RECOGNISING ACHIEVEMENT

GCE

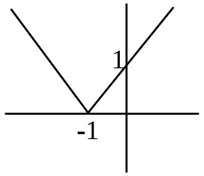
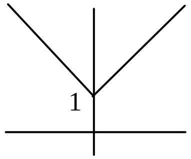
Mathematics (MEI)

Advanced GCE **4753**

Methods for Advanced Mathematics (C3)

Mark Scheme for June 2010

Section A

1 $\int_0^{\pi/6} \cos 3x \,\mathrm{d}x = \left[\frac{1}{3}\sin 3x\right]_0^{\pi/6}$ $= \frac{1}{3}\sin\frac{\pi}{2} - 0$ $= 1/3$	M1 B1 A1cao [3]	$k \sin 3x$, $k > 0$, $k \neq 3$ $k = (\pm)1/3$ 0.33 or better	or M1 for $u = 3x \Rightarrow \int \frac{1}{3}\cos u \,\mathrm{d}u$ condone 90° in limit or M1 for $\left[\frac{1}{3}\sin u\right]$ so: sin 3x : M1B0, –sin 3x: M0B0, ±3sin 3x: M0B0, –1/3 sin 3x: M0B1
2 $\mathrm{fg}(x) = \lvert x+1 \rvert$ $\mathrm{g\,f}(x) = \lvert x \rvert + 1$ 	B1 B1 B1 B1 [4]	soi from correctly-shaped graphs (i.e. without intercepts) graph of $\lvert x+1 \rvert$ only graph of $\lvert x \rvert + 1$	but must indicate which is which bod gf if negative x values are missing 'V' shape with (–1, 0) and (0, 1) labelled 'V' shape with (0, 1) labelled (0, 1)
3(i) $y = (1+3x^2)^{1/2}$ $\Rightarrow$ $dy/dx = \frac{1}{2}(1+3x^2)^{-1/2}.6x$ $= 3x(1 + 3x^2)^{-1/2}$	M1 B1 A1 [3]	chain rule ½ $u^{-1/2}$ o.e., but must be '3'	 can isw here
(ii) $y = x(1+3x^2)^{1/2}$ $\Rightarrow$ $dy/dx = x.\frac{3x}{\sqrt{1+3x^2}} + 1.(1+3x^2)^{1/2}$ $= \frac{3x^2+1+3x^2}{\sqrt{1+3x^2}}$ $= \frac{1+6x^2}{\sqrt{1+3x^2}}$ *	M1 A1ft M1 E1 [4]	product rule ft their dy/dx from (i) common denominator or factoring $(1+3x^2)^{-1/2}$ www	 must show this step for M1 E1

4 $p = 100/x = 100\,x^{-1}$ $\Rightarrow$ $dp/dx = -100x^{-2} = -100/x^2$ $dp/dt = dp/dx \times dx/dt$ $dx/dt = 10$ When $x = 50$, $dp/dx = (-100/50^2)$ $\Rightarrow dp/dt = 10 \times -0.04 = -0.4$	M1 A1 M1 B1 M1dep A1cao [6]	attempt to differentiate $-100x^{-2}$ o.e. o.e. soi soi substituting $x = 50$ into their dp/dx dep 2nd M1 o.e. e.g. decreasing at 0.4	condone poor notation if chain rule correct *or* $x = 50 + 10\,t$ B1 $\Rightarrow P = 100/x = 100/(50 + 10\,t)$ $\Rightarrow dP/dt = -100(50 + 10\,t)^{-2} \times 10 = -1000/(50 + 10t)^{-2}$ M1 A1 When $t = 0$, $dP/dt = -1000/50^2 = -0.4$ A1
5 $y^3 = xy - x^2$ $\Rightarrow$ $3y^2\,dy/dx = x\,dy/dx + y - 2x$ $\Rightarrow$ $3y^2\,dy/dx - x\,dy/dx = y - 2x$ $\Rightarrow$ $(3y^2 - x)\,dy/dx = y - 2x$ $\Rightarrow$ $dy/dx = (y - 2x)/(3y^2 - x)$ * TP when $dy/dx = 0 \Rightarrow y - 2x = 0$ $\Rightarrow$ $y = 2x$ $\Rightarrow$ $(2x)^3 = x.2x - x^2$ $\Rightarrow$ $8x^3 = x^2$ $\Rightarrow$ $x = 1/8$ *(or 0)	B1 B1 M1 E1 M1 M1 E1 [7]	$3y^2dy/dx$ $x\,dy/dx + y - 2x$ collecting terms in dy/dx only or $x = 1/8$ and $dy/dx = 0 \Rightarrow y = ¼$ or $(1/4)^3 = (1/8)(1/4) - (1/8)^2$ or verifying e.g. $1/64 = 1/64$	must show '$x\,dy/dx + y$' on one side or $x = 1/8 \Rightarrow y^3 = (1/8)y - 1/64$ M1 verifying that $y = ¼$ is a solution (must show evidence*) M1 $\Rightarrow dy/dx = (¼ - 2(1/8))/(\ldots) = 0$ E1 *just stating that $y = ¼$ is M1 M0 E0
6 $f(x) = 1 + 2\sin 3x = y$ $x \leftrightarrow y$ $x = 1 + 2\sin 3y$ $\Rightarrow$ $\sin 3y = (x - 1)/2$ $\Rightarrow$ $3y = \arcsin[(x - 1)/2]$ $\Rightarrow y = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ so $f^{-1}(x) = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ Range of f is -1 to 3 $\Rightarrow$ $-1 \le x \le 3$	M1 A1 A1 A1 M1 A1 [6]	attempt to invert must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or $-1 \le (x - 1)/2 \le 1$ must be 'x', not y or $f(x)$	at least one step attempted, or reasonable attempt at flow chart inversion (or any other variable provided same used on each side) condone <'s for M1 allow unsupported correct answers; -1 to 3 is M1 A0
7 (*A*) True , (*B*) True , (C) False Counterexample, e.g. $\sqrt{2} + (-\sqrt{2}) = 0$	B2,1,0 B1 [3]		

