

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Differentiate:

(a) $(1 + 2x)^4$

(b) $(3 - 2x^2)^{-5}$

(c) $(3 + 4x)^{\frac{1}{2}}$

(d) $(6x + x^2)^7$

(e) $\frac{1}{3 + 2x}$

(f) $\sqrt{7 - x}$

(g) $4(2 + 8x)^4$

(h) $3(8 - x)^{-6}$

Solution:

(a) Let $u = 1 + 2x$, then $y = u^4$

$$\frac{du}{dx} = 2 \quad \text{and} \quad \frac{dy}{du} = 4u^3$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 4u^3 \times 2 = 8u^3 = 8(1 + 2x)^3$$

(b) Let $u = 3 - 2x^2$ then $y = u^{-5}$

$$\frac{du}{dx} = -4x \quad \text{and} \quad \frac{dy}{du} = -5u^{-6}$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -5u^{-6} \times -4x = 20xu^{-6} = 20x(3 - 2x^2)^{-6}$$

(c) Let $u = 3 + 4x$, then $y = u^{\frac{1}{2}}$

$$\frac{du}{dx} = 4 \quad \text{and} \quad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times 4 = 2u^{-\frac{1}{2}} = 2(3 + 4x)^{-\frac{1}{2}}$$

(d) Let $u = 6x + x^2$, then $y = u^7$

$$\frac{du}{dx} = 6 + 2x \quad \text{and} \quad \frac{dy}{du} = 7u^6$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 7u^6 \times (6 + 2x) = 7(6 + 2x)(6x + x^2)^6$$

(e) Let $u = 3 + 2x$, then $y = \frac{1}{u} = u^{-1}$

$$\frac{du}{dx} = 2 \quad \text{and} \quad \frac{dy}{du} = -u^{-2}$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -u^{-2} \times 2 = -2u^{-2} = \frac{-2}{(3 + 2x)^2}$$

(f) Let $u = 7 - x$, then $y = u^{\frac{1}{2}}$

$$\frac{du}{dx} = -1 \quad \text{and} \quad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times -1 = -\frac{1}{2}(7 - x)^{-\frac{1}{2}}$$

(g) Let $u = 2 + 8x$, then $y = 4u^4$

$$\frac{du}{dx} = 8 \quad \text{and} \quad \frac{dy}{du} = 16u^3$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 16u^3 \times 8 = 128(2 + 8x)^3$$

(h) Let $u = 8 - x$, then $y = 3u^{-6}$

$$\frac{du}{dx} = -1 \quad \text{and} \quad \frac{dy}{du} = -18u^{-7}$$

Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -18u^{-7} \times -1 = 18(8-x)^{-7}$$

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Exercise A, Question 2

Question:

Given that $y = \frac{1}{(4x+1)^2}$ find the value of $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.

Solution:

Let $u = 4x + 1$, then $y = u^{-2}$

$$\frac{du}{dx} = 4 \quad \frac{dy}{du} = -2u^{-3}$$

$$\therefore \frac{dy}{dx} = -8u^{-3} = \frac{-8}{(4x+1)^3}$$

When $x = \frac{1}{4}$, $\frac{dy}{dx} = -1$

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Exercise A, Question 3

Question:

Given that $y = (5 - 2x)^3$ find the value of $\frac{dy}{dx}$ at (1, 27).

Solution:

Let $u = 5 - 2x$, then $y = u^3$

$$\frac{du}{dx} = -2 \quad \frac{dy}{du} = 3u^2$$

$$\therefore \frac{dy}{dx} = -6u^2 = -6(5 - 2x)^2$$

When $x = 1$, $\frac{dy}{dx} = -54$

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Exercise A, Question 4

Question:

Find the value of $\frac{dy}{dx}$ at the point (8, 2) on the curve with equation $3y^2 - 2y = x$.

Solution:

$$x = 3y^2 - 2y$$

$$\frac{dx}{dy} = 6y - 2$$

$$\frac{dy}{dx} = \frac{1}{6y - 2}$$

At (8, 2) the value of y is 2.

$$\therefore \frac{dy}{dx} = \frac{1}{12 - 2} = \frac{1}{10}$$

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Exercise A, Question 5

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(2\frac{1}{2}, 4 \right)$ on the curve with equation y

$$\frac{1}{2} + y^{-\frac{1}{2}} = x.$$

Solution:

$$x = y^{\frac{1}{2}} + y^{-\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{2}y^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{2}y^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{3}{2}}}$$

At the point $\left(2\frac{1}{2}, 4 \right)$ the value of y is 4.

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{1}{2}(4)^{-\frac{1}{2}} - \frac{1}{2}(4)^{-\frac{3}{2}}} = \frac{1}{\frac{1}{4} - \frac{1}{16}} = \frac{1}{\frac{3}{16}} = \frac{16}{3} = 5\frac{1}{3}$$

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Exercise B, Question 1

Question:

Differentiate:

(a) $x (1 + 3x) ^ 5$

(b) $2x (1 + 3x^2) ^ 3$

(c) $x^3 (2x + 6) ^ 4$

(d) $3x^2 (5x - 1) ^{-1}$

Solution:

(a) Let $y = x (1 + 3x) ^ 5$

Let $u = x$ and $v = (1 + 3x) ^ 5$

Then $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = 5 \times 3 (1 + 3x) ^ 4$ (using the chain rule)

Now use the product rule.

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= x \times 15 (1 + 3x) ^ 4 + (1 + 3x) ^ 5 \times 1 \\ &= (1 + 3x) ^ 4 (15x + 1 + 3x) \\ &= (1 + 3x) ^ 4 (1 + 18x) \end{aligned}$$

(b) Let $y = 2x (1 + 3x^2) ^ 3$

Let $u = 2x$ and $v = (1 + 3x^2) ^ 3$

Then $\frac{du}{dx} = 2$ and $\frac{dv}{dx} = 18x (1 + 3x^2) ^ 2$

Using the product rule,

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= 2x \times 18x (1 + 3x^2)^2 + 2(1 + 3x^2)^3 \\
 &= (1 + 3x^2)^2 [36x^2 + 2(1 + 3x^2)] \\
 &= (1 + 3x^2)^2 (42x^2 + 2) \\
 &= 2(1 + 3x^2)^2 (1 + 21x^2)
 \end{aligned}$$

(c) Let $y = x^3 (2x + 6)^4$

Let $u = x^3$ and $v = (2x + 6)^4$

Then $\frac{du}{dx} = 3x^2$ and $\frac{dv}{dx} = 8(2x + 6)^3$

Using the product rule,

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= x^3 \times 8(2x + 6)^3 + (2x + 6)^4 \times 3x^2 \\
 &= x^2 (2x + 6)^3 [8x + 3(2x + 6)] \\
 &= x^2 (2x + 6)^3 (14x + 18) \\
 &= 2x^2 (2x + 6)^3 (7x + 9)
 \end{aligned}$$

(d) Let $y = 3x^2 (5x - 1)^{-1}$

Let $u = 3x^2$ and $v = (5x - 1)^{-1}$

Then $\frac{du}{dx} = 6x$ and $\frac{dv}{dx} = -5(5x - 1)^{-2}$

Using the product rule,

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= 3x^2 \times -5(5x - 1)^{-2} + (5x - 1)^{-1} \times 6x \\
 &= -15x^2 (5x - 1)^{-2} + 6x (5x - 1)^{-1} \\
 &= 3x (5x - 1)^{-2} [-5x + 2(5x - 1)] \\
 &= 3x (5x - 2) (5x - 1)^{-2}
 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

(a) Find the value of $\frac{dy}{dx}$ at the point (1, 8) on the curve with equation $y = x^2 (3x - 1)^3$.

(b) Find the value of $\frac{dy}{dx}$ at the point (4, 36) on the curve with equation $y = 3x (2x + 1)^{\frac{1}{2}}$.

(c) Find the value of $\frac{dy}{dx}$ at the point $\left(2, \frac{1}{5}\right)$ on the curve with equation $y = (x - 1)(2x + 1)^{-1}$.

Solution:

$$(a) y = x^2 (3x - 1)^3$$

$$\text{Let } u = x^2, v = (3x - 1)^3$$

$$\text{Then } \frac{du}{dx} = 2x, \frac{dv}{dx} = 9(3x - 1)^2$$

Use the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ to give

$$\frac{dy}{dx} = x^2 \times 9(3x - 1)^2 + (3x - 1)^3 \times 2x$$

$$= x(3x - 1)^2 [9x + 2(3x - 1)]$$

$$= x(3x - 1)^2 (15x - 2) \quad *$$

At the point (1, 8), $x = 1$.

Substitute $x = 1$ into the expression *.

$$\text{Then } \frac{dy}{dx} = 1 \times 2^2 \times 13 = 52$$

$$(b) y = 3x (2x + 1)^{\frac{1}{2}}$$

$$\text{Let } u = 3x \text{ and } v = (2x + 1)^{\frac{1}{2}}$$

$$\text{Then } \frac{du}{dx} = 3 \text{ and } \frac{dv}{dx} = \frac{1}{2} \times 2(2x + 1)^{-\frac{1}{2}} = (2x + 1)^{-\frac{1}{2}}$$

Use the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ to give

$$\begin{aligned}\frac{dy}{dx} &= 3x(2x+1)^{-\frac{1}{2}} + 3(2x+1)^{\frac{1}{2}} \\ &= 3(2x+1)^{-\frac{1}{2}} [x + (2x+1)] \\ &= 3(3x+1)(2x+1)^{-\frac{1}{2}} \quad *\end{aligned}$$

At the point (4, 36), $x = 4$.

