C2 COORDINATE GEOMETRY *Worksheet A*

1 Write down an equation of the circle with the given centre and radius in each case.

a centre (0, 0) radius 5 **b** centre (1, 3) radius 2 **c** centre (4, –6) radius 1

d centre (–1, –8) radius 3 **e** centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ **f** centre (–3, 9) radius $2\sqrt{3}$

2 Write down the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 = 16$ **b** $(x - 6)^2 + (y - 1)^2 = 81$ **c** $(x + 1)^2 + (y - 4)^2 = 121$

d $(x - 7)^2 + y^2 = 0.09$ **e** $(x + 2)^2 + (y + 5)^2 = 32$ **f** $(x - 8)^2 + (y + 9)^2 = 108$

3 Find the coordinates of the centre and the radius of each of the following circles.

a $x^2 + y^2 - 4y + 3 = 0$ **b** $x^2 + y^2 - 2x - 10y - 23 = 0$

c $x^2 + y^2 + 12x - 8y + 36 = 0$ **d** $x^2 + y^2 - 2x + 16y = 35$

e $x^2 + y^2 = 8x - 6y$ **f** $x^2 + y^2 + 10x - 2y - 19 = 0$

g $4x^2 + 4y^2 - 4x - 24y + 1 = 0$ **h** $9x^2 + 9y^2 + 6x - 24y + 8 = 0$

4 Find an equation of the circle

a with centre (1, –2) which passes through the point (4, 2),

b with centre (–5, 7) which passes through the point (0, 5).

5 Find an equation of the circle in which AB is a diameter in each case.

a A (1, –2) B (3, –2) **b** A (–7, 2) B (1, 8) **c** A (1, 1) B (4, 0)

6 The points P (0, 1), Q (3, 10) and R (6, 9) all lie on circle C.

a Show that $\angle PQR$ is a right-angle.

b Hence, show that C has the equation $x^2 + y^2 - 6x - 10y + 9 = 0$.

7 Find in each case whether the given point lies inside, outside or on the given circle.

a (0, –9) $x^2 + y^2 = 64$ **b** (4, 7) $x^2 + y^2 - 2x - 6y - 26 = 0$

c (7, –3) $x^2 + y^2 + 10x - 4y = 140$ **d** (–4, 1) $x^2 + y^2 + 2x + 8y - 13 = 0$

8 The point P lies on the circle with equation $x^2 + y^2 + 12x - 6y + 27 = 0$ and the point Q has coordinates (8, 1). Find the minimum length of PQ giving your answer in the form $k\sqrt{2}$.

9 Find an equation of the circle which crosses the x-axis at the points (2, 0) and (8, 0) and touches the y-axis at the point (0, 4).

10 Given that the circle with equation $x^2 + y^2 + 8x - 12y + k = 0$ does not touch or cross either of the coordinate axes, find the set of possible values of the constant k.

11 The circle C passes through the points P, Q and R with coordinates (–2, –2), (2, –4) and (7, 1) respectively.

a Find an equation of the perpendicular bisector of the points P and Q.

b Find the coordinates of the centre of C.

c Find an equation of C.

12 The circle C has the equation $x^2 + y^2 - 4x - 4y - 28 = 0$.

a Find the distance of the point A (10, 8) from the centre of C.

The tangent to C at the point B passes through A.

b Find the length AB.

13 A circle has the equation $x^2 + y^2 + 6x - 2y = 0$ and passes through the point P.

Given that the tangent to the circle at P passes through the point Q (2, 6), find the exact length PQ in its simplest form.

14 The circle C has the equation $x^2 + y^2 - 6x - 10y + 16 = 0$ and passes through the point A (6, 2).

a Find the coordinates of the centre of C.

b Find the gradient of the normal to the circle at A.

c Find an equation of the normal to the circle at A.

15 Find an equation of

a the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point $(-1, 4)$,

b the tangent to the circle with equation $x^2 + y^2 + 2x + 4y - 40 = 0$ at the point (5, 1),

c the tangent to the circle with equation $x^2 + y^2 - 10x + 4y + 4 = 0$ at the point (2, 2).

16 Find the coordinates of the points where the circle with equation $x^2 + y^2 - 6x + 6y - 16 = 0$ intersects the coordinate axes.

17 Find in each case the coordinates of the points where the line l intersects the circle C.

a $l : y = x - 4$ $\quad C : x^2 + y^2 = 10$

b $l : 3x + y = 17$ $\quad C : x^2 + y^2 - 4x - 2y - 15 = 0$

c $l : y = 2x + 2$ $\quad C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$

18 The line with equation $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B.

Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$.

19 Show that the line with equation $y = 2x + 1$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 8y + 27 = 0$ and find the coordinates of the point where they touch.

20 The line with equation $y = x + k$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 8y + 17 = 0$.

Find the two possible values of k.

