General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MPC3

Unit Pure Core 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The curve $y = 3^x$ intersects the curve $y = 10 - x^3$ at the point where $x = \alpha$.

(a) Show that α lies between 1 and 2. *(2 marks)*

(b) (i) Show that the equation $3^x = 10 - x^3$ can be rearranged into the form $x = \sqrt[3]{10 - 3^x}$. *(1 mark)*

(ii) Use the iteration $x_{n+1} = \sqrt[3]{10 - 3^{x_n}}$ with $x_1 = 1$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

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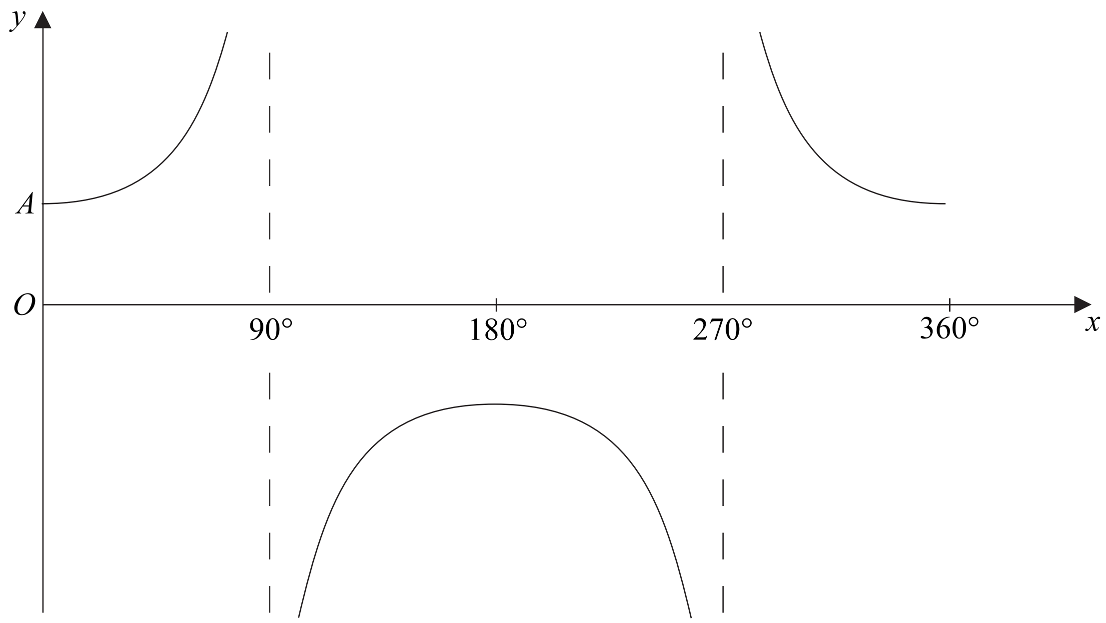
2 (a) The diagram shows the graph of $y = \sec x$ for $0° \leqslant x \leqslant 360°$.