Substitute $x = 4$ into *.

$$\text{Then } \frac{dy}{dx} = 3 \times 13 \times 9^{-\frac{1}{2}} = 3 \times 13 \times \frac{1}{3} = 13$$

$$(c) y = (x-1)(2x+1)^{-1}$$

Let $u = x-1$ and $v = (2x+1)^{-1}$

$$\text{Then } \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = -2(2x+1)^{-2}$$

Use the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ to give

$$\begin{aligned}\frac{dy}{dx} &= -2(x-1)(2x+1)^{-2} + (2x+1)^{-1} \times 1 \\ &= (2x+1)^{-2} [-2x+2 + (2x+1)] \\ &= 3(2x+1)^{-2} \quad *\end{aligned}$$

At the point $\left(2, \frac{1}{5}\right)$, $x = 2$.

Substitute $x = 2$ into *

$$\text{Then } \frac{dy}{dx} = 3 \times 5^{-2} = \frac{3}{25}$$

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Exercise B, Question 3

Question:

Find the points where the gradient is zero on the curve with equation $y = (x - 2)^2 (2x + 3)$.

Solution:

$$y = (x - 2)^2 (2x + 3)$$

$$u = (x - 2)^2 \text{ and } v = (2x + 3)$$

$$\frac{du}{dx} = 2(x - 2) \text{ and } \frac{dv}{dx} = 2$$

Use the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ to give

$$\begin{aligned} \frac{dy}{dx} &= (x - 2)^2 \times 2 + 2(x - 2)(2x + 3) \\ &= 2(x - 2)[(x - 2) + 2x + 3] \\ &= 2(x - 2)(3x + 1) \end{aligned}$$

When the gradient is zero, $\frac{dy}{dx} = 0$

$$\therefore 2(x - 2)(3x + 1) = 0$$

$$\therefore x = 2 \text{ or } x = -\frac{1}{3}$$

Substitute values for x into $y = (x - 2)^2 (2x + 3)$.

When $x = 2$, $y = 0$; when $x = -\frac{1}{3}$, $y = 12 \frac{19}{27}$.

So points of zero gradient are $(2, 0)$ and $(-\frac{1}{3}, 12 \frac{19}{27})$.

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

Differentiate:

(a) $\frac{5x}{x+1}$

(b) $\frac{2x}{3x-2}$

(c) $\frac{x+3}{2x+1}$

(d) $\frac{3x^2}{(2x-1)^2}$

(e) $\frac{6x}{(5x+3)^{\frac{1}{2}}}$

Solution:

(a) Let $u = 5x$ and $v = x + 1$

$$\therefore \frac{du}{dx} = 5 \text{ and } \frac{dv}{dx} = 1$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(x+1) \times 5 - 5x \times 1}{(x+1)^2} = \frac{5}{(x+1)^2}$$

(b) Let $u = 2x$ and $v = 3x - 2$

$$\therefore \frac{du}{dx} = 2 \text{ and } \frac{dv}{dx} = 3$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(3x-2) \times 2 - 2x \times 3}{(3x-2)^2} = \frac{6x-4-6x}{(3x-2)^2} = \frac{-4}{(3x-2)^2}$$

(c) Let $u = x + 3$ and $v = 2x + 1$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = 2$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(2x + 1) \times 1 - (x + 3) \times 2}{(2x + 1)^2} = \frac{2x + 1 - 2x - 6}{(2x + 1)^2} = \frac{-5}{(2x + 1)^2}$$

(d) Let $u = 3x^2$ and $v = (2x - 1)^2$

$$\therefore \frac{du}{dx} = 6x \text{ and } \frac{dv}{dx} = 4(2x - 1)$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2x - 1)^2 \times 6x - 3x^2 \times 4(2x - 1)}{(2x - 1)^4} \\ &= \frac{6x(2x - 1)[(2x - 1) - 2x]}{(2x - 1)^4} \\ &= \frac{-6x}{(2x - 1)^3} \end{aligned}$$

(e) Let $u = 6x$ and $v = (5x + 3)^{\frac{1}{2}}$

$$\therefore \frac{du}{dx} = 6 \text{ and } \frac{dv}{dx} = \frac{5}{2}(5x + 3)^{-\frac{1}{2}}$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\begin{aligned} \frac{dy}{dx} &= \frac{(5x + 3)^{\frac{1}{2}} \times 6 - 6x \times \frac{5}{2}(5x + 3)^{-\frac{1}{2}}}{[(5x + 3)^{\frac{1}{2}}]^2} \\ &= \frac{3(5x + 3)^{-\frac{1}{2}} [2(5x + 3) - 5x]}{(5x + 3)} \end{aligned}$$

$$= \frac{3(5x + 3) - \frac{1}{2}(10x + 6 - 5x)}{(5x + 3)}$$

$$= \frac{3(5x + 6)}{(5x + 3) \frac{3}{2}}$$

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Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(1, \frac{1}{4}\right)$ on the curve with equation $y = \frac{x}{3x+1}$.

Solution:

Let $u = x$ and $v = 3x + 1$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = 3$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(3x+1) \times 1 - x \times 3}{(3x+1)^2} = \frac{1}{(3x+1)^2} \quad *$$

At the point $\left(1, \frac{1}{4}\right)$, $x = 1$. Substitute $x = 1$ into *.

$$\text{Then } \frac{dy}{dx} = \frac{1}{16}$$

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Exercise C, Question 3

Question:

Find the value of $\frac{dy}{dx}$ at the point (12, 3) on the curve with equation $y = \frac{x+3}{(2x+1)^{\frac{1}{2}}}$.

Solution:

Let $u = x + 3$ and $v = (2x + 1)^{\frac{1}{2}}$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = (2x + 1)^{-\frac{1}{2}}$$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}} \times 1 - (x+3)(2x+1)^{-\frac{1}{2}}}{(2x+1)^{-1}}$$

$$= \frac{(2x+1)^{-\frac{1}{2}} [(2x+1) - (x+3)]}{(2x+1)^{-1}}$$

$$= (2x+1)^{-\frac{3}{2}} (x-2)$$

At the point (12, 3), $x = 12$ and

$$\frac{dy}{dx} = (25)^{-\frac{3}{2}} (10) = \frac{10}{125} = \frac{2}{25}$$

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Edexcel AS and A Level Modular Mathematics

Exercise D, Question 1

Question:

Differentiate:

(a) e^{2x}

(b) e^{-6x}

(c) e^{x+3}

(d) $4e^{3x^2}$

(e) $9e^{3-x}$

(f) xe^{2x}

(g) $(x^2 + 3)e^{-x}$

(h) $(3x - 5)e^{x^2}$

(i) $2x^4e^{1+x}$

(j) $(9x - 1)e^{3x}$

(k) $\frac{x}{e^{2x}}$

(l) $\frac{e^{x^2}}{x}$

(m) $\frac{e^x}{x+1}$

(n) $\frac{e^{-2x}}{\sqrt{x+1}}$

Solution:

(a) Let $y = e^{2x}$, then $y = e^t$ where $t = 2x$

$$\therefore \frac{dy}{dt} = e^t \text{ and } \frac{dt}{dx} = 2$$

By the chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2e^t = 2e^{2x}$$

(b) Let $y = e^{-6x}$

$$\text{If } y = e^{f(x)} \text{ then } \frac{dy}{dx} = f'(x) e^{f(x)}$$

$$\text{Let } f(x) = -6x, \text{ then } f'(x) = -6$$

$$\therefore \frac{dy}{dx} = -6e^{-6x}$$

(c) Let $y = e^{x+3}$

$$\text{Let } f(x) = x+3, \text{ then } f'(x) = 1$$

$$\text{If } y = e^{f(x)} \text{ then } \frac{dy}{dx} = f'(x) e^{f(x)}$$

$$\therefore \frac{dy}{dx} = 1 \times e^{x+3} = e^{x+3}$$

(d) Let $y = 4e^{3x^2}$

$$\text{Let } f(x) = 3x^2, \text{ then } f'(x) = 6x$$

$$\therefore \frac{dy}{dx} = 4 \times 6xe^{3x^2} = 24xe^{3x^2}$$

(e) Let $y = 9e^{3-x}$

$$\text{Let } f(x) = 3-x, \text{ then } f'(x) = -1$$

$$\therefore \frac{dy}{dx} = 9 \times -1 \times e^{3-x} = -9e^{3-x}$$

(f) Let $y = xe^{2x}$

$$\text{Let } u = x \text{ and } v = e^{2x}$$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = 2e^{2x}$$

Use product formula.