8(i) When $x = 1$, $y = 3\ln 1 + 1 - 1^2$ $= 0$	E1 [1]		
(ii) $\frac{dy}{dx} = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ At R, $\frac{dy}{dx} = 0 = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ $3 + x - 2x^2 = 0$ $\Rightarrow$ $(3 - 2x)(1 + x) = 0$ $\Rightarrow$ $x = 1.5$, (or -1) $\Rightarrow$ $y = 3\ln 1.5 + 1.5 - 1.5^2$ $= 0.466$ (3 s.f.) $\frac{d^2y}{dx^2} = -\frac{3}{x^2} - 2$ When $x = 1.5$, d^2y/dx^2 ($= -10/3$) $< 0 \Rightarrow$ max	M1 A1cao M1 M1 A1 M1 A1cao B1ft E1 [9]	d/dx (ln x) = 1/x re-arranging into a quadratic = 0 factorising or formula or completing square substituting their x ft their dy/dx on equivalent work www – don't need to calculate 10/3	 SC1 for $x = 1.5$ unsupported, SC3 if verified but condone rounding errors on 0.466
(iii) Let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1$, $v = x$ $\Rightarrow$ $\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}dx$ $= x\ln x - \int 1.dx$ $= x\ln x - x + c$ $\Rightarrow$ $A = \int_1^{2.05}(3\ln x + x - x^2)dx$ $= \left[3x\ln x - 3x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_1^{2.05}$ $= -2.5057 + 2.833..$ $= 0.33$ (2 s.f.)	 M1 A1 A1 B1 B1ft M1dep A1 cao [7]	 parts condone no c correct integral and limits (soi) $\left[3\times \text{their } 'x\ln x - x' + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]$ substituting correct limits dep 1st B1	 allow correct result to be quoted (SC3)

9(i) (0, ½)	B1 [1]	allow y = ½ , but not (x =) ½ or (½ ,0) nor P = 1/2	
(ii) $\frac{dy}{dx}=\frac{(1+e^{2x})2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ $=\frac{2e^{2x}}{(1+e^{2x})^2}$ When x = 0, dy/dx = $2e^0/(1+e^0)^2$ = ½	M1 A1 A1 B1ft [4]	Quotient or product rule correct expression – condone missing bracket cao – mark final answer follow through their derivative	product rule: $\frac{dy}{dx}=e^{2x}.2e^{2x}(-1)(1+e^{2x})^{-2}+2e^{2x}(1+e^{2x})^{-1}$ $-\frac{2e^{2x}}{(1+e^{2x})^2}$ from (udv – vdu)/v^2 SC1
(iii) $A=\int_0^1\frac{e^{2x}}{1+e^{2x}}dx$ $=\left[\frac{1}{2}\ln(1+e^{2x})\right]_0^1$ *or* let u = 1 + e^{2x}, du/dx = 2 e^{2x} $\Rightarrow$ $A=\int_2^{1+e^2}\frac{1/2}{u}du=\left[\frac{1}{2}\ln u\right]_2^{1+e^2}$ $=\frac{1}{2}\ln(1+e^2)-\frac{1}{2}\ln 2$ $=\frac{1}{2}\ln\left[\frac{1+e^2}{2}\right]$ *	B1 M1 A1 M1 A1 M1 E1 [5]	correct integral and limits (soi) k ln(1 + e^{2x}) k = ½ or v = e^{2x}, dv/dx = $2e^{2x}$ o.e. [½ ln u] or [½ ln (v + 1)] substituting correct limits www	condone no dx allow missing dx's or incompatible limits, but penalise missing brackets
(iv) $g(-x)=\frac{1}{2}\left[\frac{e^{-x}-e^x}{e^{-x}+e^x}\right]=-\frac{1}{2}\left[\frac{e^x-e^{-x}}{e^x+e^{-x}}\right]=-g(x)$ Rotational symmetry of order 2 about O	M1 E1 B1 [3]	substituting –x for x in g(x) completion www – taking out –ve must be clear must have 'rotational' 'about O', 'order 2' (oe)	not g(–x) ≠ g(x). Condone use of f for g.
(v)(A) $g(x)+\frac{1}{2}=\frac{1}{2}.\frac{e^x-e^{-x}}{e^x+e^{-x}}+\frac{1}{2}=\frac{1}{2}.(\frac{e^x-e^{-x}+e^x+e^{-x}}{e^x+e^{-x}})$ $=\frac{1}{2}.(\frac{2e^x}{e^x+e^{-x}})$ $=\frac{e^x.e^x}{e^x(e^x+e^{-x})}=\frac{e^{2x}}{e^{2x}+1}=f(x)$ (B) Translation $\begin{pmatrix}0\\1/2\end{pmatrix}$ (C) Rotational symmetry [of order 2]about P	M1 A1 E1 M1 A1 B1 [6]	combining fractions (correctly) translation in y direction up ½ unit dep 'translation' used o.e. condone omission of 180°/order 2	 allow 'shift', 'move' in correct direction for M1. $\begin{pmatrix}0\\1/2\end{pmatrix}$ alone is SC1.