21 The line with equation $y = mx$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

Find the two possible values of m.

22 The line with equation $2x + 3y = k$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y = 0$.

Find the two possible values of k.

23 The circle with equation $x^2 + y^2 - 4x - 6y = 7$ crosses the y-axis at the points A and B.

a Find the coordinates of the points A and B.

b Find the coordinates of the point where the tangent to the circle at A intersects the tangent to the circle at B.

C2 COORDINATE GEOMETRY *Worksheet B*

1 The circle C has centre (3, −2) and radius 5.

a Write down an equation of C in cartesian form.

The line $y = 2x - 3$ intersects C at the points A and B.

b Show that $AB = 4\sqrt{5}$.

2 The line AB is a diameter of circle C.

Given that A has coordinates (−5, 6) and B has coordinates (3, 8), find

a the coordinates of the centre of C,

b a cartesian equation for C,

c an equation of the tangent to C at A.

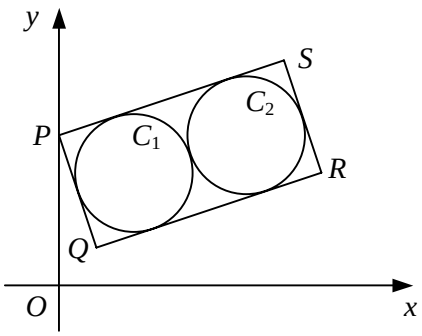
3 The circle C has equation $x^2 + y^2 + 8x - 16y + 62 = 0$.

a Find the coordinates of the centre of C and the exact radius of C.

The line l has equation $y = 2x + 1$.

b Show that the minimum distance between l and C is $3(\sqrt{5} - \sqrt{2})$.

4



The diagram shows rectangle $PQRS$ and circles C_1 and C_2.

Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are P (0, 4), Q (1, 1), R (7, 3) and S (6, 6).

a Find the radius of C_1.

b Find the coordinates of the point where the two circles touch.

c Show that C_1 has equation $2x^2 + 2y^2 - 8x - 12y + 21 = 0$.

5 The circle C touches the y-axis at the point A (0, 3) and passes through the point B (2, 7).

a Find an equation of the perpendicular bisector of AB.

b Find an equation for C.

c Show that the tangent to C at B has equation

$$3x - 4y + 22 = 0.$$

6 The point P (x, y) moves such that its distance from the point A (−3, 4) is twice its distance from the point B (0, −2).

Show that the locus of P is a circle and find the coordinates of the centre and the exact radius of this circle.

7 The points P (–4, 9) and Q (–2, –5) are such that PQ is a diameter of circle C.

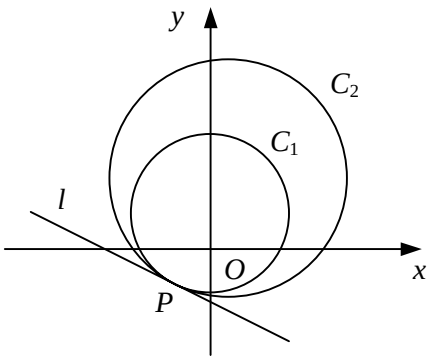
a Find the coordinates of the centre of C.

b Find an equation for C.

c Show that the point R (2, 7) lies on C.

d Hence, state the size of $\angle PRQ$, giving a reason for your answer.

8



The diagram shows circles C_1 and C_2, which both pass through the point P, and the common tangent to the circles at P, the line l.

Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.

a Find the coordinates of the centre of C_1.

Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$.

b Find an equation of the straight line passing through the centre of C_1 and the centre of C_2.

c Find an equation of line l.

9 The circle C has equation $x^2 + y^2 - 8x + 4y + 12 = 0$.

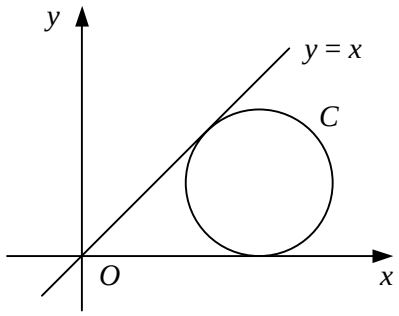
a Find the coordinates of the centre of C and the radius of C.

The point P has coordinates (3, 5) and the point Q lies on C.

b Find the largest and smallest values of the length PQ, giving your answers in the form $k\sqrt{2}$.

c Find the length of PQ correct to 3 significant figures when the line PQ is a tangent to C.

10



The diagram shows the circle C and the line $y = x$.

Given that circle C has centre (a, b), where a and b are positive constants, and that C touches the x-axis,

a find a cartesian equation for C in terms of a and b.

Given also that the line $y = x$ is a tangent to C,

b show that $a = (1 + \sqrt{2})b$.

