$x_4 = 0.2774 \text{ (4 d.p.)}$$

Work in radian mode.

c Consider $f(x) = 4x + \sin 4x - 2$

$$f(0.2775) = 0.00569\dots$$

$$f(0.2765) = -0.000087\dots$$

so k is 0.277 (3 s.f.)

As there is a sign change in the interval, k lies between the two values.