$$\frac{dy}{dx} = x \times 2e^{2x} + e^{2x} \times 1 = e^{2x} (2x + 1)$$

(g) Let $y = (x^2 + 3) e^{-x}$

$$\text{Let } u = x^2 + 3 \text{ and } v = e^{-x}$$

$$\therefore \frac{du}{dx} = 2x \text{ and } \frac{dv}{dx} = -e^{-x}$$

Use product formula.

$$\frac{dy}{dx} = (x^2 + 3) (-e^{-x}) + e^{-x} \times 2x = -e^{-x} (x^2 - 2x + 3)$$

(h) Let $y = (3x - 5) e^{x^2}$

Let $u = 3x - 5$ and $v = e^{x^2}$

$$\therefore \frac{du}{dx} = 3 \text{ and } \frac{dv}{dx} = 2xe^{x^2}$$

Use product formula.

$$\frac{dy}{dx} = (3x - 5) \times 2xe^{x^2} + e^{x^2} \times 3 = e^{x^2} (6x^2 - 10x + 3)$$

(i) Let $y = 2x^4 e^{1+x}$

Let $u = 2x^4$ and $v = e^{1+x}$

$$\therefore \frac{du}{dx} = 8x^3 \text{ and } \frac{dv}{dx} = e^{1+x}$$

Use product formula.

$$\frac{dy}{dx} = 2x^4 e^{1+x} + e^{1+x} \times 8x^3 = 2x^3 e^{1+x} (x + 4)$$

(j) Let $y = (9x - 1) e^{3x}$

Let $u = 9x - 1$ and $v = e^{3x}$

$$\therefore \frac{du}{dx} = 9 \text{ and } \frac{dv}{dx} = 3e^{3x}$$

Use product formula.

$$\frac{dy}{dx} = (9x - 1) \times 3e^{3x} + e^{3x} \times 9 = 3e^{3x} (9x + 2)$$

(k) Let $y = \frac{x}{e^{2x}}$

Let $u = x$ and $v = e^{2x}$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = 2e^{2x}$$

Use quotient rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{e^{2x} \times 1 - x \times 2e^{2x}}{e^{4x}} = e^{-2x} (1 - 2x)$$

(l) Let $y = \frac{e^{x^2}}{x}$

Let $u = e^{x^2}$ and $v = x$

$$\therefore \frac{du}{dx} = 2xe^{x^2} \text{ and } \frac{dv}{dx} = 1$$

Use quotient rule.

$$\frac{dy}{dx} = \frac{x \times 2xe^{x^2} - e^{x^2} \times 1}{x^2} = \frac{e^{x^2}(2x^2 - 1)}{x^2}$$

(m) Let $y = \frac{e^x}{x+1}$

Let $u = e^x$ and $v = x + 1$

$$\therefore \frac{du}{dx} = e^x \text{ and } \frac{dv}{dx} = 1$$

Use quotient rule.

$$\frac{dy}{dx} = \frac{(x+1)e^x - e^x \times 1}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

(n) Let $y = \frac{e^{-2x}}{\sqrt{x+1}}$

Let $u = e^{-2x}$ and $v = (x+1)^{\frac{1}{2}}$

$$\therefore \frac{du}{dx} = -2e^{-2x} \text{ and } \frac{dv}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

Use quotient rule.

$$\frac{dy}{dx} = \frac{(x+1)^{\frac{1}{2}}(-2e^{-2x}) - e^{-2x}[\frac{1}{2}(x+1)^{-\frac{1}{2}}]}{[(x+1)^{\frac{1}{2}}]^2}$$

$$= \frac{[-2(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}]e^{-2x}}{x+1}$$

$$= \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}[-4(x+1) - 1]e^{-2x}}{x+1}$$

$$= \frac{-(4x+5)e^{-2x}}{2(x+1)^{\frac{3}{2}}}$$

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Exercise D, Question 2

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(1, \frac{1}{e}\right)$ on the curve with equation

$$y = xe^{-x}.$$

Solution:

$$y = xe^{-x}$$

Let $u = x$ and $v = e^{-x}$

$$\therefore \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = -e^{-x}$$

Use the product rule to give

$$\frac{dy}{dx} = x(-e^{-x}) + e^{-x} \times 1 = e^{-x}(1 - x)$$

At $\left(1, \frac{1}{e}\right)$, $x = 1$.

$$\therefore \frac{dy}{dx} = 0$$

Solutionbank

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Exercise D, Question 3

Question:

Find the value of $\frac{dy}{dx}$ at the point (0, 3) on the curve with equation $y = (2x + 3) e^{2x}$.

Solution:

$$y = (2x + 3) e^{2x}$$

$$\text{Let } u = 2x + 3 \text{ and } v = e^{2x}$$

$$\therefore \frac{du}{dx} = 2 \text{ and } \frac{dv}{dx} = 2e^{2x}$$

Use the product rule.

$$\frac{dy}{dx} = (2x + 3) 2e^{2x} + e^{2x} \times 2 = 2e^{2x} (2x + 3 + 1) = 2e^{2x} (2x + 4)$$

$$= 4e^{2x} \left(x + 2 \right)$$

At the point (0, 3), $x = 0$.

$$\therefore \frac{dy}{dx} = 4 \times 2 = 8$$

Solutionbank

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Exercise D, Question 4

Question:

Find the equation of the tangent to the curve $y = xe^{2x}$ at the point $\left(\frac{1}{2}, \frac{1}{2}e\right)$.

Solution:

$$y = xe^{2x}$$

$$\therefore \frac{dy}{dx} = x \left(2e^{2x} \right) + e^{2x} (1) \quad (\text{From the product rule})$$

At the point $\left(\frac{1}{2}, \frac{1}{2}e\right)$, $x = \frac{1}{2}$.

$$\therefore \frac{dy}{dx} = e + e = 2e$$

The tangent at $\left(\frac{1}{2}, \frac{1}{2}e\right)$ has gradient $2e$.

Its equation is

$$y - \frac{1}{2}e = 2e \left(x - \frac{1}{2} \right)$$

$$y - \frac{1}{2}e = 2ex - e$$

$$y = 2ex - \frac{1}{2}e$$

Solutionbank

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Exercise D, Question 5

Question:

Find the equation of the tangent to the curve $y = \frac{e^{\frac{x}{3}}}{x}$ at the point $\left(3, \frac{1}{3}e \right)$.

Solution:

$$y = \frac{e^{\frac{x}{3}}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{x \left(\frac{1}{3} \right) e^{\frac{x}{3}} - e^{\frac{x}{3}} \times 1}{x^2} \quad (\text{From the quotient rule})$$

At the point $\left(3, \frac{1}{3}e \right)$, $x = 3$.

$$\therefore \frac{dy}{dx} = \frac{e - e}{9} = 0$$

The tangent at $\left(3, \frac{1}{3}e \right)$ has gradient 0.

Its equation is

$$y - \frac{1}{3}e = 0 (x - 3)$$

$$y = \frac{1}{3}e$$

Solutionbank

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Exercise D, Question 6

Question:

Find the coordinates of the turning points on the curve $y = x^2e^{-x}$, and determine whether these points are maximum or minimum points.

Solution:

$$y = x^2e^{-x}$$

$$\frac{dy}{dx} = x^2 \left(-e^{-x} \right) + e^{-x} (2x) = xe^{-x} (2 - x) \quad (\text{From the product rule})$$

At a turning point on the curve $\frac{dy}{dx} = 0$.

$$\therefore xe^{-x} (2 - x) = 0$$

$$\therefore x = 0 \text{ or } x = 2$$

Substitute these values of x into the equation $y = x^2e^{-x}$.

$$\text{When } x = 0, y = 0$$

$$\text{When } x = 2, y = 4e^{-2}$$

The turning points are at $(0, 0)$ and $\left(2, \frac{4}{e^2} \right)$.

To establish the nature of the points find $\frac{d^2y}{dx^2}$.

$$\text{As } \frac{dy}{dx} = xe^{-x} (2 - x) = e^{-x} \left(2x - x^2 \right)$$

$$\frac{d^2y}{dx^2} = e^{-x} (2 - 2x) + (2x - x^2) (-e^{-x}) = e^{-x} (2 - 4x + x^2)$$

(From the product rule)

$$\text{When } x = 0, \frac{d^2y}{dx^2} = 2 > 0 \quad \therefore (0, 0) \text{ is a } \mathbf{minimum} \text{ point}$$

$$\text{When } x = 2, \frac{d^2y}{dx^2} = -2e^{-2} < 0 \quad \therefore \left(2, \frac{4}{e^2} \right) \text{ is a } \mathbf{maximum} \text{ point}$$

Solutionbank

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Exercise D, Question 7

Question:

Given that $y = \frac{e^{3x}}{x}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, simplifying your answers.

Use these answers to find the coordinates of the turning point on the curve with equation $y = \frac{e^{3x}}{x}$, $x > 0$, and determine the nature of this turning point.