C2 COORDINATE GEOMETRY *Worksheet C*

1 A circle has the equation $x^2 + y^2 - 8x + 7 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Find the radius of the circle. **(2)**

2 A circle has the equation $x^2 + y^2 - 6x + 2y - 15 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Find the radius of the circle. **(1)**

c Show that the tangent to the circle at the point (7, 2) has equation

$$4x + 3y - 34 = 0.$$ **(4)**

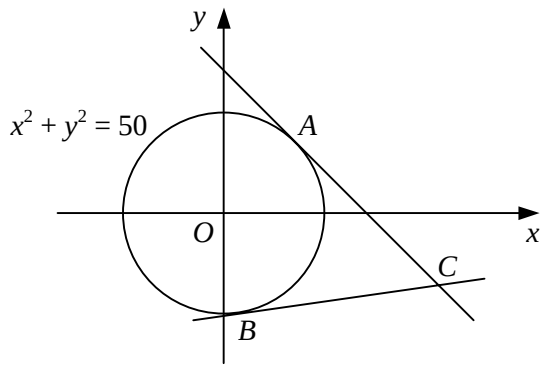
3 A circle has the equation $x^2 + y^2 + 6x - 8y + 21 = 0$.

a Find the coordinates of the centre and the radius of the circle. **(3)**

The point P lies on the circle.

b Find the greatest distance of P from the origin. **(2)**

4



The diagram shows the circle with equation $x^2 + y^2 = 50$ and the tangents to the circle at the points A (5, 5) and B (1, −7).

a Find an equation of the tangent to the circle at A. **(3)**

b Show that the tangent to the circle at B has the equation

$$x - 7y - 50 = 0.$$ **(3)**

c Find the coordinates of the point C where the tangents to the circle at A and B intersect. **(2)**

5 Circle C_1 has the equation $x^2 + y^2 - 2ay = 0$, where a is a positive constant.

a Find the coordinates of the centre and the radius of C_1. **(4)**

Circle C_2 has the equation $x^2 + y^2 - 2bx = 0$, where b is a constant and $b > a$.

b Sketch C_1 and C_2 on the same diagram. **(4)**

6 The circle C has the equation $x^2 + y^2 + 2x - 14y + 30 = 0$.

a Find the coordinates of the centre of C. **(2)**

b Find the radius of C, giving your answer in the form $k\sqrt{5}$. **(2)**

c Show that the line $y = 2x - 1$ is a tangent to C and find the coordinates of the point of contact. **(4)**

7 The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.

a Find the coordinates of the centre of C. **(2)**

The line $y = x - 2$ intersects C at the points A and B.

b Find the length AB in the form $k\sqrt{2}$. **(6)**

8 The circle C has centre $(8, -1)$ and passes through the point $(4, 1)$.

a Find an equation for C. **(3)**

b Show that the line with equation $x + 2y + 4 = 0$ is a tangent to C. **(3)**

9 The points P $(-10, 2)$, Q $(8, 14)$ and R $(-2, -10)$ all lie on circle C.

a Show that PR is perpendicular to PQ. **(2)**

b Hence, show that C has the equation $x^2 + y^2 - 6x - 4y - 156 = 0$. **(5)**

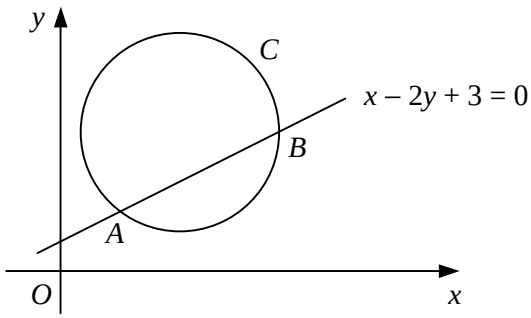
10 A circle has the equation $x^2 + y^2 - 2x - 7y - 16 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Show that the radius of the circle is $k\sqrt{13}$, where k is an exact fraction to be found. **(2)**

c Find an equation of the tangent to the circle at the point $(4, 8)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

11



The line with equation $x - 2y + 3 = 0$ intersects the circle C at the points A and B as shown in the diagram above. Given that the centre of C has coordinates $(6, 7)$,

a find the coordinates of the mid-point of the chord AB. **(6)**

Given also that the x-coordinate of the point A is 3,

b find the coordinates of the point B, **(3)**

c find an equation for C. **(2)**

12 The circle C has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

a Find the coordinates of the centre and the radius of C. **(3)**

b Find the distance of the centre of C from the origin in the form $k\sqrt{5}$. **(2)**

The point A lies on C and the tangent to C at A passes through the origin O.

c Show that $OA = 6\sqrt{2}$. **(3)**

13 The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.

a Show that l passes through the centre of C. **(3)**

b Find an equation for each tangent to C that is parallel to l. **(6)**