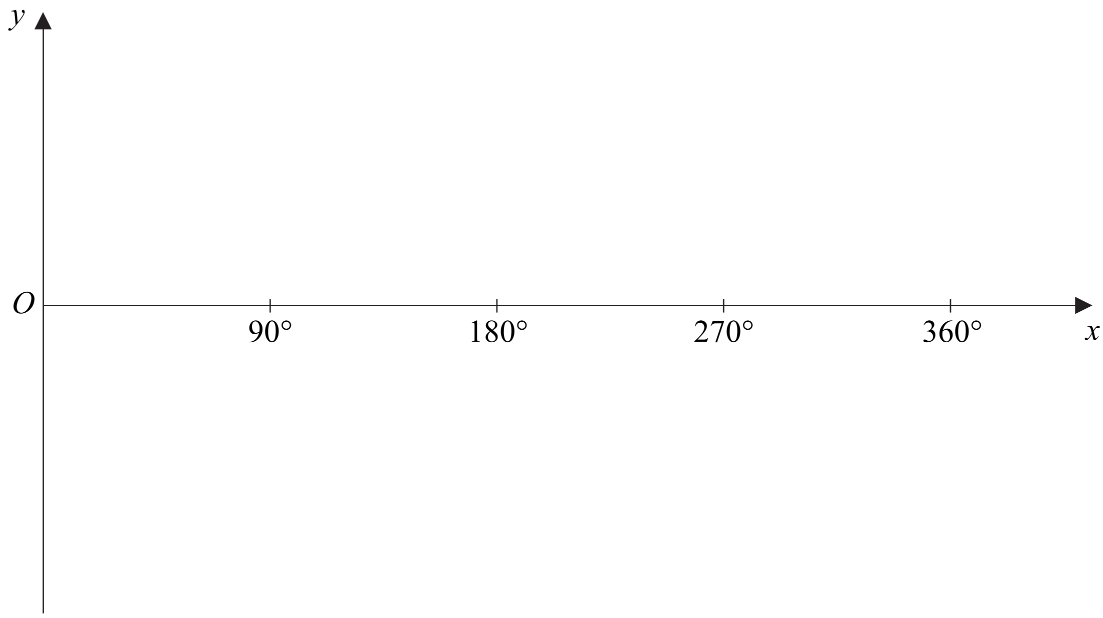


(i) The point A on the curve is where $x = 0$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch, on the axes given on page **3**, the graph of $y = |\sec 2x|$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) Solve the equation $|\sec(2x - 10°)| = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*



3 (a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when:

(i) $y = \ln(5x - 2)$; *(2 marks)*

(ii) $y = \sin 2x$. *(2 marks)*

(b) The functions f and g are defined with their respective domains by

$\mathrm{f}(x) = \ln(5x - 2)$, for real values of x such that $x \geqslant \frac{1}{2}$

$\mathrm{g}(x) = \sin 2x$, for real values of x in the interval $-\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$

(i) Find the range of f. *(2 marks)*

(ii) Find an expression for $\mathrm{gf}(x)$. *(1 mark)*

(iii) Solve the equation $\mathrm{gf}(x) = 0$. *(3 marks)*

(iv) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(2 marks)*

Turn over ▶

4 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to $\int_{0.5}^{2} \frac{x}{1+x^3}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Find the exact value of $\int_{0}^{1} \frac{x^2}{1+x^3}\,dx$. *(4 marks)*

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5 (a) Show that the equation

$$10\,\text{cosec}^2 x = 16 - 11 \cot x$$

can be written in the form

$$10 \cot^2 x + 11 \cot x - 6 = 0$$ *(1 mark)*

(b) Hence, given that $10\,\text{cosec}^2 x = 16 - 11 \cot x$, find the possible values of $\tan x$. *(4 marks)*

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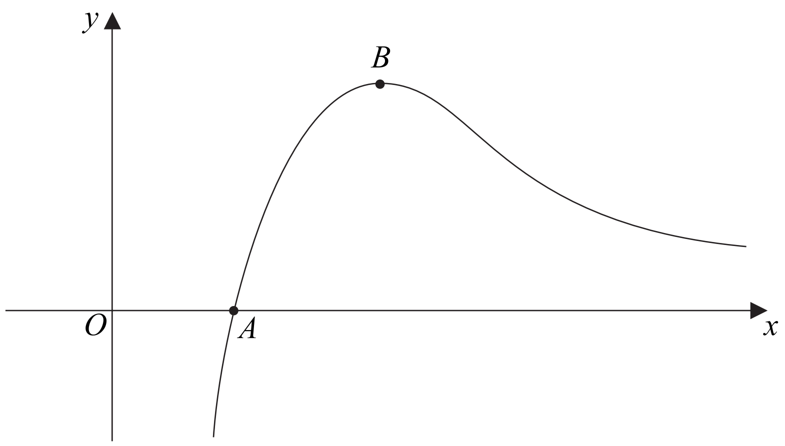
6 The diagram shows the curve $y = \frac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(a) State the coordinates of A. *(1 mark)*

(b) Find the coordinates of the stationary point, B, of the curve, giving your answer in an exact form. *(5 marks)*