Solution:

$$y = \frac{e^{3x}}{x}$$

$$\frac{dy}{dx} = \frac{x(3e^{3x}) - e^{3x} \times 1}{x^2} = \frac{(3x - 1)e^{3x}}{x^2} \quad (\text{From the quotient rule})$$

To determine $\frac{d^2y}{dx^2}$ use the quotient rule again with

$$\begin{aligned} u &= 3xe^{3x} - e^{3x} \text{ and } v = x^2 \\ \frac{d^2y}{dx^2} &= \frac{x^2(9xe^{3x} + 3e^{3x} - 3e^{3x}) - (3xe^{3x} - e^{3x})(2x)}{x^4} \\ &= \frac{9x^3e^{3x} - 6x^2e^{3x} + 2xe^{3x}}{x^4} \\ &= \frac{xe^{3x}(9x^2 - 6x + 2)}{x^4} \\ &= \frac{e^{3x}(9x^2 - 6x + 2)}{x^3} \end{aligned}$$

At the turning point $\frac{dy}{dx} = 0$.

$$\therefore \frac{(3x - 1)e^{3x}}{x^2} = 0$$

$$\therefore x = \frac{1}{3}$$

Substitute $x = \frac{1}{3}$ into $y = \frac{e^{3x}}{x}$.

$$\therefore y = \frac{e}{\frac{1}{3}} = 3e$$

So $\left(\frac{1}{3}, 3e \right)$ are the coordinates of the point with zero gradient.

To determine the nature of this point, substitute $x = \frac{1}{3}$ into

$$\frac{d^2y}{dx^2} = \frac{e^{3x}(9x^2 - 6x + 2)}{x^3}$$

$$\text{When } x = \frac{1}{3}, \frac{d^2y}{dx^2} = \frac{e(1 - 2 + 2)}{\frac{1}{27}} = 27e > 0$$

So there is a **minimum** point at $\left(\frac{1}{3}, 3e \right)$.

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Exercise E, Question 1

Question:

Find the function $f'(x)$ where $f(x)$ is

(a) $\ln(x + 1)$

(b) $\ln 2x$

(c) $\ln 3x$

(d) $\ln(5x - 4)$

(e) $3 \ln x$

(f) $4 \ln 2x$

(g) $5 \ln(x + 4)$

(h) $x \ln x$

(i) $\frac{\ln x}{x + 1}$

(j) $\ln(x^2 - 5)$

(k) $(3 + x) \ln x$

(l) $e^x \ln x$

Solution:

(a) $f(x) = \ln(x + 1)$

$F(x) = x + 1, f'(x) = 1$

$$\therefore f'(x) = \frac{1}{x + 1}$$

(b) $f(x) = \ln 2x$

$F(x) = 2x, f'(x) = 2$

$$\therefore f'(x) = \frac{2}{2x} = \frac{1}{x}$$

$$(c) f(x) = \ln 3x$$

$$f'(x) = \frac{3}{3x} = \frac{1}{x}$$

$$(d) f(x) = \ln(5x - 4)$$

$$f'(x) = \frac{5}{5x - 4}$$

$$(e) f(x) = 3 \ln x$$

$$f'(x) = 3 \times \frac{1}{x} = \frac{3}{x}$$

$$(f) f(x) = 4 \ln 2x$$

$$f'(x) = 4 \times \frac{2}{2x} = \frac{4}{x}$$

$$(g) f(x) = 5 \ln(x + 4)$$

$$f'(x) = 5 \times \frac{1}{x + 4} = \frac{5}{x + 4}$$

$$(h) f(x) = x \ln x$$

Use the product rule with $u = x$ and $v = \ln x$.

$$\text{Then } \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = \frac{1}{x}$$

$$\therefore f'(x) = x \times \frac{1}{x} + \ln x \times 1 = 1 + \ln x$$

$$(i) f(x) = \frac{\ln x}{x + 1}$$

Use the quotient rule with $u = \ln x$ and $v = x + 1$.

$$\text{Then } \frac{du}{dx} = \frac{1}{x} \text{ and } \frac{dv}{dx} = 1$$

$$\therefore f'(x) = \frac{(x + 1) \left(\frac{1}{x} \right) - \ln x \times 1}{(x + 1)^2} = \frac{x + 1 - x \ln x}{x(x + 1)^2}$$

$$(j) f(x) = \ln(x^2 - 5)$$

$$F(x) = x^2 - 5, \quad f'(x) = 2x$$

$$\text{If } f(x) = \ln F(x) \quad \text{then } f'(x) = \frac{f'(x)}{F(x)}$$

$$\therefore f'(x) = \frac{2x}{x^2 - 5}$$

$$(k) f(x) = (3 + x) \ln x$$

Use the product rule with $u = 3 + x$ and $v = \ln x$.

$$\text{Then } \frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = \frac{1}{x}$$

$$\therefore f'(x) = (3 + x) \frac{1}{x} + \ln x = \frac{3 + x}{x} + \ln x$$

$$(l) f(x) = e^x \ln x$$

Use the product rule with $u = e^x$ and $v = \ln x$.

$$\text{Then } \frac{du}{dx} = e^x \text{ and } \frac{dv}{dx} = \frac{1}{x}$$

$$\therefore f'(x) = e^x \times \frac{1}{x} + \ln x \times e^x = e^x \ln x + \frac{e^x}{x}$$

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Exercise F, Question 1

Question:

Differentiate:

(a) $y = \sin 5x$

(b) $y = 2 \sin \frac{1}{2}x$

(c) $y = 3 \sin^2 x$

(d) $y = \sin (2x + 1)$

(e) $y = \sin 8x$

(f) $y = 6 \sin \frac{2}{3}x$

(g) $y = \sin^3 x$

(h) $y = \sin^5 x$

Solution:

(a) $y = \sin 5x$

If $y = \sin f (x)$, then $\frac{dy}{dx} = f' (x) \cos f (x)$.

Let $f (x) = 5x$, then $f' (x) = 5$

$$\therefore \frac{dy}{dx} = 5 \cos 5x$$

(b) $y = 2 \sin \frac{1}{2}x$

$$\frac{dy}{dx} = 2 \times \left(\frac{1}{2} \cos \frac{1}{2}x \right) = \cos \frac{1}{2}x$$

(c) $y = 3 \sin^2 x = 3 (\sin x)^2$

Let $u = \sin x$, then $y = 3u^2$

$$\frac{du}{dx} = \cos x \text{ and } \frac{dy}{du} = 6u$$

From the chain rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 6u \cos x \\ &= 6 \sin x \cos x \\ &= 3 (2 \sin x \cos x) \\ &= 3 \sin 2x \quad (\text{From the double angle formula}) \end{aligned}$$

$$(d) y = \sin (2x + 1)$$

Let $f (x) = 2x + 1$, then $f' (x) = 2$

$$\therefore \frac{dy}{dx} = 2 \cos (2x + 1)$$

$$(e) y = \sin 8x$$

$$\therefore \frac{dy}{dx} = 8 \cos 8x$$

$$(f) y = 6 \sin \frac{2}{3}x$$

$$\therefore \frac{dy}{dx} = 6 \times \frac{2}{3} \cos \frac{2}{3}x = 4 \cos \frac{2}{3}x$$

$$(g) y = \sin^3 x$$

Let $u = \sin x$, then $y = u^3$

$$\frac{du}{dx} = \cos x \text{ and } \frac{dy}{du} = 3u^2$$

$$\therefore \frac{dy}{dx} = 3u^2 \cos x = 3 \sin^2 x \cos x \quad (\text{From the chain rule})$$

$$(h) y = \sin^5 x$$

Let $u = \sin x$, then $y = u^5$

$$\frac{du}{dx} = \cos x \text{ and } \frac{dy}{du} = 5u^4$$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times \cos x = 5 \sin^4 x \cos x$$

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Exercise G, Question 1

Question:

Differentiate:

(a) $y = 2 \cos x$

(b) $y = \cos^5 x$

(c) $y = 6 \cos \frac{5}{6} x$

(d) $y = 4 \cos (3x + 2)$

(e) $y = \cos 4x$

(f) $y = 3 \cos^2 x$

(g) $y = 4 \cos \frac{1}{2} x$

(h) $y = 3 \cos 2x$

Solution:

(a) $y = 2 \cos x$

$$\therefore \frac{dy}{dx} = -2 \sin x$$

(b) $y = \cos^5 x$

Let $u = \cos x$, then $y = u^5$

$$\frac{du}{dx} = -\sin x \text{ and } \frac{dy}{du} = 5u^4$$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times (-\sin x) = -5 \cos^4 x \sin x$$

(c) $y = 6 \cos \frac{5}{6} x$

$$\therefore \frac{dy}{dx} = 6 \times -\frac{5}{6} \sin \frac{5}{6} x = -5 \sin \frac{5}{6} x$$

$$(d) y = 4 \cos (3x + 2)$$

$$\therefore \frac{dy}{dx} = 4 \times -3 \sin (3x + 2) = -12 \sin (3x + 2)$$

$$(e) y = \cos 4x$$

$$\therefore \frac{dy}{dx} = -4 \sin 4x$$

$$(f) y = 3 \cos^2 x$$

$$\text{Let } u = \cos x, \text{ then } y = 3u^2$$

$$\frac{du}{dx} = -\sin x \text{ and } \frac{dy}{du} = 6u$$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 6u (-\sin x)$$

$$= -6 \cos x \sin x$$

$$= -3 (2 \sin x \cos x)$$

$$= -3 \sin 2x \quad (\text{From the double angle formula})$$

$$(g) y = 4 \cos \frac{1}{2}x$$

$$\therefore \frac{dy}{dx} = 4 \left(-\frac{1}{2} \sin \frac{1}{2}x \right) = -2 \sin \frac{1}{2}x$$

$$(h) y = 3 \cos 2x$$

$$\therefore \frac{dy}{dx} = 3 (-2 \sin 2x) = -6 \sin 2x$$

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Exercise H, Question 1

Question:

Differentiate:

(a) $y = \tan 3x$

(b) $y = 4 \tan^3 x$

(c) $y = \tan (x - 1)$

(d) $y = x^2 \tan \frac{1}{2}x + \tan \left(x - \frac{1}{2} \right)$

Solution:

(a) $y = \tan 3x$

$$\therefore \frac{dy}{dx} = 3 \sec^2 3x$$

(b) $y = 4 \tan^3 x$

Let $u = \tan x$, then $y = 4u^3$

$$\frac{du}{dx} = \sec^2 x \text{ and } \frac{dy}{du} = 12u^2$$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 12u^2 \sec^2 x = 12 \tan^2 x \sec^2 x$$

(c) $y = \tan (x - 1)$

$$\therefore \frac{dy}{dx} = \sec^2 (x - 1)$$

(d) $y = x^2 \tan \frac{1}{2}x + \tan \left(x - \frac{1}{2} \right)$

The first term is a product with $u = x^2$ and $v = \tan \frac{1}{2}x$

$$\therefore \frac{du}{dx} = 2x \text{ and } \frac{dv}{dx} = \frac{1}{2} \sec^2 \frac{1}{2}x$$

$$\begin{aligned}\frac{dy}{dx} &= \left[x^2 \left(\frac{1}{2} \sec^2 \frac{1}{2}x \right) + \tan \frac{1}{2}x \times 2x \right] + \sec^2 \left(x - \frac{1}{2} \right) \\ &= \frac{1}{2}x^2 \sec^2 \frac{1}{2}x + 2x \tan \frac{1}{2}x + \sec^2 \left(x - \frac{1}{2} \right)\end{aligned}$$

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Exercise I, Question 1

Question:

Differentiate

(a) $\cot 4x$

(b) $\sec 5x$

(c) $\operatorname{cosec} 4x$

(d) $\sec^2 3x$

(e) $x \cot 3x$

(f) $\frac{\sec^2 x}{x}$

(g) $\operatorname{cosec}^3 2x$

(h) $\cot^2 (2x - 1)$

Solution:

(a) $y = \cot 4x$

Let $u = 4x$, then $y = \cot u$

$$\frac{du}{dx} = 4 \text{ and } \frac{dy}{du} = -\operatorname{cosec}^2 u$$

$$\therefore \frac{dy}{dx} = -\operatorname{cosec}^2 u \times 4 = -4\operatorname{cosec}^2 4x \quad (\text{From the chain rule})$$

(b) $y = \sec 5x$

Let $u = 5x$, then $y = \sec u$

$$\frac{du}{dx} = 5 \text{ and } \frac{dy}{du} = \sec u \tan u$$

$$\therefore \frac{dy}{dx} = 5 \sec u \tan u = 5 \sec 5x \tan 5x$$

(c) $y = \operatorname{cosec} 4x$

Let $u = 4x$, then $y = \operatorname{cosec} u$

$$\frac{du}{dx} = 4 \text{ and } \frac{dy}{du} = -\operatorname{cosec} u \cot u$$

$$\therefore \frac{dy}{dx} = -4 \operatorname{cosec} u \cot u = -4 \operatorname{cosec} 4x \cot 4x$$

(d) $y = \sec^2 3x$

Let $u = \sec 3x$, then $y = u^2$

$$\frac{du}{dx} = 3 \sec 3x \tan 3x \text{ and } \frac{dy}{du} = 2u$$

From the chain rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 2u \times 3 \sec 3x \tan 3x \\ &= 2 \sec 3x \times 3 \sec 3x \tan 3x \\ &= 6 \sec^2 3x \tan 3x \end{aligned}$$

(e) $y = x \cot 3x$

This is a product so use the product formula.

Let $u = x$ and $v = \cot 3x$

$$\frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = -3 \operatorname{cosec}^2 3x$$

$$\therefore \frac{dy}{dx} = x (-3 \operatorname{cosec}^2 3x) + \cot 3x \times 1 = \cot 3x - 3x \operatorname{cosec}^2 3x$$

(f) $y = \frac{\sec^2 x}{x}$

This is a quotient so use the quotient rule.

Let $u = \sec^2 x$ and $v = x$

$$\frac{du}{dx} = 2 \sec x (\sec x \tan x) \text{ and } \frac{dv}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{x (2 \sec^2 x \tan x) - \sec^2 x \times 1}{x^2} = \frac{\sec^2 x (2x \tan x - 1)}{x^2}$$

(g) $y = \operatorname{cosec}^3 2x$

Let $u = \operatorname{cosec} 2x$, then $y = u^3$

$$\frac{du}{dx} = -2 \operatorname{cosec} 2x \cot 2x \text{ and } \frac{dy}{du} = 3u^2$$

From the chain rule

$$\begin{aligned}\frac{dy}{dx} &= 3u^2 (- 2 \operatorname{cosec} 2x \cot 2x) \\ &= - 6 \operatorname{cosec}^2 2x \operatorname{cosec} 2x \cot 2x \\ &= - 6 \operatorname{cosec}^3 2x \cot 2x\end{aligned}$$

$$(h) y = \cot^2 (2x - 1)$$

Let $u = \cot (2x - 1)$ then $y = u^2$

$$\frac{du}{dx} = - 2 \operatorname{cosec}^2 (2x - 1) \quad \text{and} \quad \frac{dy}{du} = 2u$$

From the chain rule

$$\frac{dy}{dx} = 2u [- 2 \operatorname{cosec}^2 (2x - 1)] = - 4 \cot (2x - 1) \operatorname{cosec}^2 (2x - 1)$$

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Exercise J, Question 1

Question:

Find the function $f'(x)$ where $f(x)$ is

(a) $\sin 3x$

(b) $\cos 4x$

(c) $\tan 5x$

(d) $\sec 7x$

(e) $\operatorname{cosec} 2x$

(f) $\cot 3x$

(g) $\sin \frac{2x}{5}$

(h) $\cos \frac{3x}{7}$

(i) $\tan \frac{2x}{5}$

(j) $\operatorname{cosec} \frac{x}{2}$

(k) $\cot \frac{1}{3}x$

(l) $\sec \frac{3x}{2}$

Solution:

(a) $f(x) = \sin 3x$
 $f'(x) = 3 \cos 3x$

(b) $f(x) = \cos 4x$
 $f'(x) = -4 \sin 4x$

$$(c) f(x) = \tan 5x$$

$$f'(x) = 5 \sec^2 5x$$

$$(d) f(x) = \sec 7x$$

$$f'(x) = 7 \sec 7x \tan 7x$$

$$(e) f(x) = \operatorname{cosec} 2x$$

$$f'(x) = -2 \operatorname{cosec} 2x \cot 2x$$

$$(f) f(x) = \cot 3x$$

$$f'(x) = -3 \operatorname{cosec}^2 3x$$

$$(g) f(x) = \sin \frac{2x}{5}$$

$$f'(x) = \frac{2}{5} \cos \frac{2x}{5}$$

$$(h) f(x) = \cos \frac{3x}{7}$$

$$f'(x) = -\frac{3}{7} \sin \frac{3x}{7}$$

$$(i) f(x) = \tan \frac{2x}{5}$$

$$f'(x) = \frac{2}{5} \sec^2 \frac{2x}{5}$$

$$(j) f(x) = \operatorname{cosec} \frac{x}{2}$$

$$f'(x) = -\frac{1}{2} \operatorname{cosec} \frac{x}{2} \cot \frac{x}{2}$$

$$(k) f(x) = \cot \frac{1}{3}x$$

$$f'(x) = -\frac{1}{3} \operatorname{cosec}^2 \frac{1}{3}x$$

$$(l) f(x) = \sec \frac{3x}{2}$$

$$f'(x) = \frac{3}{2} \sec \frac{3x}{2} \tan \frac{3x}{2}$$

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Exercise J, Question 2

Question:

Find the function $f'(x)$ where $f(x)$ is

(a) $\sin^2 x$

(b) $\cos^3 x$

(c) $\tan^4 x$

(d) $(\sec x)^{\frac{1}{2}}$

(e) $\sqrt{\cot x}$

(f) $\operatorname{cosec}^2 x$

(g) $\sin^3 x$

(h) $\cos^4 x$

(i) $\tan^2 x$

(j) $\sec^3 x$

(k) $\cot^3 x$

(l) $\operatorname{cosec}^4 x$

Solution:

(a) $f(x) = \sin^2 x = (\sin x)^2$

$f'(x) = 2(\sin x)^1 \cos x = 2 \sin x \cos x = \sin 2x$

(b) $f(x) = \cos^3 x = (\cos x)^3$

$f'(x) = 3(\cos x)^2 (-\sin x) = -3 \cos^2 x \sin x$

(c) $f(x) = \tan^4 x = (\tan x)^4$

$f'(x) = 4(\tan x)^3 (\sec^2 x) = 4 \tan^3 x \sec^2 x$

$$(d) f(x) = (\sec x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (\sec x)^{-\frac{1}{2}} \times \sec x \tan x = \frac{1}{2} (\sec x)^{\frac{1}{2}} \tan x$$

$$(e) f(x) = (\cot x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (\cot x)^{-\frac{1}{2}} \times (-\operatorname{cosec}^2 x) = -\frac{1}{2} (\cot x)^{-\frac{1}{2}} \operatorname{cosec}^2 x$$

$$(f) f(x) = \operatorname{cosec}^2 x = (\operatorname{cosec} x)^2$$

$$f'(x) = 2 (\operatorname{cosec} x)^1 (-\operatorname{cosec} x \cot x) = -2 \operatorname{cosec}^2 x \cot x$$

$$(g) f(x) = (\sin x)^3$$

$$f'(x) = 3 (\sin x)^2 \cos x = 3 \sin^2 x \cos x$$

$$(h) f(x) = (\cos x)^4$$

$$f'(x) = 4 (\cos x)^3 (-\sin x) = -4 \cos^3 x \sin x$$

$$(i) f(x) = (\tan x)^2$$

$$f'(x) = 2 \tan x \times \sec^2 x = 2 \tan x \sec^2 x$$

$$(j) f(x) = (\sec x)^3$$

$$f'(x) = 3 (\sec x)^2 \sec x \tan x = 3 \sec^3 x \tan x$$

$$(k) f(x) = (\cot x)^3$$

$$f'(x) = 3 (\cot x)^2 (-\operatorname{cosec}^2 x) = -3 \operatorname{cosec}^2 x \cot^2 x$$

$$(l) f(x) = (\operatorname{cosec} x)^4$$

$$f'(x) = 4 (\operatorname{cosec} x)^3 (-\operatorname{cosec} x \cot x) = -4 \operatorname{cosec}^4 x \cot x$$

Solutionbank

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Exercise J, Question 3

Question:

Find the function $f'(x)$ where $f(x)$ is

(a) $x \cos x$

(b) $x^2 \sec 3x$

(c) $\frac{\tan 2x}{x}$

(d) $\sin^3 x \cos x$

(e) $\frac{x^2}{\tan x}$

(f) $\frac{1 + \sin x}{\cos x}$

(g) $e^{2x} \cos x$

(h) $e^x \sec 3x$

(i) $\frac{\sin 3x}{e^x}$

(j) $e^x \sin^2 x$

(k) $\frac{\ln x}{\tan x}$

(l) $\frac{e^{\sin x}}{\cos x}$

Solution:

(a) $f(x) = x \cos x$

$$f'(x) = x(-\sin x) + \cos x(1) \quad (\text{Product rule})$$

$$= -x \sin x + \cos x$$

$$(b) f(x) = x^2 \sec 3x$$

$$f'(x) = x^2 (3 \sec 3x \tan 3x) + \sec 3x (2x) \quad (\text{Product rule})$$

$$= x \sec 3x (3x \tan 3x + 2)$$

$$(c) f(x) = \frac{\tan 2x}{x}$$

$$f'(x) = \frac{x(2 \sec^2 2x) - \tan 2x(1)}{x^2} \quad (\text{Quotient rule})$$

$$= \frac{2x \sec^2 2x - \tan 2x}{x^2}$$

$$(d) f(x) = \sin^3 x \cos x$$

$$f'(x) = \sin^3 x (-\sin x) + \cos x (3 \sin^2 x \cos x) \quad (\text{Product rule})$$

$$= 3 \sin^2 x \cos^2 x - \sin^4 x$$

$$(e) f(x) = \frac{x^2}{\tan x}$$

$$f'(x) = \frac{\tan x (2x) - x^2 (\sec^2 x)}{\tan^2 x} \quad (\text{Quotient rule})$$

$$= \frac{2x \tan x - x^2 \sec^2 x}{\tan^2 x}$$

$$(f) f(x) = \frac{1 + \sin x}{\cos x}$$

$$f'(x) = \frac{\cos x (\cos x) - (1 + \sin x) (-\sin x)}{\cos^2 x} \quad (\text{Quotient rule})$$

$$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$$

$$= \frac{(\cos^2 x + \sin^2 x) + \sin x}{\cos^2 x} \quad (\text{Use } \cos^2 x + \sin^2 x \equiv 1)$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$(g) f(x) = e^{2x} \cos x$$

$$f'(x) = e^{2x} (-\sin x) + \cos x (2e^{2x}) \quad (\text{Product rule})$$

$$= e^{2x} (2 \cos x - \sin x)$$

$$(h) f(x) = e^x \sec 3x$$

$$f'(x) = e^x (3 \sec 3x \tan 3x) + \sec 3x (e^x) \quad (\text{Product rule})$$

$$= e^x \sec 3x (3 \tan 3x + 1)$$

$$(i) f(x) = \frac{\sin 3x}{e^x}$$

$$f'(x) = \frac{e^x (3 \cos 3x) - \sin 3x (e^x)}{(e^x)^2} \quad (\text{Quotient rule})$$

$$= \frac{e^x (3 \cos 3x - \sin 3x)}{(e^x)^2}$$

$$= \frac{3 \cos 3x - \sin 3x}{e^x}$$

$$(j) f(x) = e^x \sin^2 x$$

$$f'(x) = e^x (2 \sin x \cos x) + \sin^2 x (e^x) \quad (\text{Product rule})$$

$$= e^x \sin x (2 \cos x + \sin x)$$

$$(k) f(x) = \frac{\ln x}{\tan x}$$

$$f'(x) = \frac{\tan x \left(\frac{1}{x}\right) - \ln x (\sec^2 x)}{\tan^2 x} \quad (\text{Quotient rule})$$

$$= \frac{\tan x - x \sec^2 x \ln x}{x \tan^2 x} \quad (\text{Multiply numerator and denominator by } x)$$

$$(l) f(x) = \frac{e^{\sin x}}{\cos x}$$

$$f'(x) = \frac{\cos x [e^{\sin x} \times \cos x] - e^{\sin x} (-\sin x)}{\cos^2 x} \quad (\text{Quotient rule})$$

$$= \frac{e^{\sin x} (\cos^2 x + \sin x)}{\cos^2 x}$$

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Exercise K, Question 1

Question:

Differentiate with respect to x :

(a) $\ln x^2$

(b) $x^2 \sin 3x$

[E]

Solution:

(a) $y = \ln x^2 = 2 \ln x$ (This uses properties of logs)

$$\therefore \frac{dy}{dx} = 2 \times \frac{1}{x} = \frac{2}{x}$$

Alternative method

When $y = \ln f(x)$, $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$ (From the chain rule)

$$\therefore y = \ln x^2 \Rightarrow \frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x}$$

(b) $y = x^2 \sin 3x$

$$\frac{dy}{dx} = x^2 (3 \cos 3x) + \sin 3x (2x) \quad (\text{Product rule})$$

$$= 3x^2 \cos 3x + 2x \sin 3x$$

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Exercise K, Question 2

Question:

Given that

$$f(x) \equiv 3 - \frac{x^2}{4} + \ln \frac{x}{2}, x > 0$$

find $f'(x)$.

[E]

Solution:

$$f(x) = 3 - \frac{x^2}{4} + \ln \frac{x}{2}$$

$$f'(x) = 0 - \frac{2x}{4} + \frac{\frac{1}{2}}{\frac{1}{2}x} = -\frac{x}{2} + \frac{1}{x}$$

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Exercise K, Question 3

Question:

Given that $2y = x - \sin x \cos x$, show that $\frac{dy}{dx} = \sin^2 x$.

[E]

Solution:

$$2y = x - \sin x \cos x$$

$$\therefore y = \frac{x}{2} - \frac{1}{2} \sin x \cos x$$

$$\frac{dy}{dx} = \frac{1}{2} - \left[\frac{1}{2} \sin x (- \sin x) + \cos x \left(\frac{1}{2} \cos x \right) \right] \quad (\text{Product rule})$$

$$= \frac{1}{2} + \frac{1}{2} (\sin^2 x) - \frac{1}{2} \cos^2 x$$

$$= \frac{1}{2} (1 - \cos^2 x) + \frac{1}{2} \sin^2 x$$

$$= \frac{1}{2} \sin^2 x + \frac{1}{2} \sin^2 x \quad (\text{Using } \cos^2 x + \sin^2 x \equiv 1)$$

$$= \sin^2 x$$

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Exercise K, Question 4

Question:

Differentiate, with respect to x ,

(a) $\frac{\sin x}{x}, x > 0$

(b) $\ln \frac{1}{x^2 + 9}$

[E]

Solution:

(a) $y = \frac{\sin x}{x}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{x \cos x - \sin x \times 1}{x^2} && \text{(Using the quotient rule)} \\ &= \frac{x \cos x - \sin x}{x^2}\end{aligned}$$

(b) $y = \ln \frac{1}{x^2 + 9} = \ln 1 - \ln (x^2 + 9)$ (Using laws of logarithms)

$$\Rightarrow y = -\ln (x^2 + 9)$$

$$\therefore \frac{dy}{dx} = \frac{-2x}{x^2 + 9}$$

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Exercise K, Question 5

Question:

Use the derivatives of $\sin x$ and $\cos x$ to prove that the derivative of $\tan x$ is $\sec^2 x$.

