

Worked Solutions

Edexcel C3 Paper K

$$1. \frac{(2x-3)(x+2)}{(2x-3)(2x+3)} \times \frac{(x-1)}{(x+2)^2} = \frac{x-1}{(2x+3)(x+2)} \quad (5)$$

$$2. (a) fg(x) = \frac{2}{x-3} + 2 = \frac{2x-4}{x-3}, \text{ domain } x \in \mathbb{R}, x \neq 3$$

$$gf(x) = \frac{2}{x+2-3} = \frac{2}{x-1}, \text{ domain } x \in \mathbb{R}, x \neq 1 \quad (4)$$

$$(b) \frac{2x-4}{x-3} = \frac{2}{x-1} \Rightarrow 2x^2 - 6x + 4 = 2x - 6 \quad x^2 - 4x + 5 = 0$$

$$"b^2 - 4ac < 0" \therefore \text{no real solution.} \quad (3)$$

$$3. (a) \frac{(1-x^2) - x(-2x)}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2} \quad (3)$$

$$(b) x^2 \cdot \frac{1}{x} + 2x \cdot \ln x = x(1 + 2 \ln x) \quad (3)$$

$$(c) \cos x \cdot e^{\sin x} \quad (3)$$

$$4. (a) \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$$

$$\cos^2 \theta \geq 0 \quad \therefore \text{squared term}$$

$$\sin \theta > 0 \text{ because } 0 < \theta < 180^\circ \quad (4)$$

$$(b) 1 + \tan^2 x - 4 \tan x + 2 = 0$$

$$\tan^2 x - 4 \tan x + 3 = 0$$

$$(\tan x - 3)(\tan x - 1) = 0$$

$$\therefore \begin{cases} \tan x = 3 \Rightarrow x = 71.6^\circ (1 \text{ d.p.}), 251.6^\circ (1 \text{ d.p.}) \\ \tan x = 1 \Rightarrow x = 45^\circ, 225^\circ \end{cases} \quad (5)$$

$$5. (a) \frac{dy}{dx} = x^3 + 3x^2 - 3$$

$$\text{Let } f(x) = x^3 + 3x^2 - 3$$

$$\left. \begin{matrix} f(0) = -3 \\ f(1) = 1 \end{matrix} \right\} \text{sign change } \therefore \alpha \text{ lies in } [0, 1] \quad (4)$$

$$(b) x^3 + 3x^2 - 3 = 0$$

$$x^3 + 3x^2 = 3$$

$$x^2(x+3) = 3$$

$$x^2 = \frac{3}{x+3}$$

$$x = \sqrt{\frac{3}{x+3}} \quad (2)$$

$$(c) x_1 = 0.8660, x_2 = 0.8809, x_3 = 0.8792, x_4 = 0.8794 \quad (2)$$

$$(d) \alpha = 0.879 \quad (1)$$

$$6. (a) \text{asymptote to } \ln(x+2) \text{ is } x = -2, \text{ asymptote to } \ln 3x \text{ is } x = 0. \quad (2)$$

$$(b) B(-1, 0); C\left(\frac{1}{3}, 0\right) \quad (2)$$

$$(c) 3x = x + 2, A \text{ is } (1, \ln 3) \quad (1)$$

$$(d) \frac{dy}{dx} = \frac{1}{x} \text{ and } \frac{1}{x+2} \text{ gradients of curves at } A : 1 \text{ \& } \frac{1}{3}$$

$$\tan \theta = \frac{1 - \frac{1}{3}}{1 + 1 \times \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}}$$

$$= \frac{1}{2} \quad (6)$$

7. (a) $2A = P + Q, 2B = P - Q$

$$\sin(A + B) - \sin(A - B)$$

$$= \sin A \cos B + \cos A \sin B - \sin A \cos B + \cos A \sin B$$

$$= 2 \cos A \sin B$$

$$= 2 \cos \left(\frac{P + Q}{2} \right) \sin \left(\frac{P - Q}{2} \right)$$

(5)

(b) $\sin 4\theta - \sin 2\theta + \cos 3\theta = 0$

$$2 \cos 3\theta \sin \theta + \cos 3\theta = 0$$

$$\cos 3\theta(2 \sin \theta + 1) = 0$$

$$\cos 3\theta = 0 \Rightarrow 3\theta = 90, 270, 450, 630, 810, 990$$

$$\theta = 30, 90, 150, 210, 270, 330$$

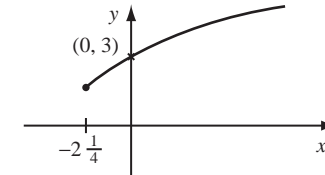
$$\sin \theta = -\frac{1}{2} \Rightarrow \theta = 210, 330$$

(6)

8. (a) $f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} \quad \therefore f(x) \geq -2\frac{1}{4}$ (1)

(b) $f^{-1} : (\text{domain}) x \geq -2\frac{1}{4}, (\text{range}) f^{-1} \geq 1\frac{1}{2}$ (2)

(c)



(4)

(d) $\text{gf: } x \mapsto |x^2 - 3x - 4|, x \in \mathbb{R}, x \geq \frac{3}{2}$ (2)

(e) either $x^2 - 3x - 4 = 6$

$$x^2 - 3x - 10 = 0 \quad (x - 5)(x + 2) = 0 \quad x = 5, x = -2$$

or $x^2 - 3x - 4 = -6 \quad x^2 - 3x + 2 = 0$

$$(x - 2)(x - 1) = 0 \quad x = 2, x = 1$$

solutions are $x = 2, 5$ (since $x \geq \frac{3}{2}$) (5)