(c) Find the exact value of the gradient of the normal to the curve at the point where $x = e^3$. *(3 marks)*

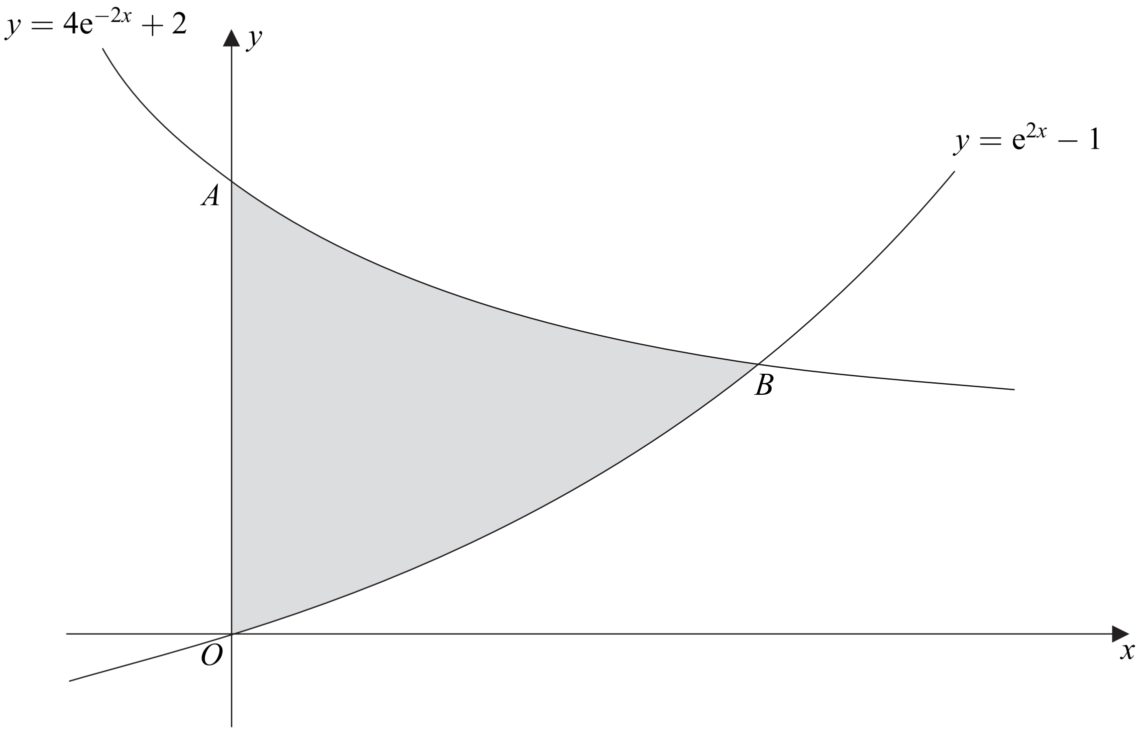
7 (a) Use integration by parts to find:

(i) $\int x \cos 4x \, dx$; *(4 marks)*

(ii) $\int x^2 \sin 4x \, dx$. *(4 marks)*

(b) The region bounded by the curve $y = 8x\sqrt{(\sin 4x)}$ and the lines $x = 0$ and $x = 0.2$ is rotated through 2π radians about the x-axis. Find the value of the volume of the solid generated, giving your answer to three significant figures. *(3 marks)*

8 The diagram shows the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$.



The curve $y = 4e^{-2x} + 2$ crosses the y-axis at the point A and the curves intersect at the point B.

(a) Describe a sequence of two geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x} - 1$. *(4 marks)*

(b) Write down the coordinates of the point A. *(1 mark)*

(c) (i) Show that the x-coordinate of the point B satisfies the equation

$$(e^{2x})^2 - 3e^{2x} - 4 = 0$$ *(2 marks)*

(ii) Hence find the exact value of the x-coordinate of the point B. *(3 marks)*

(d) Find the exact value of the area of the shaded region bounded by the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$ and the y-axis. *(5 marks)*

END OF QUESTIONS