[E]

Solution:

$$\text{Let } y = \tan x = \frac{\sin x}{\cos x}$$

Use the quotient rule with $u = \sin x$ and $v = \cos x$.

$$\text{Then } \frac{du}{dx} = \cos x \text{ and } \frac{dv}{dx} = -\sin x$$

As

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{\cos x \cos x - \sin x (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad (\text{Use } \cos^2 x + \sin^2 x \equiv 1)$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x \quad \left(\text{As } \sec x = \frac{1}{\cos x} \right)$$

So derivative of $\tan x$ is $\sec^2 x$.

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Exercise K, Question 6

Question:

$$f(x) = \frac{x}{x^2 + 2}, x \in \mathbb{R}$$

Find the set of values of x for which $f'(x) < 0$.

[E]

Solution:

$$f(x) = \frac{x}{x^2 + 2}$$

$$f'(x) = \frac{(x^2 + 2)(1) - x(2x)}{(x^2 + 2)^2} = \frac{x^2 + 2 - 2x^2}{(x^2 + 2)^2} = \frac{2 - x^2}{(x^2 + 2)^2}$$

$$\text{When } f'(x) < 0, \frac{2 - x^2}{(x^2 + 2)^2} < 0$$

$$\therefore 2 - x^2 < 0 \quad [\text{As } (x^2 + 2)^2 > 0]$$

$$\therefore x^2 > 2$$

$$\therefore x < -\sqrt{2}, x > \sqrt{2}$$

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Exercise K, Question 7

Question:

The function f is defined for positive real values of x by

$$f(x) = 12 \ln x - x^{\frac{3}{2}}$$

Write down the set of values of x for which $f(x)$ is an increasing function of x

[E].

Solution:

$$f(x) = 12 \ln x - x^{\frac{3}{2}}$$

$$f'(x) = 12 \times \frac{1}{x} - \frac{3}{2}x^{\frac{1}{2}}$$

When $f(x)$ is an increasing function, $f'(x) > 0$.

$$\therefore \frac{12}{x} - \frac{3}{2}x^{\frac{1}{2}} > 0$$

$$\therefore \frac{12}{x} > \frac{3}{2}x^{\frac{1}{2}}$$

As $x > 0$, multiply both sides by x to give

$$12 > \frac{3}{2}x^{1\frac{1}{2}}$$

$$\therefore x^{\frac{3}{2}} < 8$$

$$\therefore x < 8^{\frac{2}{3}}$$

i.e. $x < 4$

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Exercise K, Question 8

Question:

Given that $y = \cos 2x + \sin x$, $0 < x < 2\pi$, and x is in radians, find, to 2 decimal places, the values of x for which $\frac{dy}{dx} = 0$.

[E]

Solution:

$$y = \cos 2x + \sin x$$

$$\frac{dy}{dx} = -2 \sin 2x + \cos x$$

$$\text{Put } \frac{dy}{dx} = 0$$

$$\therefore \cos x - 2 \sin 2x = 0$$

$$\therefore \cos x - 4 \sin x \cos x = 0 \quad (\text{Using double angle formula})$$

$$\text{i.e. } \cos x (1 - 4 \sin x) = 0.$$

$$\therefore \cos x = 0 \text{ or } \sin x = \frac{1}{4}$$

$$\therefore x = 1.57 \text{ or } 4.71 \text{ or } 0.25 \text{ or } 2.89$$

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Exercise K, Question 9

Question:

The maximum point on the curve with equation $y = x\sqrt{\sin x}$, $0 < x < \pi$, is the point A. Show that the x -coordinate of point A satisfies the equation $2 \tan x + x = 0$.

[E]

Solution:

$$y = x\sqrt{\sin x}$$

$$\frac{dy}{dx} = x \left[\frac{1}{2} (\sin x)^{-\frac{1}{2}} \times \cos x \right] + (\sin x)^{\frac{1}{2}} \times 1$$

At the maximum point, $\frac{dy}{dx} = 0$.

$$\therefore \frac{x}{2} (\sin x)^{-\frac{1}{2}} \cos x + (\sin x)^{\frac{1}{2}} = 0$$

Multiply equation by $2 (\sin x)^{\frac{1}{2}}$:

$$x \cos x + 2 \sin x = 0$$

Divide equation by $\cos x$:

$$x + 2 \tan x = 0$$

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Exercise K, Question 10

Question:

$$f(x) = e^{0.5x} - x^2, x \in \mathbb{R}$$

(a) Find $f'(x)$.

(b) By evaluating $f'(6)$ and $f'(7)$, show that the curve with equation $y = f(x)$ has a stationary point at $x = p$, where $6 < p < 7$.

[E]

Solution:

$$\begin{aligned} \text{(a) } f(x) &= e^{0.5x} - x^2 \\ \therefore f'(x) &= 0.5e^{0.5x} - 2x \end{aligned}$$

$$\begin{aligned} \text{(b) } f'(6) &= -1.96 < 0 \\ f'(7) &= 2.56 > 0 \end{aligned}$$

As the sign changes and the function is continuous, $f'(x) = 0$ has a root p where $6 < p < 7$.

So $y = f(x)$ has a stationary point at $x = p$.

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Exercise K, Question 11

Question:

$$f(x) \equiv e^{2x} \sin 2x, 0 \leq x \leq \pi$$

(a) Use calculus to find the coordinates of the turning points on the graph of $y = f(x)$.

(b) Show that $f''(x) = 8e^{2x} \cos 2x$.

(c) Hence, or otherwise, determine which turning point is a maximum and which is a minimum.

[E]

Solution:

$$(a) f(x) = e^{2x} \sin 2x$$

$$\therefore f'(x) = e^{2x} (2 \cos 2x) + \sin 2x (2e^{2x})$$

$$\text{When } f'(x) = 0, 2e^{2x} (\cos 2x + \sin 2x) = 0$$

$$\therefore \sin 2x = -\cos 2x$$

Divide both sides by $\cos 2x$:

$$\tan 2x = -1$$

$$\therefore 2x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$$

$$\therefore x = \frac{3\pi}{8} \text{ or } \frac{7\pi}{8} \quad (\text{As } 0 \leq x \leq \pi)$$

$$\text{As } y = f(x), \text{ when } x = \frac{3\pi}{8}, y = \frac{1}{\sqrt{2}} e^{\frac{3\pi}{4}}$$

$$\text{and when } x = \frac{7\pi}{8}, y = -\frac{1}{\sqrt{2}} e^{\frac{7\pi}{4}}$$

So $\left(\frac{3\pi}{8}, \frac{1}{\sqrt{2}} e^{\frac{3\pi}{4}} \right)$ and $\left(\frac{7\pi}{8}, -\frac{1}{\sqrt{2}} e^{\frac{7\pi}{4}} \right)$ are stationary values.

$$(b) \text{ As } f'(x) = 2e^{2x} (\cos 2x + \sin 2x)$$

$$\begin{aligned} f''(x) &= 2e^{2x} (-2 \sin 2x + 2 \cos 2x) + 4e^{2x} (\cos 2x + \sin 2x) \\ &= 8e^{2x} \cos 2x \end{aligned}$$

$$(c) f'' \left(\frac{3\pi}{8} \right) = 8e^{\frac{3\pi}{4}} \cos \frac{3\pi}{4} = -4\sqrt{2}e^{\frac{3\pi}{4}} < 0$$

$$\therefore \text{maximum at } \left(\frac{3\pi}{8}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}} \right)$$

$$f'' \left(\frac{7\pi}{8} \right) = 8e^{\frac{7\pi}{4}} \cos \frac{7\pi}{4} = +4\sqrt{2}e^{\frac{7\pi}{4}} > 0$$

$$\therefore \text{minimum at } \left(\frac{7\pi}{8}, \frac{-1}{\sqrt{2}}e^{\frac{7\pi}{4}} \right).$$

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Exercise K, Question 12

Question:

The curve **C** has equation $y = 2e^x + 3x^2 + 2$. The point **A** with coordinates (0, 4) lies on **C**. Find the equation of the tangent to **C** at **A**.

[E]

Solution:

$$y = 2e^x + 3x^2 + 2$$

$$\frac{dy}{dx} = 2e^x + 6x$$

At the point **A** (0, 4), $x = 0$, so the gradient of the tangent at **A** is $2e^0 + 6 \times 0 = 2$.

\therefore the equation of the tangent at **A** is

$$y - 4 = 2 (x - 0)$$

$$\text{i.e. } y = 2x + 4$$

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Exercise K, Question 13

Question:

The curve **C** has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, x > 0$$

The point **P** is a stationary point on **C**.

