

Core Mathematics C3 For Edexcel Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (a) Simplify $\frac{x^2 + 7x + 12}{x^2 + 3x}$. (3)

(b) Find the value of x for which

$$\log_3(x^2 + 7x + 12) - \log_3(x^2 + 3x) = 2. \quad (4)$$

2. The functions f and g are defined by

$$f: x \mapsto \frac{1}{x-1}, \quad x \in \mathbb{R}, \quad x > 1$$

$$g: x \mapsto 3^x + k, \quad x \in \mathbb{R} \text{ and } k \text{ is a constant.}$$

(a) Find the range of f . (2)

(b) Given that $gf\left(\frac{3}{2}\right) = 14$, find the value of k . (4)

3. (a) Prove that

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1. \quad (4)$$

(b) Using part (a) solve the equation

$$8 \cos^2 x - 8 \cos^4 x = 1, \quad \text{for } 0 < x < 180^\circ. \quad (3)$$

4. (a) Differentiate with respect to x

(i) $x^3 e^{3x}$ (3)

(ii) $\frac{2x}{\cos x}$ (3)

(b) Given that $x = \cos y^2$,

find $\frac{dy}{dx}$ in terms of y . (4)

5. (a) Sketch on the same set of axes the graphs of

$$y = \ln(x - 3) \quad x \in \mathbb{R}, \quad x > 3.$$

$$\text{and } y = \frac{2}{x}, \quad x \neq 0 \quad (3)$$

Given that $f(x) = \ln(x - 3) - \frac{2}{x}$, $x > 3$,

- (b) Explain how your graphs show that the equation $f(x) = 0$ has only one solution. (1)

- (c) Show that the solution of $f(x) = 0$ lies between $x = 4$ and $x = 5$. (2)

- (d) The iterative formula

$$x_{n+1} = e^{\frac{2}{x_n}} + 3$$

is used to solve the equation $f(x) = 0$.

Taking $x_0 = 5$, find the values of x_1, x_2, x_3 and x_4 and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 2 decimal places. (3)

6. (a) Given that $\cot x = \frac{4}{3}$ and that x is a reflex angle, find the exact value of $\operatorname{cosec} x$. (4)

- (b) Find the exact value of $\cos 2x$. (4)

- (c) Find the exact value of $\tan 2x$. (3)
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7. (a) Sketch on the same pair of axes the graphs of

$$y = |x^2 - a^2| \quad \text{and} \quad y = x + a; \quad a > 1$$

Mark clearly on your graphs the coordinates of the points of intersection with the x and y axes. (4)

- (b) The curve and straight line intersect at three points.

Show that one of these points has coordinates $(-a, 0)$. (1)

- (c) Show that the x coordinate of one of the other points satisfies the equation

$$x^2 - x - a^2 - a = 0 \quad (2)$$

- (d) Write down an equation which is satisfied by the third point of intersection. (2)

- (e) Given that $a = 3$, find the coordinates of the points of intersection of the curve and the straight line. (2)
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8. Show that $x = e^{-t}(\cos t + \sin t)$ satisfies the equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0. \quad (6)$$

(a) Find the value of x , when $t = 0$, and show that $\frac{dx}{dt} = 0$, when $t = 0$. (2)

(b) Find the next positive value for t for which $\frac{dx}{dt} = 0$. (2)

(c) Hence find a maximum and minimum value for x . (2)

(d) Justify your answers to (c) by evaluating $\frac{d^2x}{dt^2}$. (2)

END

TOTAL 75 MARKS