

5.

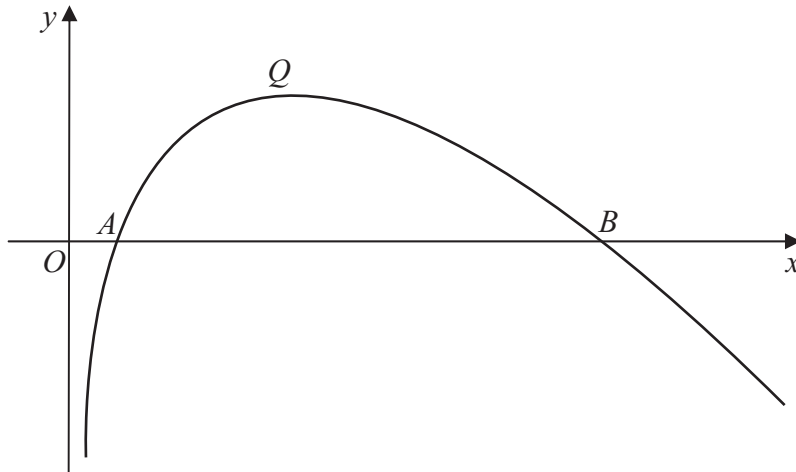


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of
$$x = \frac{8}{1 + \ln x}$$
 (3)

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)



6. The function f is defined by

$$f: x \mapsto \frac{3 - 2x}{x - 5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

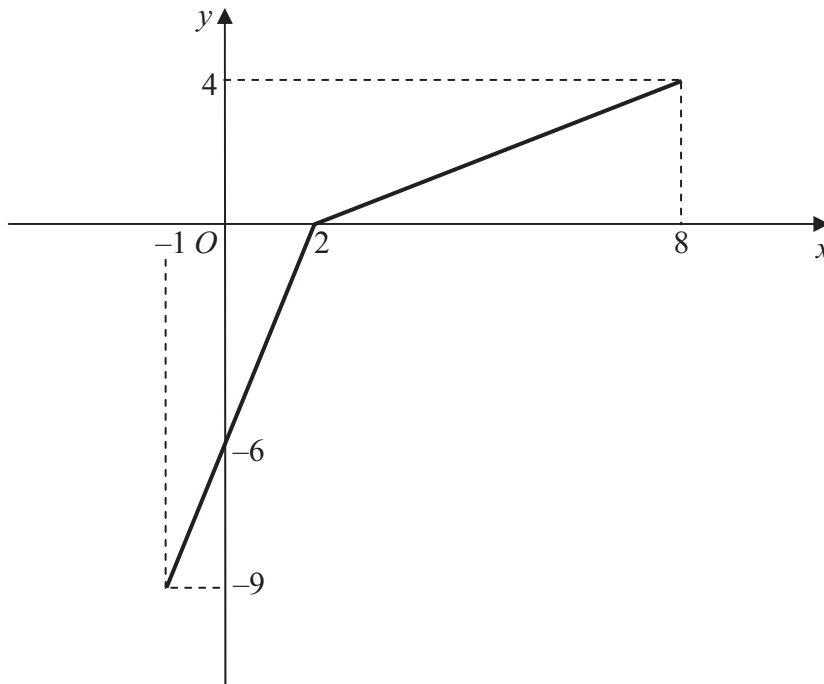


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|,$

(ii) $y = g^{-1}(x).$

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)



7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

Handwriting lines for the answer.



8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y .

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x .

(4)