(a) Calculate the x -coordinate of **P**.

The point **Q** on **C** has x -coordinate 1.

(b) Find an equation for the normal to **C** at **Q**.

[E]

Solution:

$$(a) f(x) = 3 \ln x + \frac{1}{x}$$

$$f'(x) = \frac{3}{x} - \frac{1}{x^2}$$

$$\text{When } f'(x) = 0, \frac{3}{x} - \frac{1}{x^2} = 0$$

Multiply equation by x^2 :

$$3x - 1 = 0$$

$$3x = 1$$

$$x = \frac{1}{3}$$

So the x -coordinate of the stationary point **P** is $\frac{1}{3}$.

(b) At the point **Q**, $x = 1$. $\therefore y = f(1) = 1$

The gradient of the curve at point **Q** is $f'(1) = 3 - 1 = 2$.

So the gradient of the normal to the curve at **Q** is $-\frac{1}{2}$.

\therefore the equation of the normal is $y - 1 = -\frac{1}{2}(x - 1)$

$$\text{i.e. } y = -\frac{1}{2}x + 1\frac{1}{2}$$

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Exercise K, Question 14

Question:

Differentiate $e^{2x} \cos x$ with respect to x .

The curve **C** has equation $y = e^{2x} \cos x$.

- (a) Show that the turning points on **C** occur when $\tan x = 2$.
- (b) Find an equation of the tangent to **C** at the point where $x = 0$.

[E]

Solution:

Let $f(x) = e^{2x} \cos x$

Then $f'(x) = e^{2x}(-\sin x) + \cos x(2e^{2x})$

(a) The turning points occur when $f'(x) = 0$.

$$\therefore e^{2x}(2\cos x - \sin x) = 0$$

$$\therefore \sin x = 2\cos x$$

Divide both sides by $\cos x$:

$$\tan x = 2$$

(b) When $x = 0$, $y = f(0) = e^0 \cos 0 = 1$

The gradient of the curve at $(0, 1)$ is $f'(0)$.

$$f'(0) = 0 + 2 = 2$$

This is the gradient of the tangent at $(0, 1)$ also.

So the equation of the tangent at $(0, 1)$ is

$$y - 1 = 2(x - 0)$$

$$\therefore y = 2x + 1$$

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Exercise K, Question 15

Question:

Given that $x = y^2 \ln y$, $y > 0$,

(a) find $\frac{dx}{dy}$

(b) use your answer to part (a) to find in terms of e , the value of $\frac{dy}{dx}$ at $y = e$.

[E]

Solution:

(a) $x = y^2 \ln y$

Use the product rule to give

$$\frac{dx}{dy} = y^2 \left(\frac{1}{y} \right) + \ln y \times 2y = y + 2y \ln y$$

(b) $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{y + 2y \ln y}$

when $y = e$,

$$\frac{dy}{dx} = \frac{1}{e + 2e \ln e} = \frac{1}{3e}$$

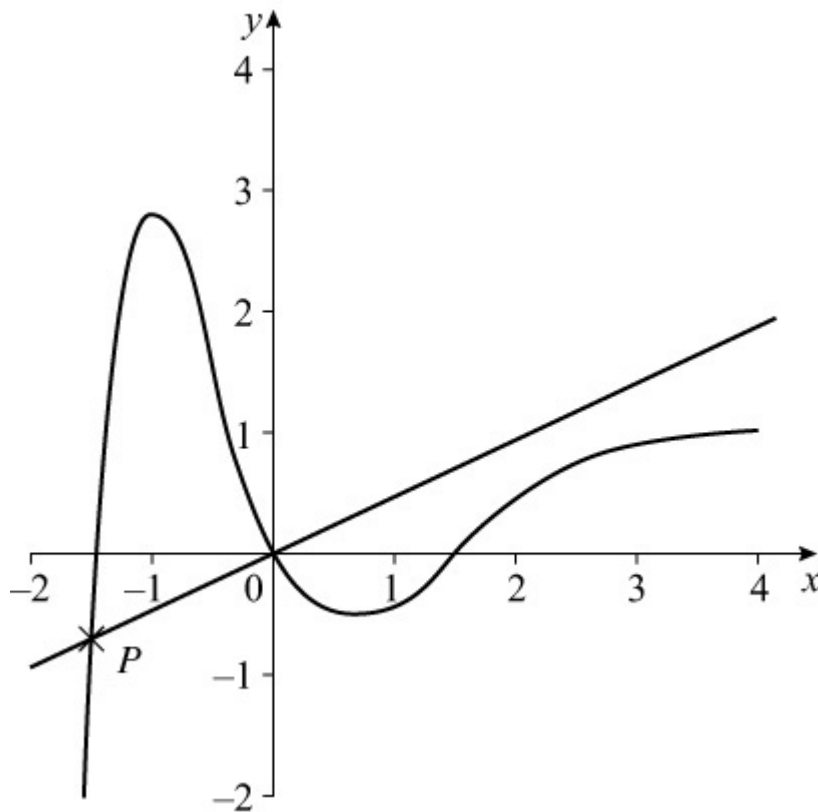
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Exercise K, Question 16

Question:



The figure shows part of the curve **C** with equation $y = f(x)$, where $f(x) = (x^3 - 2x)e^{-x}$

(a) Find $f'(x)$.

The normal to **C** at the origin **O** intersects **C** at a point **P**, as shown in the figure.

(b) Show that the x -coordinate of **P** is the solution of the equation

$$2x^2 = e^x + 4.$$

[E]

Solution:

$$(a) f(x) = (x^3 - 2x)e^{-x}$$

$$\begin{aligned} \therefore f'(x) &= (x^3 - 2x)(-e^{-x}) + e^{-x}(3x^2 - 2) = e^{-x} \\ &(-x^3 + 3x^2 + 2x - 2) \end{aligned}$$

(b) The gradient of the curve at $(0, 0)$ is $f'(0) = -2$

The normal at the origin has gradient $\frac{1}{2}$.

So the equation of the normal at the origin is $y = \frac{1}{2}x$

This normal meets the curve $y = (x^3 - 2x)e^{-x}$ at the point P .

\therefore the x -coordinate of P satisfies

$$\frac{1}{2}x = (x^3 - 2x)e^{-x}$$

Multiply both sides by $2e^x$:

$$xe^x = 2x^3 - 4x$$

Divide both sides by x and rearrange to give

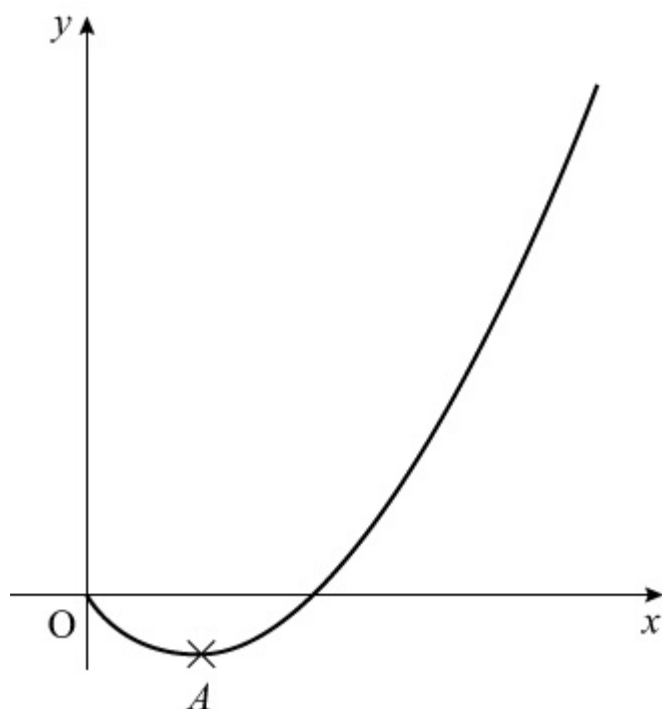
$$2x^2 = e^x + 4$$

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Exercise K, Question 17

Question:



The diagram shows part of the curve with equation $y = f(x)$ where $f(x) = x(1+x)\ln x$ $\{x > 0\}$

The point A is the minimum point of the curve.

(a) Find $f'(x)$.

(b) Hence show that the x -coordinate of A is the solution of the equation $x = g(x)$, where

$$g(x) = e^{-\frac{1+x}{1+2x}}$$

[E]

Solution:

$$(a) f(x) = x(1+x)\ln x = (x+x^2)\ln x$$

$$f'(x) = (x+x^2) \times \frac{1}{x} + \ln x(1+2x) = (1+x) + (1+2x)\ln x$$

(b) A is the minimum point on the curve $y = f(x)$

$$\therefore f'(x) = 0 \text{ at point A.}$$

$$\therefore (1+x) + (1+2x) \ln x = 0$$

$$\therefore (1+2x) \ln x = -(1+x)$$

$$\therefore \ln x = -\frac{1+x}{1+2x}$$

$$\therefore x = e^{-\frac{1+x}{1+2x}}$$

i.e. the x -coordinate of A is a solution of $x = g(x)$ where $g(x) = e^{-\frac{1+x}{1+2x}}$.